Optimum Accumulation and International Trade*

I

Since Frank Ramsey's pioneering article in 1928 [9] the problem of optimum capital accumulation has fascinated a number of economists. None of them, to our knowledge, has discussed the problem in the context of international trade. This is part of a more general lack of any serious exercise in intertemporal welfare economics in the literature on international trade theory. This paper has the very limited purpose of extending the standard Ramsey model to the case of a two-sector open economy, relating some of the results to those in the optimum-tariffs literature, and finally indicating some of the problems in relaxing the assumptions of the model.

We take the Ramsey model, with an infinite planning horizon, with the crucial boundedness assumption of " finite bliss " (defined as the maximum economically obtainable state of satisfaction) to take care of the problem of the convergence of the utility integral,[1] with constant number, composition, technical knowledge and tastes of the country's population, with utility in one period depending on the level of aggregate national consumption in that period alone, with the instantaneous utility function $U(C)$, C being consumption, having a positive first and a negative second derivative for $0 < C < \infty$, and with $\lim\limits_{c \to 0} U'(C) = \infty$ and $\lim\limits_{c \to 0} U'(C) = 0$ (in order to avoid corner solutions). We have two sectors in the economy, sector 1 producing an homogeneous consumer good and sector 2 producing an homogeneous (and immortal) capital good. We assume the foreign country to be static and the balance of trade (and payments: since foreign investment is absent) [2] to be always in equilibrium.

With well-behaved neo-classical production functions and with given amounts of capital and labour for a period we can easily draw the familiar concave-from-below transformation curve for that period relating F_1, the current output of sector 1 to F_2, the current output of sector 2. If we superimpose the foreign country's offer curve on each point of this transformation curve, the resulting envelope gives us the well-known Baldwin-envelope curve [1] which is actually the open-economy availability frontier for that period. Given the foreign offer curve this envelope shifts in a determinate way with the stock of capital and labour.

The total labour supply L for the economy is constant; without loss of generality we can assume it to be equal to 1 (so that total capital-labour ratio $k = K$ the amount of capital, and $\dot{k} = \dot{K}$). So from the Baldwin-envelope curve AA' in Fig. 1 we can define

$$C = \varphi(k, \dot{k}). \qquad (1)$$

* In preparing this paper I owe much to frequent discussions with Dr. C. C. von Weizsäcker. I am also grateful to Professor R. M. Solow, Dr. F. H. Hahn, Dr. J. A. Mirlees and to the referee of this Review for valuable comments on an earlier draft. All errors are, of course, exclusively mine.

[1] Strictly speaking, to assume a bounded utility function is not sufficient for the convergence of the utility integral. To avoid awkward mathematical difficulties I shall simply assume that the utility integral converges for the Ramsey-optimum policy.

[2] I have elsewhere treated the problem of optimum foreign investment in terms of the Ramsey model.

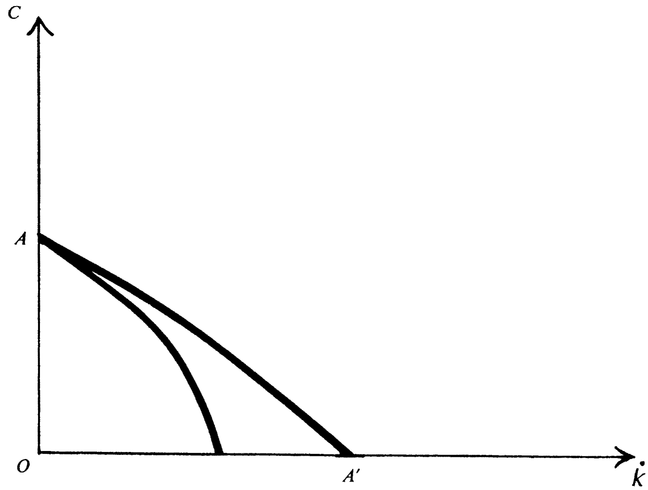

FIGURE 1

Following Ramsey, our criterion of eligibility of a consumption path is a sufficiently rapid approach over time to the state of bliss. So our problem is to minimise the sum of the differences between the bliss-level of utility and the actual utility of consumption over time under our side condition (1). Now let us put the bliss-level of utility to be equal to $\bar{U}$; so long as the model is independent of historical time and there are constant returns to scale $\bar{U}$ is a constant. We are to minimise

$$\int_0^\infty \{\bar{U} - U[\varphi(k, \dot{k})]\} dt. \qquad (2)$$

Changing the variable of integration (2) becomes

$$\int_{k(0)}^{\hat{k}} \frac{\bar{U} - U[\varphi(k, \dot{k})]}{\dot{k}} \, dk \qquad (3)$$

where $k(o)$ is the given value of the initial capital stock and $\hat{k}$ is the value of the least amount of capital with which (finite) bliss can be attained. It will be always right to invest at a positive rate so long as $k < \hat{k}$; hence if we denote the optimum path of accumulation by $k^*(t)$, $k^*(t)$ is an increasing function of t, strictly increasing until k^* reaches the value of k, say at $T \leqq \infty$. So it is permissible to change the variable of integration to k in (o, T); for the range (T, ∞) the integral is zero. Given k, the integral (3) is minimised if we simply minimise the integrand with respect to $\dot{k}$. This gives us

$$-\frac{\partial \varphi}{\partial \dot{k}} \cdot \dot{k} = \frac{\bar{U} - U(C)}{U'(C)}. \qquad (4)$$

Now $-\dfrac{\partial \varphi}{\partial \dot{k}}$ = the slope of the Baldwin-envelope curve AA' in Fig. 1.

= the marginal rate of transformation through foreign trade.

From the optimum-tariffs literature [3], [4], [6], [5] we know that the marginal rate of transformation through foreign trade is equal to

$$P \cdot \frac{1 + \dfrac{1}{\eta_s}}{1 - \dfrac{1}{\eta_d}}$$

where P is the prevailing international price of capital goods in terms of consumer goods (the latter being taken as numéraire); η_d is the elasticity of foreign demand for exports (taken with a minus sign to make it positive) and η_s is the elasticity of foreign supply of imports.[1] Substituting this in (4) we get the optimum rate of capital accumulation

$$\dot{k} = \frac{\bar{U} - U(C)}{U'(C)} \cdot \frac{(\eta_d - 1)\eta_s}{P(\eta_s + 1)\eta_d} \quad . \quad . \quad . \quad (5)$$

If we care to assume a special iso-elastic utility function of the form

$$U(C) = \bar{U} - C^{-m}, \text{ where } m \text{ is a positive constant,} \quad . \quad . \quad . \quad . \quad (6)$$

we can easily find out S, the optimum savings ratio in the open economy. Substituting (6) into (5),

$$P \, . \, \dot{k} = \frac{C \, . \, (\eta_d - 1)\eta_s}{m \, . \, (\eta_s + 1)\eta_d}.$$

So

$$S = \frac{P\dot{k}}{C + P\dot{k}} = \frac{\eta_s(\eta_d - 1)}{m\eta_d(1 + \eta_s) + \eta_s(\eta_d - 1)}. \quad . \quad . \quad . \quad (7)$$

If we assume that η_d, the foreign elasticity of demand for exports and η_s, the elasticity of foreign supply of imports are constant,[2] we have a constant optimum savings ratio. It can be noted that the larger is η_d, and/or the larger is η_s (provided $\eta_d > 1$, which is taken for granted in the optimum-tariffs literature) and/or the smaller is m, the higher is S; in other words, the larger is the elasticity of foreign demand for exports and the elasticity of foreign supply of imports and/or the smaller is what can be called " the coefficient of egalitarianism " in our utility function (since m indicates the amount one would take away from the rich to give a certain amount to the poor), the higher is the optimum savings ratio in the open economy.

From the literature on optimum tariffs we know that one of the (a-temporal) efficiency conditions in such a trade model is to impose a suitable tax so as to equalise the domestic price ratio $\tilde{P}$ (supposed to reflect the marginal rate of " domestic " transformation) to the marginal rate of " foreign " transformation (which we have seen to be equal to $P \, . \, \frac{(1 + \eta_s)\eta_d}{\eta_s(\eta_d - 1)}$. So when what is known as the " optimum " tariff is levied, we can see from (5) that the " domestic " value of (Ramsey-)optimum investment ($= \tilde{P}\dot{k}$) becomes equal to $\frac{\bar{U} - U(C)}{U'(C)}$; this way of expressing it makes it look exactly like the familiar Ramsey Rule.

II

In this Section we comment on some of the simplifying assumptions used in the preceding discussion. Let us take up those regarding the static supply of labour and the static foreign country. The easiest way of relaxing these two assumptions is to take the case of labour growing at an exponential rate, λ, and assume that the foreign country is growing at a rate which is constant and which is (miraculously enough) equal to λ. In this case, if utility is now a function of per capita consumption, we can use the same Baldwin-envelope technique (since under the assumptions we have simply to redefine our

[1] This involves an implicit assumption that the home country imports capital goods and exports consumer goods. There is, however, in our model nothing to prevent a switch in the process of movement towards bliss from importing capital goods to exporting them (particularly if the rest of the world does not have any net capital accumulation).

[2] There is, of course, need for caution in assuming constant international elasticities, since difficulties might arise in view of their being essentially composite in nature, but for the problem dealt with in this paper, they are of the second order of importance.

domestic transformation curve and the foreign offer curve in per capital terms without altering any of their essential characteristics) and the same type of analysis as in our Section I in determining the optimum rate of accumulation with international trade. We can follow Koopmans [7] in using utility per head enjoyed on the exponential growth path that accords with the so-called Golden Rule of Accumulation [1] [8], [10], [11] to take the place of utility in Ramsey's state of bliss in defining eligibility of consumption paths. One can work with the difference between the utility integral for any given feasible path and its value for the Golden Rule path and study this difference over time.

[1] The Golden Rule of equating the rate of interest to the rate of growth ensures the maximum sustainable level of per capita consumption in the Golden Age. One can easily show that the Golden Rule holds good for an open economy as much as it does for the closed economy, which is obvious since the Golden Rule indicates a purely technological feature of the model that is unaffected by international trade. For a general analysis of the properties of Golden-age-equilibrium in terms of a two-country, two-sector trade model, see [2]. One interesting condition in such a model seems to be that unless the two trading countries have the same (constant) " natural " rates of growth, Golden-age-equilibrium simply does not exist.

But the above relates to only a very special case of growth in the foreign country. Letting the foreign country grow in a more general way is a much more difficult problem. Economic growth in the foreign country will shift the shape and position of the foreign offer curve and hence that of the Baldwin-envelope for the home country. Thus growth in the foreign country introduces essentially the same type of problems as the phenomenon of continuous technical progress in the closed-economy optimum-savings models.

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