Variable Rare Disasters: An Exactly Solved Framework for Ten Puzzles in Macro-Finance

Xavier Gabaix

NYU Stern, CEPR and NBER

March 6, 2010*

Abstract

This paper incorporates a time-varying severity of disasters into the hypothesis proposed by Rietz (1988) and Barro (2006) that risk premia result from the possibility of rare large disasters. During a disaster an asset's fundamental value falls by a time-varying amount. This in turn generates time-varying risk premia and thus volatile asset prices and return predictability. Using the recent technique of linearity-generating processes, the model is tractable and all prices are exactly solved in closed form. In this paper's framework, the following empirical regularities can be understood quantitatively: (i) equity premium puzzle; (ii) risk-free rate puzzle; (iii) excess volatility puzzle; (iv) predictability of aggregate stock market returns with price-dividend ratios; (v) often greater explanatory power of characteristics than covariances for asset returns; (vi) upward sloping nominal yield curve; (vii) predictability of future bond excess returns and long term rates via the slope of the yield curve; (viii) corporate bond spread puzzle; (ix) high price of deep out-of-the-money puts; and (x) high put prices being followed by high stock returns. The calibration passes a variance bound test, as normal-times market volatility is consistent with the wide dispersion of disaster outcomes in the historical record. The model also extends to Epstein-Zin-Weil preferences and to a setting with many factors.

---

*xgabaix@stern.nyu.edu. I thank Alex Chinco, Esben Hedegaard and Rob Tumarkin for excellent research assistance. For helpful conversations and comments, I thank five referees, the editor and the coeditor, David Chapman, Alex Edmans, Emmanuel Farhi, Francois Gourio, Christian Julliard, Sydney Ludvigson, Anthony Lynch, Thomas Philippon, José Scheinkman, José Ursua, Stijn Van Nieuwerburgh, Adrien Verdelhan, Stan Zin, and seminar participants at AFA, Chicago GSB, Duke, Harvard, Minnesota Workshop in Macro Theory, MIT, NBER, NYU, Paris School of Economics, Princeton, Texas Finance Festival, UCLA, and Washington University at Saint Louis. I thank Robert Barro, Stephen Figlewski, Arvind Krishnamurthy, José Ursua, and Annette Vissing-Jorgensen for their data. I thank the NSF for support.

1 Introduction

Lately, there has been a revival of a hypothesis proposed by Rietz (1988) that the possibility of rare disasters, such as economic depressions or wars, is a major determinant of asset risk premia. Indeed, Barro (2006) has shown that, internationally, disasters have been sufficiently frequent and large to make Rietz's proposal viable and account for the high risk premium on equities.

The rare disaster hypothesis is almost always formulated with constant severity of disasters. This is useful for thinking about averages but cannot account for some key features of asset markets such as volatile price-dividend ratios for stocks, volatile bond risk premia, and return predictability. In this paper, I formulate a variable-severity version of the rare disasters hypothesis and investigate the impact of time-varying disaster severity on the prices of stocks and bonds as well as the predictability of their returns.[1]

I show that many asset puzzles can be qualitatively understood using this model. I then demonstrate that a parsimonious calibration allows one to understand the puzzles quantitatively, provided that real and nominal variables have a sufficiently variable sensitivity to disasters (something I will argue is plausible below).

The proposed framework allows for a very tractable model of stocks and bonds in which all prices are in closed forms. In this setting, the following patterns are not puzzles but emerge naturally when the present model has just two shocks: one real for stocks and one nominal for bonds.[2]

A. Stock market: Puzzles about the aggregates

1. Equity premium puzzle: The standard consumption-based model with reasonable relative risk aversion (less than 10) predicts a too-low equity premium (Mehra and Prescott 1985).

2. Risk-free rate puzzle: Increasing risk aversion leads to a too-high risk-free rate in the standard model (Weil 1989).[3]

3. Excess volatility puzzle: Stock prices seem more volatile than warranted by a model with a constant discount rate (Shiller 1981).

4. Aggregate return predictability: Future aggregate stock market returns are partly predicted by price/dividend (P/D) and similar ratios (Campbell and Shiller 1988).

B. Stock market: Puzzles about the cross-section of stocks

[1] A later companion paper, Farhi and Gabaix (2009) studies exchange rates. A brief introduction is Gabaix (2008), but almost all results appear here for the first time.

[2] I mention just a few references, but most puzzles have been documented by numerous authors.

[3] For this and the above puzzle, the paper simply imports from Rietz (1988), Longstaff and Piazzesi (2004) and Barro (2006).

5. Characteristics vs. Covariances puzzle: Stock characteristics (e.g. the P/D ratio) often predict future returns as well as or better than covariances with risk factors (Daniel and Titman 1997).

C. Nominal bond puzzles

6. Yield curve slope puzzle: The nominal yield curve slopes up on average. The premium of long-term yields over short-term yields is too high to be explained by a traditional RBC model. This is the bond version of the equity premium puzzle (Campbell 2003).

7. Long term bond return predictability: a high slope of the yield curve predicts high excess returns on long term bonds (Macaulay (1938), Fama-Bliss (1987), Campbell-Shiller (1991)).

8. Credit spread puzzle: Corporate bond spreads are higher than seemingly warranted by historical default rates (Huang and Huang 2003).

D. Options puzzles

9. Deep out-of-the-money puts have higher prices than predicted by the Black-Scholes model (Jackwerth and Rubinstein 1996).

10. When prices of puts on the stock market index are high, its future returns are high (Bollerslev, Tauchen and Zhou forth.).

To understand the economics of the model, first consider bonds. Consistent with the empirical evidence reviewed below, a disaster leads on average to a positive jump in inflation in the model. This has a greater detrimental impact on long-term bonds, so they command a high risk premium relative to short-term bonds. This explains the upward slope of the nominal yield curve. Next, suppose that the size of the expected jump in inflation itself varies. Then the slope of the yield curve will vary and will predict excess bond returns. A high slope will mean-revert and thus predicts a fall in the long rate and high returns on long term bonds. This mechanism accounts for many stylized facts on bonds.

The same mechanism is at work for stocks. Suppose that a disaster reduces the fundamental value of a stock by a time-varying amount. This yields a time-varying risk premium which generates a time-varying price-dividend ratio and the "excess volatility" of stock prices. It also makes stock returns predictable via measures such as the dividend-price ratio. When agents perceive the severity of disasters as low, price-dividend ratios are high and future returns are low.

The model's mechanism also impacts disaster-related assets such as corporate bonds and options. If high-quality corporate bonds default mostly during disasters, then they should command a high premium that cannot be accounted for by their behavior during normal times. The model also generates option prices with a “volatility smirk,” i.e. a high put price (hence implied volatility) for deep out-of-the-money puts.

After laying out the framework and solving it in closed form, I calibrate it. The values for disasters are essentially taken from Barro and Ursua (2008)'s analysis of many countries' disasters, defined as falls in GDP or consumption of 10% or more. The calibration gives results for stocks, bonds and options consistent with empirical values. The volatilities of the expectation about disaster sizes are very hard to measure directly. However, the calibration generates a steady state dispersion of anticipations that is lower than the dispersion of realized values. This is shown by “dispersion ratio tests” in the spirit of Shiller (1982), which are passed by the disaster model. By that criterion, the calibrated values in the model appear reasonable. Importantly, they generate a series of fine quantitative predictions. Hence, the model calibrates quite well.

Throughout this paper, I use the class of “linearity-generating” (LG) processes (Gabaix 2009). That class keeps all expressions in closed form. The entire paper could be rewritten with other processes (e.g. affine-yield models) albeit with considerably more complicated algebra and the need to resort to numerical solutions. The LG class and the affine class give the same expression to a first order approximation. Hence, there is little economic consequence in the use of LG processes and their use should be viewed as an analytical convenience.

Relation to the literature A few papers address the issue of time-varying disasters. Longstaff and Piazzesi (2004) consider an economy with constant intensity of disasters, but in which stock dividends are a variable, mean-reverting share of consumption. They find a high equity premium, and highly volatile stock returns. Veronesi (2004) considers a model in which investors learn about a world economy that follows a Markov chain through two possible economic states, one of which may be a disaster state. He finds GARCH effects and apparent “overreaction.” Weitzman (2007) provides a Bayesian view that the main risk is model uncertainty, as the true volatility of consumption may be much higher than the sample volatility. Unlike the present work, those papers do not consider bonds, nor study return predictability.

After the present paper was circulated, Wachter (2009) proposed a different model, based on Epstein-Zin utilities, where valuation movements come solely from the stochastic probability of disaster, and which analyzes stocks and the short term rate, but not nominal bonds. The present paper, in contrast, allows the stochasticity to come both from movements in the probability of disaster and from the expected recovery rate of various assets, and can work with CRRA as well as Epstein-Zin utility. Importantly, it is conceived to easily handle several assets, such as nominal bonds and stocks (here), stocks with different timing of cash flows (Binsbergen, Brandt and Koijen 2009), particular sectors of the stock market (Ghandi and Lustig 2009) and exchange rates (Farhi and Gabaix 2009). This choice is motivated by the empirical evidence, which shows that several factors are needed to explain risk premia (Fama and French 1993) across stocks and bonds. It is useful to have asset-specific shocks, as single-factor models generate perfect correlations of risk-premia correlations across assets, while empirically valuation ratios are not very correlated across assets (see section 4).

Within the class of rational, representative-agents frameworks that deliver time-varying risk premia, the variable rare disasters model may be a third workable framework, along with the external-habit model of Campbell-Cochrane (CC, 1999) and the long run risk model of Bansal-Yaron (BY, 2004). They have proven to be two very useful and influential models. Still, the reader might ask, why do we need another model of time-varying risk premia? The variable rare disasters framework has several useful features.

First, as emphasized by Barro (2006), the model uses the traditional iso-elastic expected utility framework like the majority of macroeconomic theory. CC and BY use more complex utility functions with external habit and Epstein-Zin (1989)-Weil (1990) utility, which are harder to embed in macroeconomic models. In Gabaix (2009b) (see also Gourio 2009), I show how the present model (which is in an endowment economy) can be directly mapped into a production economy with traditional real-business cycle features. Hence, the rare disasters idea brings us close to the long-sought unification of macroeconomics and finance (see Jermann (1998), Boldrin, Christiano and Fisher (2001), and Uhlig (2007) for attacks of this problem using habit formation). Second, the model makes different predictions for the behavior of "tail-sensitive" assets, such as deep out of the money options, and high-yield corporate bonds – broadly speaking, it of course predicts they command very high premia. Third, the model is particularly tractable. Stock and bond prices have linear closed forms. As a result, asset prices and premia can be derived and analytically understood without recourse to simulations. Fourth, the model easily accounts for some facts that are hard to generate in the CC and BY models. In the model, "characteristics" (such as price-dividend ratios) predict future stock returns better than market covariances, something that it is next to impossible to generate in the CC and BY models. The model also generates a low correlation between consumption growth and stock market returns, which is hard for CC and BY to achieve, as emphasized by Lustig, van Nieuwerburgh, and Verdelhan (2008).

There is a well-developed literature that studies jumps particularly with option pricing in mind. Using options, Liu, Pan and Wang (2004) calibrate models with constant risk premia and uncertainty aversion demonstrating the empirical relevance of rare events in asset pricing. Santa-Clara and Yan (forth.) also use options to calibrate a model with frequent jumps. Typically, the jumps in these papers happen every few days or few months and affect consumption by moderate amounts, whereas the jumps in the rare-disasters literature happen perhaps once every 50 years, and are larger. Those authors do not study the impact of jumps on bonds and return predictability.

Section 2 presents the macroeconomic environment and the cash-flow processes for stocks and bonds. Section 3 derives equilibrium prices. Section 4 proposes a calibration, and reports the model's implications for stocks, options and bonds. Section 5 discusses various extensions of the model, in particular to an Epstein-Zin-Weil economy. The Appendix contains the notations of the paper and some derivations. An online appendix contains supplementary information.

2 Model Setup

2.1 Macroeconomic Environment

The environment follows Rietz (1988) and Barro (2006) and adds a stochastic probability and severity of disasters. There is a representative agent with utility $E_0 \left[\sum_{t=0}^{\infty} e^{-\rho t} \left(C_t^{1-\gamma} - 1\right) / (1-\gamma)\right]$, where $\gamma \geq 0$ is the coefficient of relative risk aversion and $\rho > 0$ is the rate of time preference. She receives a consumption endowment C_t. At each period $t+1$, a disaster may happen with a probability p_t. If a disaster does not happen $C_{t+1}/C_t = e^{g_C}$ where g_C is the normal-time growth rate of the economy. If a disaster happens $C_{t+1}/C_t = e^{g_C} B_{t+1}$, where $B_{t+1} > 0$ is a random variable.[4] For instance, if $B_{t+1} = 0.8$, consumption falls by 20%. To sum up:[5]

$$\frac{C_{t+1}}{C_t} = e^{g_C} \times \begin{cases} 1 & \text{if there is no disaster at } t+1 \\ B_{t+1} & \text{if there is a disaster at } t+1 \end{cases} \tag{1}$$

The pricing kernel is the marginal utility of consumption $M_t = e^{-\rho t} C_t^{-\gamma}$, and follows:

$$\frac{M_{t+1}}{M_t} = e^{-\delta} \times \begin{cases} 1 & \text{if there is no disaster at } t+1 \\ B_{t+1}^{-\gamma} & \text{if there is a disaster at } t+1 \end{cases} \tag{2}$$

where $\delta = \rho + \gamma g_c$, the "Ramsey" discount rate, is the risk-free rate in an economy that would have a zero probability of disasters. The price at t of an asset yielding a stream of dividends $(D_s)_{s \geq t}$ is: $P_t = E_t \left[\sum_{s \geq t} M_s D_s\right] / M_t$.

[4] Typically, extra i.i.d. noise is added, but given that it never materially affects asset prices it is omitted here. It could be added without difficulty. Also, a countercyclicality of risk premia could be easily added to the model without hurting its tractability.

[5] The consumption drop is permanent. One can add mean-reversion after a disaster as in Gourio (2008a).

2.2 Setup for Stocks

I consider a typical stock i which is a claim on a stream of dividends $(D_{it})_{t\geq 0}$, that follows:[6]

$$\frac{D_{i,t+1}}{D_{it}} = e^{g_{iD}}\left(1+\varepsilon^D_{i.t+1}\right) \times \begin{cases} 1 & \text{if there is no disaster at } t+1 \\ F_{i,t+1} & \text{if there is a disaster at } t+1 \end{cases} \tag{3}$$

where $\varepsilon^D_{i,t+1} > -1$ is a mean zero shock that is independent of the disaster event. It matters only for the calibration of dividend volatility. In normal times, D_{it} grows at an expected rate of g_{iD}. But, if there is a disaster, the dividend of the asset is partially wiped out following Longstaff and Piazzesi (2004) and Barro (2006): the dividend is multiplied by a random variable $F_{i,t+1} \geq 0$. $F_{i,t+1}$ is the recovery rate of the stock. When $F_{i,t+1} = 0$ the asset is completely destroyed or expropriated. When $F_{i,t+1} = 1$, there is no loss in dividend.

To model the time-variation in the asset's recovery rate, I introduce the notion of "resilience" H_{it} of asset i,

$$H_{it} = p_t E^D_t\left[B^{-\gamma}_{t+1} F_{i,t+1} - 1\right]. \tag{4}$$

where E^D (resp. E^{ND}) is the expected value conditionally on a *d*isaster happening at $t+1$ (resp. *no d*isaster).[7]

In (4) p_t and $B^{-\gamma}_{t+1}$ are economy-wide variables while the resilience and recovery rate $F_{i,t+1}$ are stock-specific though typically correlated with the rest of the economy. When the asset is expected to do well in a disaster (high $F_{i,t+1}$), H_{it} is high – investors are optimistic about the asset. In the cross-section an asset with higher resilience H_{it} is safer than one with low resilience.

I specify the dynamics of H_{it} directly rather than specify the individual components p_t, B_{t+1} and $F_{i,t+1}$. I split resilience H_{it} into a constant part H_{i*} and a variable part $\widehat{H}_{it}$:

$$H_{it} = H_{i*} + \widehat{H}_{it}$$

and postulate the following linearity-generating (LG) process for the variable part $\widehat{H}_{it}$:

$$\widehat{H}_{i,t+1} = \frac{1+H_{i*}}{1+H_{it}} e^{-\phi_H}\widehat{H}_{it} + \varepsilon^H_{i,t+1} \tag{5}$$

where $E_t\varepsilon^H_{i,t+1} = 0$, and $\varepsilon^H_{i,t+1}, \varepsilon^D_{t+1}$ and the disaster event are uncorrelated variables. Economically, $\widehat{H}_{it}$ does not jump if there is a disaster, but that could be changed with little consequence.[8]

[6] There can be many stocks. The aggregate stock market is a priori not aggregate consumption, because the whole economy is not securitized in the stock market. Indeed, stock dividends are more volatile that aggregate consumption, and so are their prices (Lustig, van Nieuwerburgh, Verdelhan, 2008).

[7] Later in the paper, when there is no ambiguity (e.g., for $E\left[B^{-\gamma}_{t+1}\right]$), I will drop the D.

[8] ε^H_{t+1} can be heteroskedastic – but, its variance need not be spelled out, as it does not enter into the prices.

Eq. 5 means that $\widehat{H}_{it}$ mean-reverts to 0 but as a "twisted" autoregressive process (Gabaix 2009a develops these twisted or LG processes). As H_{it} hovers around H_{i*}, $\frac{1+H_{i*}}{1+H_{it}}$ is close to 1 and the process is an AR(1) up to second order terms: $\widehat{H}_{i,t+1} = e^{-\phi_H}\widehat{H}_{it} + \varepsilon^H_{i,t+1} + O\left(\widehat{H}^2_{it}\right)$. Gabaix (2009a) shows that the process economically behaves like an AR(1). The "twist" term $\frac{1+H_{i*}}{1+H_{it}}$ makes prices linear in the factors and independent of the functional form of the noise. I next turn to bonds.

2.3 Setup for Bonds

The two most salient facts on nominal bonds are arguably the following. First, the nominal yield curve slopes up on average; i.e., long term rates are higher than short term rates (e.g., Campbell 2003, Table 6). Second, there are stochastic bond risk premia. The risk premium on long term bonds increases with the difference between the long term rate and the short term rate. (Campbell and Shiller 1991, Cochrane and Piazzesi 2005, Fama and Bliss 1987). These facts are considered to be puzzles, because they are not generated by the standard macroeconomic models, which generate risk premia that are too small (Mehra and Prescott 1985).

I propose the following explanation. When a disaster occurs, inflation increases (on average). Since very short term bills are essentially immune to inflation risk while long term bonds lose value when inflation is higher, long term bonds are riskier, so they get a higher risk premium. Hence, the yield curve slopes up. Moreover, the magnitude of the surge in inflation is time-varying, which generates a time-varying bond premium. If that bond premium is mean-reverting, it generates the Fama-Bliss puzzle. Note that this explanation does not hinge on the specifics of the disaster mechanism. The advantage of the disaster framework is that it allows for formalizing and quantifying the idea in a simple way.

Several authors have models where inflation is higher in bad times, which makes the yield curve slope up. An earlier unification of several puzzles is provided by Wachter (2006), who studies a Campbell-Cochrane (1999) model with extra nominal shocks, and concludes that it explains an upward sloping yield curve and the Campbell-Shiller (1991) findings. The Brandt and Wang (2003) study is also a Campbell-Cochrane (1999) model, but in which risk-aversion depends directly on inflation. Bansal and Shaliastovich (2009) build on Bansal and Yaron (2004). In Piazzesi and Schneider (2007) inflation also rises in bad times, although in a very different model. Finally, Duffee (2002) and Dai and Singleton (2002) show econometric frameworks that deliver the Fama-Bliss and Campbell-Shiller results.

However, the process needs to satisfy $\widehat{H}_{it}/\left(1+H_{i*}\right) \geq e^{-\phi_H}-1$, so the process is stable, and also $\widehat{H}_{it} \geq -p-H_{i*}$ to ensure $F_{it} \geq 0$. Hence, the variance needs to vanish in a right neighborhood $\max\left(\left(e^{-\phi_H}-1\right)\left(1+H_{i*}\right), -p-H_{i*}\right)$, see Gabaix (2009a).

I decompose trend inflation I_t as $I_t = I_* + \widehat{I}_t$, where I_* is its constant part and $\widehat{I}_t$ is its variable part. The variable part of inflation follows the process:

$$\widehat{I}_{t+1} = \frac{1 - I_*}{1 - I_t} \cdot \left(e^{-\phi_I} \widehat{I}_t + 1_{\{\text{Disaster at } t+1\}} J_t \right) + \varepsilon^I_{t+1} \tag{6}$$

where ε^I_{t+1} has mean 0 and is uncorrelated with the realization of a disaster. This equation means first, that if there is no disaster, $E_t \widehat{I}_{t+1} = \frac{1-I_*}{1-I_t} e^{-\phi_I} I_t$, i.e., inflation follows the LG twisted autoregressive process (Gabaix 2009a). Inflation mean-reverts at a rate ϕ_I, with the LG twist $\frac{1-I_*}{1-I_t}$ to ensure tractability. In addition, in case of a disaster, inflation jumps by an amount J_t, decomposed into $J_t = J_* + \widehat{J}_t$, where J_* is the baseline jump in inflation, $\widehat{J}_t$ is the mean-reverting deviation of the jump size from baseline. This jump in inflation makes long term bonds particularly risky. It follows a twisted auto-regressive process and, for simplicity, does not jump during crises:

$$\widehat{J}_{t+1} = \frac{1 - I_*}{1 - I_t} e^{-\phi_J} \widehat{J}_t + \varepsilon^J_{t+1} \tag{7}$$

where ε^J_{t+1} has mean 0. ε^J_{t+1} is uncorrelated with disasters but can be correlated with innovations in I_t.

A few more concepts are useful. I define $H_\$ = p_t E_t \left[F_{\$,t+1} B^{-\gamma}_{t+1} - 1 \right]$, where $F_{\$,t+1}$ is one minus the default rate on bonds (later, this will be useful to differenciate government from corporate bonds). For simplicity I assume that $H_\$$ is a constant: there will be much economics coming solely from the variations of I_t. I call π_t the variable part of the bond risk premium:

$$\pi_t \equiv \frac{p_t E_t \left[B^{-\gamma}_{t+1} F_{\$,t+1} \right]}{1 + H_\$} \widehat{J}_t. \tag{8}$$

The second notation is only useful when the typical jump in inflation J_* is not zero, and the reader is invited to skip it in the first reading. I parametrize J_* in terms of a variable $\kappa \leq \left(1 - e^{-\phi_I}\right)/2$, called the inflation disaster risk premium:[9]

$$\frac{p_t E_t \left[B^{-\gamma}_{t+1} F_{\$,t+1} \right] J_*}{1 + H_\$} = (1 - I_*) \, \kappa \left(1 - e^{-\phi_I} - \kappa\right) \tag{9}$$

i.e., in the continuous time limit: $p_t E_t \left[B^{-\gamma}_{t+1} F_{\$,t+1} \right] J_* = \kappa \left(\phi_I - \kappa\right)$. A high κ means a high central jump in inflation if there is a disaster. For most of the paper it is enough to think that $J_* = \kappa = 0$.

[9] Calculating bond prices in a Linearity-Generating process sometimes involves calculating the eigenvalues of its generator. I presolve by parameterizing j_* by κ. The upper bound on κ implicitly assumes that j_* is not too large.

2.4 Expected Returns

I conclude the presentation of the economy by stating a general Lemma about the expected returns.

Lemma 1 *(Expected returns) Consider an asset i and call $r_{i,t+1}$ the asset's return. Then, the expected return of the asset at t, conditional on no disasters, is:*

$$r_{it}^{e} = \frac{1}{1-p_t}\left(e^{\delta} - p_t E_t^D\left[B_{t+1}^{-\gamma}\left(1+r_{i,t+1}\right)\right]\right) - 1. \tag{10}$$

In the limit of small time intervals,

$$r_{it}^{e} = \delta - p_t E_t^D\left[B_{t+1}^{-\gamma}\left(1+r_{i,t+1}\right) - 1\right] = r_f - p_t E_t^D\left[B_{t+1}^{-\gamma} r_{i,t+1}\right] \tag{11}$$

where r_f is the real risk-free rate in the economy:

$$r_f = \delta - p_t E_t^D\left[B_{t+1}^{-\gamma} - 1\right]. \tag{12}$$

The unconditional expected return is $\left(1-p_t\right) r_{it}^{e} + p_t E_t^D\left[r_{i,t+1}\right]$.

Proof. It comes from the Euler equation, $1 = E_t\left[\left(1+r_{i,t+1}\right) M_{t+1}/M_t\right]$, i.e.:

$$1 = e^{-\delta}\{\underbrace{\left(1-p_t\right)\cdot\left(1+r_{it}^{e}\right)}_{\text{No disaster term}} + \underbrace{p_t \cdot E_t^D\left[B_{t+1}^{-\gamma}\left(1+r_{i,t+1}\right)\right]}_{\text{Disaster term}}\}.$$

■

Equation 10 indicates that only the behavior in disasters (the $r_{i,t+1}$ term) creates a risk premium. It is equal to the risk-adjusted (by $B_{t+1}^{-\gamma}$) expected capital loss of the asset if there is a disaster.

The unconditional expected return on the asset (i.e., without conditioning on no disasters) in the continuous time limit is $r_{it}^{e} - p_t E_t^D\left[r_{i,t+1}\right]$. Barro (2006) observes that the unconditional expected return and the expected return conditional on no disasters are very close. The possibility of disaster affects primarily the risk premium, and much less the expected loss.

3 Asset Prices and Returns

3.1 Stocks

Theorem 1 *(Stock prices) Let* $h_{i*} = \ln(1 + H_{i*})$ *and define* $\delta_i = \delta - g_{iD} - h_{i*}$*, which will be called the stock's effective discount rate. The price of stock* i *is:*

$$P_{it} = \frac{D_{it}}{1 - e^{-\delta_i}} \left(1 + \frac{e^{-\delta_i - h_{i*}} \widehat{H}_{it}}{1 - e^{-\delta_i - \phi_H}} \right). \tag{13}$$

In the limit of short time periods, the price is:

$$P_{it} = \frac{D_{it}}{\delta_i} \left(1 + \frac{\widehat{H}_{it}}{\delta_i + \phi_H} \right). \tag{14}$$

The next proposition links resilience H_{it} and the equity premium.

Proposition 1 *(Expected stock returns) The expected returns on stock* i*, conditional on no disasters, are:*

$$r_{it}^e = \delta - H_{it} \tag{15}$$

The equity premium (conditional on no disasters) is $r_{it}^e - r_f = p_t E_t \left[B_{t+1}^{-\gamma} (1 - F_{i,t+1}) \right]$ *where* r_f *is the risk-free rate derived in (12). To obtain the unconditional values of those two quantities, subtract* $p_t E_t^D [1 - F_{i,t+1}]$*.*

Proof. If a disaster occurs, dividends are multiplied by F_{it}. As $\widehat{H}_{it}$ does not change, $1 + r_{it} = F_{it}$. So returns are, by Eq. 11, $r_{it}^e = \delta - p_t \left(E_t \left[B_{t+1}^{-\gamma} F_{i,t+1} \right] - 1 \right) = \delta - H_{it}$. ■

As expected, more resilient stocks (assets that do better in a disaster) have a lower ex ante risk premium (a higher H_{it}). When resilience is constant ($\widehat{H}_{it} \equiv 0$), Equation 14 is Barro (2006)'s expression. The price-dividend ratio is increasing in the stock's resiliency of the asset h_{i*}.

The key advance in Theorem 1 is that it derives the stock price with a stochastic resilience $\widehat{H}_{it}$. More resilient stocks (high $\widehat{H}_{it}$) have a higher valuation. Since resilience $\widehat{H}_{it}$ is volatile, price-dividend ratios are volatile, in a way that is potentially independent of innovations to dividends. Hence, the model generates a time-varying equity premium and there is "excess volatility," i.e. volatility of the stocks unrelated to cash-flow news. As the P/D ratio is stationary, it mean-reverts. Thus, the model generates predictability in stock prices. Stocks with a high P/D ratio will have low returns and stocks with a low P/D ratio will have high returns. Section 4.2 quantifies this predictability. Proposition 11 extends equation (14) to a world that has variable expected growth rates of cash-flows in addition to variable risk premia.

3.2 Nominal Government Bonds

Theorem 2 *(Bond prices) In the limit of small time intervals, the nominal short term rate is $r_t = \delta - H_\$ + I_t$, and the price of a nominal zero-coupon bond of maturity T is:*

$$Z_{\$t}(T) = e^{-(\delta - H_\$ + I_{**})T}\left(1 - \frac{1-e^{-\psi_I T}}{\psi_I}(I_t - I_{**}) - K_T \pi_t\right), \qquad K_T \equiv \frac{\frac{1-e^{-\psi_I T}}{\psi_I} - \frac{1-e^{-\psi_J T}}{\psi_J}}{\psi_J - \psi_I} \tag{16}$$

where I_t is inflation, π_t is the bond risk premium, $I_{**} \equiv I_* + \kappa$, $\psi_I \equiv \phi_I - 2\kappa$, $\psi_J \equiv \phi_J - \kappa$. The discrete-time expression is in (39).

Theorem 2 gives a closed-form expression for bond prices. As expected, bond prices decrease with inflation and with the bond risk premium. Indeed, expressions $\frac{1-e^{-\psi_I T}}{\psi_I}$ and K_T are non-negative and increasing in T. The term $\frac{1-e^{-\psi_I T}}{\psi_I} I_t$ simply expresses that inflation depresses nominal bond prices and mean-reverts at a (risk-neutral) rate ψ_I. The bond risk premium π_t affects all bonds but not the short-term rate.

When $\kappa > 0$ (resp. $\kappa < 0$) inflation typically increases (resp. decreases) during disasters. While ϕ_I (resp. ϕ_J) is the speed of mean-reversion of inflation (resp. of the bond risk premium, which is proportional to J_t) under the physical probability, ψ_I (resp. ψ_J) is the speed of mean-reversion of inflation (resp. of the bond risk premium) under the risk-neutral probability.

I next calculate expected bond returns, bond forward rates, and yields.

Proposition 2 *(Expected bond returns) Conditional on no disasters, the short-term real return on a short-term bill is: $r^e_{\$t}(0) = \delta - H_\$$ and the real excess return on the bond of maturity T is:*

$$r^e_{\$t}(T) - r^e_{\$t}(0) = \frac{\frac{1-e^{-\psi_I T}}{\psi_I}(\kappa(\psi_I + \kappa) + \pi_t)}{1 - \frac{1-e^{-\psi_I T}}{\psi_I}(I_t - I_{**}) + K_T \pi_t} \tag{17}$$

$$= T(\kappa(\psi_I + \kappa) + \pi_t) + O(T^2) + O(\pi_t, I_t, \kappa)^2 \tag{18}$$

$$= T p_t E_t\left[B_{t+1}^{-\gamma} F_{\$,t+1}\right] J_t + O(T^2) + O(\pi_t, I_t, \kappa)^2. \tag{19}$$

Proof. After a disaster inflation jumps by J_t and π_t by 0. The bond holder suffers a capital loss equal to $e^{-(\delta - H_\$ + I_{**})T} \cdot \frac{1-e^{-\psi_I T}}{\psi_I} J_t$. Lemma 1 gives the risk premia, using $p_t E_t\left[B_{t+1}^{-\gamma} F_{\$,t+1} J_t\right] = \kappa(\phi_I - \kappa) + \pi_t = \kappa(\psi_I + \kappa) + \pi_t$. ■

Expression (19) shows the first order value of the bond risk premium for bonds of maturity T. It is the maturity T of the bond multiplied by an inflation premium, $p_t E_t\left[B_{t+1}^{-\gamma} F_{\$,t+1}\right] J_t$. The inflation premium is equal to the risk-neutral probability of disasters (adjusting for the recovery rate), $p_t E_t\left[B_{t+1}^{-\gamma} F_{\$,t+1}\right]$, times the expected jump in inflation if there is a disaster, J_t. We note that a lower recovery rate shrinks risk premia, a general feature we will explore in more detail in Section

3.4.

Lemma 2 *(Bond yields and forward rates) The forward rate, $f_t(T) \equiv -\partial \ln Z_{\$t}(T)/\partial T$ is:*

$$
\begin{aligned}
f_t(T) &= \delta - H_\$ + I_{**} + \frac{e^{-\psi_I T}(I_t - I_{**}) + \frac{e^{-\psi_I T} - e^{-\psi_J T}}{\psi_J - \psi_I}\pi_t}{1 - \frac{1-e^{-\psi_I T}}{\psi_I}(I_t - I_{**}) - K_T \pi_t} && (20)\\
&= \delta - H_\$ + I_{**} + e^{-\psi_I T}(I_t - I_{**}) + \frac{e^{-\psi_I T} - e^{-\psi_J T}}{\psi_J - \psi_I}\pi_t + O(I_t - I_{**}, \pi_t)^2 && (21)\\
&= \delta - H_\$ + I_{**} + \left(1 - \psi_I T + \frac{\psi_I T^2}{2}\right)(I_t - I_{**}) + \left(T - \frac{\psi_I + \psi_J}{2}T^2\right)\pi_t && (22)\\
&\quad + O\left(T^3\right) + O(I_t - I_{**}, \pi_t)^2 .
\end{aligned}
$$

The bond yield is $y_t(T) = -(\ln Z_{\$t}(T))/T$ with $Z_{\$t}(T)$ given by (16), and its Taylor expansion is given in Eq. 40-41.

The forward rate increases with inflation and the bond risk premia. The coefficient of inflation decays with the speed of mean-reversion of inflation, ψ_I, in the "risk-neutral" probability. The coefficient of the bond premium, π_t, is $\frac{e^{-\psi_I T} - e^{-\psi_J T}}{\psi_J - \psi_I}$, hence has value 0 at both very short and very long maturities and has a positive hump-shape in between. Very short term bills, being safe, do not command a risk premium, and long term forward rates also are essentially constant (Dybvig, Ingersoll and Ross 1996). Thus, the time-varying risk premium only affects intermediate maturities of forwards.

3.3 Options

Let us next study options, which offer a potential way to measure disasters. The price of a European one-period put on a stock i with strike K expressed as a ratio to the initial price is: $V_t = E_t\left[\frac{M_{t+1}}{M_t}\max(0, K - P_{i,t+1}/P_{it})\right]$. Recall that Theorem 1 yielded $P_{it}/D_{it} = a + b\widehat{H}_{it}$ for two constants a and b. Hence, $E_t^{ND}[P_{i,t+1}/P_{it}] = e^{\mu_{it}}$ with $\mu_{it} = g_{iD} + \ln \frac{a + b\frac{e^{-\phi_H}\widehat{H}_{it}}{1+e^{-h^*}\widehat{H}_{it}}}{a + b\widehat{H}_{it}}$. Therefore I parametrize the noise according to:

$$
\frac{P_{i,t+1}}{P_{it}} = e^{\mu_{it}} \times \begin{cases} e^{\sigma u_{i,t+1} - \sigma^2/2} & \text{if there is no disaster at } t+1 \\ F_{i,t+1} & \text{if there is a disaster at } t+1 \end{cases} \tag{23}
$$

where $u_{i,t+1}$ is a standard Gaussian variable and $F_{i,t+1}$ is as in (). This parametrization ensures that the option price has a closed form, and at the same time conform to the essence of the economics. Economically, I assume that in a disaster most of the option value comes from the disaster, not from "normal times" volatility. In normal times returns are log-normal. However, if there is a disaster, stochasticity comes entirely from the disaster (there is no Gaussian u_{t+1} noise). The above structure takes advantage of the flexibility in the modelling of the noise in $\widehat{H}_{it}$ and D_{it}. Rather than modelling them separately, I assume that their aggregate gives exactly a log normal noise (the online appendix provides a way to ensure that this is possible). At the same time, (23) is consistent with the processes and prices in the rest of the paper.

Proposition 3 *(Put price) The value of a put with strike K (the fraction of the initial price at which the put is in the money) and a one-period maturity is $V_{it} = V_{it}^{ND} + V_{it}^{D}$ with V_{it}^{ND} and V_{it}^{D} corresponding to the events with no disasters and with disasters respectively:*

$$V_{it}^{ND} = e^{-\delta+\mu_{it}} (1 - p_t) V_{Put}^{BS} \left(Ke^{-\mu_{it}}, \sigma\right) \quad (24)$$

$$V_{it}^{D} = e^{-\delta+\mu_{it}} p_t E_t \left[B_{t+1}^{-\gamma} \max\left(0, Ke^{-\mu_{it}} - F_{i,t+1}\right)\right] \quad (25)$$

where $V_{Put}^{BS}(K, \sigma)$ is the Black-Scholes value of a put with strike K, volatility σ, initial price 1, maturity 1, and interest rate 0.

3.4 Corporate Spread, Government Debt and Inflation Risk

Consider the corporate spread, which is the difference between the yield on the corporate bonds issued by the safest corporations (such as AAA firms) and government bonds. The "corporate spread puzzle" is that the spread is too high compared to the historical rate of default (Huang and Huang 2003). It has a very natural explanation under the disaster view. It is mostly during disasters (in bad states of the world) that very safe corporations will default. Hence, the risk premia on default risk will be very high. To explore quantitatively this effect, I consider the case of a constant severity of disasters. The following Proposition summarizes the effects, which are analyzed quantitatively in the next section.

Proposition 4 *(Corporate bond spread, disasters, and expected inflation) Consider a corporation i, call F_i the recovery rate of its bond,[10] and λ_i the default rate conditional on no disaster, the yield on debt is $y_i = \delta + \lambda_i - pE^D\left[B^{-\gamma} F_\$ F_i\right]$. So, calling y_G the yield on government bonds, the corporate spread is:*

$$y_i - y_G = \lambda_i + pE^D\left[B^{-\gamma} F_\$ (1 - F_i)\right]. \quad (26)$$

In particular, when inflation is expected to be high during disasters (i.e. $F_\$$ is low, perhaps because current Debt / GDP is high), then (i) the spread $|y_i - y_j|$ between two nominal assets i, j, is low,

[10] In the assumptions of Chen, Collin-Dufresne and Goldstein (2009) and Cremers, Driessen and Maenhout (2008), the loss rate conditional on a default, λ^d, is the same across firms but only their probability of defaulting in a disater state, p_i varies. Then $F_i = 1 - p_i\lambda^d$, which is a particular case of this paper.

and (ii) the yield on nominal assets is high.

Proof. The Euler equation is $1 = e^{-\delta}(1+y_i)\left[(1-p)(1-\lambda_i) + pE\left[B^{-\gamma}F_{\$,t+1}F_i\right]\right]$, and the Proposition follows by taking the limit of small time intervals. ■

4 A Calibration

4.1 Calibrated Parameters

I propose the following calibration of the model's parameters, expressed in annualized units. I assume that time-variation of disaster risk enters through the recovery rate F_{it} for stocks and through the potential jump in inflation J_t for bonds. The calibration's inputs are summarized in Table I, while the results from the calibration are in Table II–VI and Figure I.

Section 5.3 will shows that, with the calibration, the variation of realized disaster risk varies enough compared to the volatility of resilience, so that by that criterion the calibrated numbers are reasonable.

Macroeconomy

In normal times, consumption grows at rate $g_c = 2.5\%$. To keep things parsimonious, the probability and conditional severity of macroeconomic disasters are taken to be constant over time. This implies that the real rate is constant. The disaster probability is $p = 3.63\%$, Barro and Ursua (2008)'s estimate. I take $\gamma = 4$, for which Barro and Ursua's evaluation of the probability distribution of B_{t+1} gives $E\left[B^{-\gamma}\right] = 5.29$ so that the utility-weighted mean recovery rate of consumption is $\overline{B} = E\left[B^{-\gamma}\right]^{-1/\gamma} = 0.66$. Because of risk aversion, bad events get a high weight: the modal loss is less severe. There is an active literature centering around the basic disaster parameters: see Barro and Ursua (2008), and Barro, Nakamura, Steinsson and Ursua (2009) who find estimates consistent with the initial Barro (2006) numbers.

The key number is the risk-neutral probability of disasters, $pE\left[B^{-\gamma}\right] = 19.2$. This high risk-neutral probability allows the model to calibrate a host of high risk premia. Following Barro and Ursua, I set the rate of time preference to match a risk free rate of 1%, so, in virtue of Eq. 12, the rate of time preference is $\rho = 6.6\%$.

Stocks

I take a growth rate of dividends $g_{iD} = g_C$, consistent with the international evidence (Campbell 2003, Table 3). The volatility of the dividend is $\sigma_D = 11\%$, as in Campbell and Cochrane (1999). The speed of mean-reversion of resilience ϕ_H, is the speed of mean-reversion of the price/dividend ratio. It has been carefully examined in two recent studies based on US data. Lettau and van Nieuwerburgh (2008) find $\phi_H = 9.4\%$. However, they find $\phi_H = 26\%$ when allowing for a structural break in the time series, which they propose is warranted. Cochrane (1988) finds $\phi_H = 6.1\%$, with

Table I: Variables Used in the Calibration.

Variables	Values
Time preference, risk aversion	$\rho = 6.6\%, \gamma = 4$
Growth rate of consumption and dividends	$g = g_{iD} = 2.5\%$
Volatility of dividends	$\sigma_D = 11\%$
Probability of disaster, Recovery rate of C after disaster	$p = 3.63\%, \overline{B} = 0.66$
Stocks' recovery rate: Typical value, Volatility, Speed of mean-reversion	$F_{i*} = \overline{B}, \sigma_F = 10\%, \phi_H = 13\%$
Inflation: Typical value, Volatility, Speed of mean-reversion	$I_* = 3.7\%, \sigma_I = 1.5\%, \phi_I = 18\%$
Jump in Inflation: Typical value, Volatility, Speed of mean-reversion	$J_* = 2.1\%, \sigma_J = 15\%, \phi_J = 92\%$

an s.e. of 4.7%. I take the mean of those three estimates, which leads to $\phi_H = 13\%$. Given these ingredients, the online appendix specifies a volatility process for H_{it}.

To specify the volatility of the recovery rate F_{it}, I specify that it has a baseline value $F_{i*} = \overline{B}$, and support $F_{it} \in [F_{\min}, F_{\max}] = [0, 1]$. That is, if there is a disaster, dividends can do anything between losing all their value and losing no value. The process for H_{it} then implies that the corresponding average volatility for F_{it}, the expected recovery rate of stocks in a disaster, is 10%. This may be considered to be a high volatility. Economically, it reflects the fact that it seems easy for stock market investors to alternatively feel extreme pessimism and optimism (e.g., during the large turning points around 1980, around 2001 and around 2008). In any case, this perception of the risk for F_{it} is not observable directly, so the calibration does not appear to contradict any known fact about observable quantities.

The disaster model implies a high covariance of stock prices with consumption. Is that true empirically? First, it is clear that we need multi-country data, as e.g. a purely US-based sample would not represent the whole distribution of outcomes, as it would contain too few disasters. Using such multi-country data, Ghosh and Julliard (2008) find a low importance of disaster. On the other hand, Barro and Ursua (2009) find a high covariance between consumption and stock returns during a disaster, which warrants the basic disaster model. The methodological debate, which involves missing observations, for instance due to closed stock markets, price controls, the measurement of consumption, and the very definition of disasters, is likely to continue for years to come. My reading of the Barro and Ursua (2009) paper is that the covariance between consumption and stock returns, once we include disaster returns, is large enough to vindicate the disaster model.

Inflation and Nominal Bonds

For simplicity and parsimony, I consider the case when inflation does not burst during disasters, $F_{\$,t+1} = 1$. Bond and inflation data come from CRSP. Bond data are monthly prices of zero-coupon

Table II: Some Variables Generated by the Calibration.

Variables	Values
Ramsey discount rate	$\delta = 16.6\%$
Risk-adjusted probability of disaster	$pE\left[B_{t+1}^{-\gamma}\right] = 19.2\%$
Stocks: Effective discount rate	$\delta_i = 5\%$,
Stock resilience: Typical value, volatility	$H_{i*} = 9\%$, $\sigma_H = 1.9\%$
Stocks: Equity premium, conditional on no disasters, uncond.	6.5%, 5.3%
Real short term rate	1%
Resilience of one nominal dollar	$H_\$ = 15.6\%$
5-year nominal slope $y_t(5) - y_t(1)$: Mean and volatility	0.57%, 0.92%
Long run $-$ short run yield: Typical value	$\kappa = 2.6\%$
Inflation Parameters	$I_{**} = 6.3\%, \psi_I = 12.9\%, \psi_J = 89.4\%$
Bond risk premium: Volatility	$\sigma_\pi = 2.9\%$

Notes. The main other objects generated by the model are in Tables III–VI and Figure I.

bonds with maturities of 1 to 5 years, from June 1952 through September 2007. In the same time sample, I estimate the inflation process as follows. First, I linearize the LG process for inflation, which becomes: $I_{t+1} - I_* = e^{-\phi_I \Delta t}(I_t - I_*) + \varepsilon_{t+1}^I$. Next, it is well-known that inflation, observed at the monthly frequency, contains a substantial high-frequency and transitory component, which in part is due to measurement error. The model accommodates this. Call $\widetilde{I}_t = I_t + \eta_t$ the measured inflation (which can be thought of as trend inflation plus mean zero noise), while I_t is the trend inflation. I estimate inflation using the Kalman filter, with $I_{t+1} = C_1 + C_2 I_t + \varepsilon_{t+1}^I$ for the trend inflation, and $\widetilde{I}_t = I_t + \eta_t$ for the noisy measurement of inflation. Estimation is at the quarterly frequency, and yields $C_2 = 0.954$ (s.e. 0.020), i.e. the speed of mean-reversion of inflation is $\phi_I = 0.18$ in annualized values. Also, the annualized volatility of innovations in trend inflation is $\sigma_I = 1.5\%$. I have also checked that estimating the process for I_t on the nominal short rate (as recently done by Fama 2006) yields substantially the same conclusion. Finally, I set I_* at the mean inflation, 3.7%. (The small nonlinearity in the LG term process makes I_* differ from the mean of I_t by only a trivial amount).

To assess the process for J_t, I consider the 5-year slope, $s_t = y_t(5) - y_t(1)$. Eq. 41 shows that, conditional on no disasters, it follows (up to second order terms), $s_{t+1} = a + e^{-\phi_J \Delta t} s_t + bI_t + \varepsilon_{t+1}^s$, where Δt is the length of "a period" (e.g., a quarter means $\Delta t = 1/4$). I estimate this process at a quarterly frequency. The coefficient on s_t is 0.795 (s.e. 0.043). This yields $\phi_J = 0.92$. The standard deviation of innovations to the slope is 0.92%.

To calibrate κ I consider the baseline value of the yield, which from (16) is $y_t(T) = y_t(0) + \kappa +$

$\ln\left(1-\frac{1-e^{-\psi_I T}}{\psi_I}\kappa\right)/T$, with $\psi_I = \phi_I - 2\kappa$, and I compute that value of κ that ensures $y_t(5) - y_t(1) = 0.0057$, the empirical mean of the 5-year slope. This gives $\kappa = 2.6\%$. By (9), this implies an inflation jump during disasters of $J_* = 2.1\%$.

As a comparison, Barro and Ursua (2008, p.304) find a median increase of inflation during disasters of 2.4%. They find a median inflation rate of 6.6% during disasters, compared to 4.2% for long samples taken together. This is heartening, but one must keep in mind that Barro and Ursua find that the average increase in inflation during disasters is equal to 109% – because of hyperinflations, inflation is very skewed.[11] I conclude that a jump in inflation of 2.1% is consistent with the historical experience. Investors do not know ex ante if disasters will bring about inflation or deflation; on average however, they expect more inflation.

As there is considerable variation in the actual jump in inflation, there is much room for variations in the perceived jump in inflation, $J_t = J_* + \widehat{J}_t$ – something that the calibration indeed will deliver. We saw that empirically, the standard deviation of the innovations to the 5-year spread is 0.92% (in annualized values), while in the model it is: $(K_5 - K_1)\,\sigma_\pi$. Hence we calibrate $\sigma_\pi = 2.9\%$. As a result, the standard deviation of the 5-year spread is $(K_5 - K_1)\,\sigma_\pi/\sqrt{2\phi_J} = 0.68\%$, while in the data it is 0.79%. Hence, the model is reasonable in terms of observables.

An important non-observable is the perceived jump of inflation during a disaster, J_t. Its volatility is $\sigma_J = \sigma_\pi/\left(pE\left[B^{-\gamma}\right]\right) = 15.4\%$, and its population standard deviation is $\sigma_J/\sqrt{2\phi_J} = 11\%$. This is arguably high – though it does not violate the constraint that the actual jump in inflation should be more dispersed than its expectation (section 5.3). One explanation is that the yield spread has some high-frequency transitory variation that leads to a very high measurement of ϕ_J; with a lower value one would obtain a considerably lower value of σ_J. Another interpretation is that the demand for bonds shifts at a high frequency (perhaps for liquidity reasons). While this is captured by the model as a change in perceived inflation risk, it could be linked to other factors. In any case, we shall see that the model does well in a series of dimensions explored in Section 4.3.

On the degree of parsimony of this calibration

This paper is chiefly concerned with the value of stocks and government bonds. It uses two latent measures of riskiness, one for real quantities (the stock resilience H_{it}), one for nominal quantities (the bond risk premium π_t), that load on just one macro shock, the disaster shock. The model is agnostic about their correlation – their shocks could be very correlated, or not. This assumption of at least one nominal factor and one real factors is used by most authors, e.g. Bansal and Shaliastovich (2009), Lettau and Wachter (2007), Piazzesi and Schneider (2007), Wachter (2006).

[11] There is a difference between wars and financial disasters: wars very rarely lead to deflations, but financial disasters often do, especially during the Great Depression. The inflation jump is a bit higher during wars than financial disasters, by about 1% of 4%, depending on whether one takes the median or the mean of windorized values. It is useful to note that financial disasters in non-OECD are typically inflationary.

I think that it is hardly possible to be more parsimonious and still account for the basic facts of asset prices. Indeed, a tempting, though ultimately inadequate, idea would be the following: nominal bonds and stocks are driven by just one factor, perhaps the disaster probability. However, there is much evidence that risk premia are driven by more than one factor. Fama and French (1993) find that five factors are necessary to account for stocks and bonds.[12]. Hence, the framework in this paper using two factors (a nominal, and a real one) is in a sense the minimal framework to make sense of asset price puzzles on stocks and nominal bonds.

I next turn to the return predictability generated by the model. Sometimes, I use simulations, which the online appendix details.

4.2 Stocks: Predictability and Options

4.2.1 Average Levels

The equity premium (conditional on no disasters) is $r_{it}^{e} - r_f = p\left(E\left[B^{-\gamma}\right]\left(1 - F_{i*}\right)\right) = 6.5\%$. The unconditional equity premium is 5.3% (the above value, minus $p\left(1 - F_{i*}\right)$). So, as in Barro (2006), the excess returns of stocks mostly reflect a risk premium, not a peso problem.[13] The mean value of the price/dividend ratio is 18.2 (and is close to Eq. 14, evaluated at $\widehat{H}_{it} = 0$), in line with the empirical evidence reported in Table III. The central value of the D/P ratio is $\delta_i = 5.0\%$.

4.2.2 Aggregate Stock Market Returns: Excess Volatility and Predictability

"Excess" Volatility The model generates "excess volatility" and predictability. Consider (14), $P_{it}/D_{it} = \left(1 + \widehat{H}_{it}/\left(\delta_i + \phi_H\right)\right)/\delta_i$. As stock market resilience $\widehat{H}_{it}$ is volatile so are stock market prices and P/D ratios. Table III reports the numbers. The standard deviation of $\ln\left(P/D\right)$ is 0.27. Volatile resilience yields a volatility of the log of the price / dividend ratio equal to 10%. For parsimony, I assume that innovations to dividends and resilience are uncorrelated. The volatility of equity returns is 15%. I conclude that the model can quantitatively account for an "excess" volatility of stocks through a stochastic risk-adjusted severity of disasters. In addition, in a sample with rare disasters, changes in the P/D ratio mean only change in future returns, not future dividends. This is in line with the empirical findings of Campbell and Cochrane (1999).

Predictability Consider (14) and (15). When $\widehat{H}_{it}$ is high, (15) implies that the risk premium is low and P/D ratios (14) are high. Hence, the model generates above average subsequent stock

[12] In addition, the correlation between stocks and nominal bond premia appears to be very small. Viceira (2007) reports that the correlation between bond returns and stock returns is 3%. The correlation between the change in the Cochrane-Piazzesi (CP, 2005) factor and stock market returns is also 3%, while the correlation between the level of CP and the change in stock market returns also 3%. This means that at least two factors are necessary.

[13] Note that this explanation for the equity premium is very different from the one proposed in Brown, Goetzmann and Ross (1995), which centers around survivorship bias.

Table III: Some Stock Market Moments.

	Data	Model
Mean P/D	23	18.2
Stdev $\ln P/D$	0.33	0.30
Stdev of stock returns	0.18	0.15

Explanation: Stock market moments. The data are Campbell (2003, Table 1 and 10)'s calculation for the USA 1891–1997.

market returns when the market-wide P/D ratio is below average. This is the view held by many (e.g. Campbell and Shiller 1988, Cochrane 2008) though not all (Goyal and Welch 2008). The model predicts the following magnitudes for regression coefficients.

Proposition 5 *(Predicting stock returns via P/D ratios) Consider the predictive regressions of the return from holding the stock from t to $t+T$, $r^e_{it\to t+T}$ on the initial price-dividend ratio,* $\ln(D_{it}/P_{it})$*:*

$$r^e_{it\to t+T} = \alpha_T + \beta_T \ln(D_{it}/P_{it}) + noise. \quad (27)$$

$$r^e_{it\to t+T} = \alpha'_T + \beta'_T (D_{it}/P_{it}) + noise \quad (28)$$

In the model for small holding horizons T the slopes are, to the leading order: $\beta_T = (\delta_i + \phi_H) T$ and $\beta'_T = (1 + \phi_H/\delta_i) T$.

This intuition for the value of β_T is thus. First, the slope is proportional to T simply because returns over a horizon T are proportional to T. Second, when the P/D ratio is lower than baseline by 1%, it increases returns through two channels: the dividend yield is higher by δ_i% and mean-reversion of the price-dividend ratio creates capital gains of ϕ%.

Table IV: Predicting Returns with the Dividend-Price Ratio

	Data			Model	
Horizon	Slope	s.e.	R^2	Slope	R^2
1	0.11	(0.053)	0.04	0.17	0.06
4	0.42	(0.18)	0.12	0.45	0.19
8	0.85	(0.20)	0.29	0.79	0.30

Explanation: Predictive regression for the expected stock return $r^e_{it\to t+T} = \alpha_T + \beta_T \ln(D_{it}/P_{it})$, at horizon T (annual frequency). The data are Campbell (2003, Table 10 and 11B)'s calculation for the US 1891–1997.

Using the paper's calibration of $\delta_i = 5\%$ and $\phi_H = 13\%$, Proposition 5 predicts a slope coefficient $\beta_1 = 0.18$ at a one-year horizon. This prediction is in line with the careful estimates of Lettau and van Nieuwerburgh (2008) who find a β_1 value of 0.23 in their preferred specification. Also, Cochrane (2008) runs regression (28) at the annual horizon and finds $\beta_1' = 3.8$ with a standard error of 1.6. Proposition 5 predicts $\beta_1' = 3.6$. We note that the approximation in Proposition 5, valid for "small" T, appears to be valid up to approximately a 1-year holding period.

I conclude that the model is successful not only at matching the level, but also the variation and predictability of the stock market.

Characteristics vs Covariances In a rare disaster economy, characteristics tend to predict returns better than covariances, something that a strand of research argues is true (Daniel and Titman 1997), although this is not uncontroversial (Davis, Fama and French 2000). Indeed, in a sample without disasters, betas will only reflect the covariance during "normal times." But, risk premia are only due to the covariance with consumption in disasters. The two can be entirely different. Hence, the "normal times" betas can have no relation with risk premia. However, "characteristics," like the P/D ratio, imbed measures of risk premia (as in 14). Hence, characteristics will predict returns better than covariances.

However, there could be some spurious links if stocks with low H_{i*} have higher cash-flow betas. One could conclude that a cash-flow beta commands a risk premium, but this is not because cash-flow betas cause a risk premium, simply because stocks with high cash-flow beta happen to be stocks that have a large loading on the disaster risk.

These points may help explain the somewhat contradictory findings in the debate about whether characteristics or covariances explain returns. When normal-times covariances badly measure the true risk, as is the case in a disaster model, characteristics will often predict expected returns better than covariances.

4.3 Bond Premia and Yield Curve Puzzles

4.3.1 Excess Returns and Time-Varying Risk Premia

Bonds carry a time-varying risk premium. Eq. 18 indicates that bond premia are (to a first order) proportional to bond maturity T. This is the finding of Cochrane and Piazzesi (2005). The one factor here is the inflation premium π_t which is compensation for a jump in inflation if a disaster happens. The model delivers this because a bond's loading of inflation risk is proportional to its maturity T.

The nominal yield curve slopes up on average. Suppose that when the disaster happens, inflation jumps by $J_* > 0$. This leads to a positive parametrization κ of the bond premia (Eq. 9).

The typical nominal short term rate (i.e., the one corresponding to $I_t = I_*$) is $y(0) = \delta - H_\$ + I_*$ while the long term rate is $y(0) + \kappa$ (i.e., $-\lim_{T\to\infty} \ln Z_{\$t}(T)/T$). Hence, the long term rate is above the short term rate by $\kappa > 0$. The yield curve slopes up. Economically this is because long maturity bonds are more sensitive than short-term bonds to inflation risk, so they command a risk premium.

4.3.2 The Forward Spread Predicts Bond Excess Returns (Fama-Bliss)

Fama and Bliss (1987) regress short-term excess bond returns on the forward spread, i.e. the forward rate minus the short-term rate:

$$\text{Fama-Bliss regression: Excess return on bond of maturity } T = \alpha_T + \beta_T \cdot (f_t(T) - r_t) + \text{noise.} \quad (29)$$

The expectation hypothesis yields constant bond premia, hence predicts $\beta_T = 0$. I next derive the model's prediction. As in the calibration $var(I_t)\,\psi_I^2/var(\pi_t) = 0.023$, I highlight the case where this quantity is small, which means that changes in the slope of the yield curve come from changes in the bond risk premium rather than changes in the drift of the short term rate.

Proposition 6 *(Coefficient in the Fama-Bliss regression) The slope coefficient β_T of the Fama-Bliss regression (29) is given in (42). When $var(I_t)\,\psi_I^2/var(\pi_t) \ll 1$,*

$$\beta_T = 1 + \frac{\psi_J}{2} T + O\left(T^2\right). \quad (30)$$

When $var(\pi_t) = 0$ (no risk premium shocks) the expectation hypothesis holds and $\beta_T = 0$. In all cases, the slope β_T is nonnegative and eventually goes to 0, $\lim_{T\to\infty} \beta_T = 0$.

To understand the economics of the previous proposition, consider the variable part of the two sides of the Fama-Bliss regression (29). The excess return on a $T-$maturity bond is approximately $T\pi_t$ (see Eq. 18) while the forward spread is $f_t(T) - r_t \simeq T\pi_t$ (see Eq. 22). Both sides are proportional to $\pi_t T$. Thus, the Fama-Bliss regression (29) has a slope equal to 1 which is the leading term of (30).

This value β_T above 1 is precisely what Fama and Bliss have found, a finding confirmed by Cochrane and Piazzesi (2005). This is quite heartening for the model. Table V reports the results. We also see that as maturity increases, coefficients initially rise but then fall at long horizons, as predicted by Proposition 6. Economically, most of the variations in the slope of the yield curve are due to variations in risk-premium, not to the expected change of inflation.

Table V: Fama-Bliss Excess Return Regression

	Data			Model	
Maturity T	β	(s.e.)	R^2	β	R^2
2	0.99	(0.33)	0.16	1.33	0.34
3	1.35	(0.41)	0.17	1.71	0.23
4	1.61	(0.48)	0.18	1.84	0.14
5	1.27	(0.64)	0.09	1.69	0.08

Explanation: The regressions are the excess returns on a zero-coupon bond of maturity T, regressed on the spread between the T forward rate and the short term rate: $rx_{t+1}(T) = \alpha + \beta(f_t(T) - f_t(1)) + \varepsilon_{t+1}(T)$. The unit of time is one year. The empirical results are from Cochrane and Piazzesi (2005, Table 2). The expectation hypothesis implies $\beta = 0$.

4.3.3 The Slope of the Yield Curve Predicts Future Movements in Long Rates (Campbell Shiller)

Campbell and Shiller (CS, 1991) find that a high slope of the yield curve predicts that future long term rates will fall. CS regress changes in yields on the spread between the yield and the short-term rate:

$$\text{Campbell-Shiller regression: } \frac{y_{t+\Delta t}(T-\Delta t) - y_t(T)}{\Delta t} = a + \beta_T \cdot \frac{y_t(T) - y_t(0)}{T} + \text{noise} \quad (31)$$

The expectation hypothesis predicts $\beta_T = 1$. However, CS find negative β_T's, with a roughly affine shape as a function of maturity (see Table VI). This empirical result is predicted by the model, as the next Proposition shows. As in the calibration $var(I_t)\phi_I^2/var(\pi_t) = 0.045$, I highlight the case where those quantities are small.

Proposition 7 *(Coefficient in the Campbell-Shiller regression) The slope coefficient β_T in the Campbell-Shiller (1991) regression (31) is given by (43). When $\phi_I^2 var(I_t)/var(\pi_t) \ll 1$, $\kappa T \ll 1$,*

$$\begin{aligned} \beta_T &= -\left(1 + \frac{2\psi_j - \psi_I}{3}T\right) + o(T) \text{ when } T \to 0 \\ \beta_T &= -\psi_J T + o(T) \text{ when } T \gg 1 \end{aligned} \quad (32)$$

Table VI also contains simulation results of the model's predictions. They are in line with CS's results. To understand the economics better, I use a Taylor expansion in the case where inflation is minimal. The slope of the yield curve is, to the leading order, $(y_t(T) - y_t(0))/T = \frac{\pi_t}{2} + O(T)$. Hence, to a first order approximation (when inflation changes are not very predictable) the slope of

Table VI: Campbell-Shiller Yield Change Regression

	Data		Model
Maturity T	β	(s.e.)	β
3	-0.15	(0.28)	-1.03
6	-0.83	(0.44)	-1.16
12	-1.43	(0.60)	-1.41
24	-1.45	(1.00)	-1.92
48	-2.27	(1.46)	-2.83

Explanation: The regressions are the change in bond yield on the slope of the yield curve: $y_{t+1}(T-1) - y_t(T) = \alpha + \frac{\beta}{T-1}(y_t(T) - y_t(1)) + \varepsilon_{t+1}(T)$ The time unit is one month. The empirical results are from Campbell, Lo, MacKinlay (1997, Table 10.3). The expectation hypothesis implies $\beta = 1$.

the yield curve reflects the bond risk premium. The change in yield is (the proof of Proposition 7 justifies this):

$$\frac{y_{t+\Delta t}(T-\Delta t) - y_t(T)}{\Delta t} \simeq -\frac{\partial y_t(T)}{\partial T} = \frac{-\pi_t}{2} + O(T).$$

Hence, the CS regression yields a coefficient of -1, to the leading order. Economically, it means that a high bond premium increases the slope of the yield curve (by $\pi_t/2$).

As bond maturity increases, Proposition 7 predicts that the coefficient in the CS regression becomes more and more negative. The economic reason is the following. For long maturities, yields have vanishing sensitivity to the risk premium (as in Dybvig, Ingersoll and Ross 1996) which the model says has the shape $y_t(T) = a + b\pi_t/T + o(1/T)$ for some constants a, b. So the slope of the yield curve varies as $b\pi_t/T^2$, and the expected change in the yield is $-b\phi_J\pi_t/T$. So the slope in the CS regression (31) is $\beta_T \sim -\phi_J T$. On the other hand, the expression for β_T shows that when the predictability due to inflation is non-negligible, the CS coefficient should go to 1 for very large maturities.

In Table VI we see that the fit between theory and evidence is rather good. The only poor fit is at small maturity. The CS coefficient is closer to 0 than in the model. The short term rate has a larger predictable component at short term horizons than in the model. For instance, this could reflect a short-term forecastability in Fed Funds rate changes. That feature could be added to the model as in the online appendix. Given the small errors in fit, it is arguably better not to change the baseline model which broadly accounts for the CS finding. Economically, the CS finding reflects the existence of a stochastic one-factor bond risk premium.

4.3.4 Explaining Cochrane and Piazzesi (2005)

Cochrane and Piazzesi (CP, 2005) establish that (i) a parsimonious description of bond premia is given by a stochastic one-factor risk premium, (ii) (zero-coupon) bond premia are proportional to bond maturity, and (iii) this risk premium is well proxied empirically by a "tent-shape" linear combination of forward rates. Eq. 18 delivers their first two findings: there is a single bond risk factor π_t, and the loading on it is proportional to bond maturity. Economically, it is because a bond of maturity T has a sensitivity to inflation risk approximately proportional to T.

To understand CP's third finding, rewrite (21) as:

$$f_t(T) = F(T) + e^{-\psi_I T} I_t + \Lambda(T)\pi_t, \qquad \Lambda(T) \equiv \frac{e^{-\psi_I T} - e^{-\psi_J T}}{\psi_J - \psi_I}.$$

The economic intuition for the CP "tent shape" effect is as follows. The forward rate of maturity T has a loading $\Lambda(T)$ on the bond risk premium π_t. $\Lambda(T)$ has a tent-shape: $\Lambda(0) = \Lambda(\infty) = 0$, and $\Lambda(T) > 0$ for $T > 0$. We saw earlier (after Lemma 2) that the economic reason for this tent-shape of $\Lambda(T)$ is that short term bonds have no inflation risk premium, and long term forwards are constant (in this model, $f_t(\infty) = \delta - H_{\$} + I_{**}$), so that only intermediate maturity forwards have a loading on the bond risk premium. So, to capture the bond risk premium, a tent-shape $\sum_{T=1}^{5} w_T f_t(T)$ combination for forwards predicts the bond risk premium. The simple $(\sum_{T=1}^{5} w_T)$ and maturity-weighted $(\sum_{T=1}^{5} T w_T)$ sum of the weights should be roughly 0, so as to eliminate $e^{-\psi_I T} I_t$ up to second order terms.

This reasoning leads one to ask if there is a simple combination of forward rates which one might expect to robustly proxy for the risk premia. The next Proposition gives an answer.[14]

Proposition 8 *(Estimation-free combinations of forwards to proxy the bond risk premium) Given time horizons a and b, consider the following "estimation-free" combinations of forwards:*

$$CP_t^{EF}(a,b) \equiv \left[-f_t(a) + 2f_t(a+b) - f_t(a+2b)\right]/b^2$$

where $f_t(T)$ are the forwards of maturity T. Then, up to third order terms, for small a and b, $CP_t^{EF}(a,b) = (\psi_I + \psi_J)\pi_t$ is proportional the bond risk premium.

Proof. From (21) and (40) up to third order terms $CP_t^{EF} = (\psi_I + \psi_J)\pi_t$. The leading inflation term is $-\psi_I^2 I_t$, a third order term. ■

For instance, $CP_t^{EF}(1,2) = (-f_t(1) + 2f_t(3) - f_t(5))/4$, uses the forwards up to maturity 5 years. Proposition 8 suggests that CP_t^{EF} could be used in practice to proxy for the bond risk premia,

[14] Lettau and Wachter (2007) proposed earlier another combination of theoretical factors to obtain the risk premium, but it is not estimation-free.

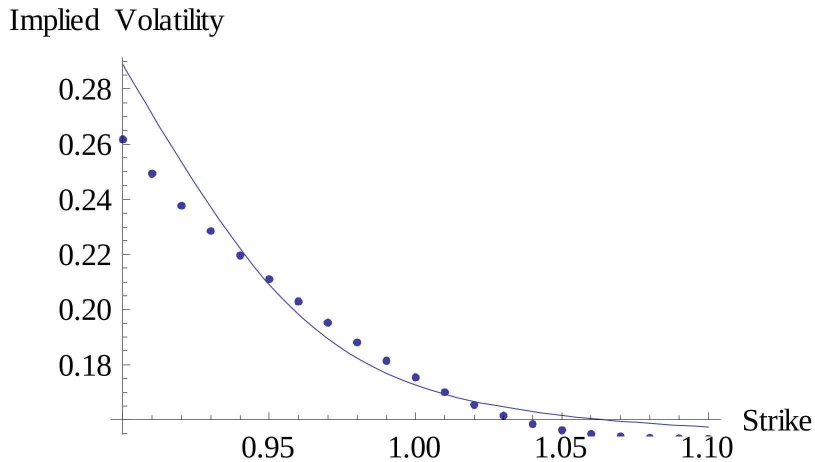

Figure I: This Figure shows the Black-Scholes annualized implied volatility of a 1-month put on the stock market. The solid line is from the model's calibration. The dots are the empirical average (January 2001 - February 2006) for the options on the S&P 500 index, calculated as in Figlewski (2008). The initial value of the market is normalized to 1. The implied volatility on deep out-of-the-money puts is higher than the implied volatility on at-the-money puts, which reflects the probability of rare disasters.

without requiring a preliminary estimation.[15] Over the 1964-2008, repeating the CP analysis gives an average R^2 of 28% to predict excess bond return, while the estimation-free $CP_t^{EF}(1,2)$ yields a R^2 of 23%. This is arguably a good performance, given the CP analysis uses five regressors, and the estimation-free CP_t^{EF} uses just one. In addition, consider a country with a short dataset: the estimate of the CP coefficients will be very noisy. Researchers could use the estimation-free CP_t^{EF} to evaluate risk-premia.

I conclude that the model explains all three CP findings, and proposes new combinations of factors to predict the bond premium. These are "estimation-free"and might be useful empirically.

4.4 Options

I now ask whether the model's calibration (which did not target any option-specific value) yields good values for options. I calculate the model's Black-Scholes implied volatility of puts with a 1 month maturity. I am very grateful to Stephen Figlewski for providing the empirical implied volatility of 1-month options on the S&P 500, from January 2001 to February 2006, obtained with the interpolation method described in Figlewski (2008).

Figure I reports the implied volatility, from the data, as well as the in the calibration. The correspondence is quite good, despite the fact that no extra parameter was tuned to match options

[15] Another interesting combination is $8CP_t^{EF}(1,2) - \frac{1}{2}CP_t^{EF}(2,1)$, which is $-2f(1) + 0.5f(2) + 3f(3) + 0.5f(4) - 2f(5)$, and is very close to what CP estimate.

prices. Hence, I conclude that in a first pass and for the maturity presented here, the variable rare disasters model gets options prices correct. Du (forth.) finds other parametrizations of jumps that match option prices. Of course, a more systematic study would be desirable. Farhi et al. (2009) and Jurek (2009) investigate the link between currency option prices and currency levels, finding support for the existence of a disaster risk premium.

Backus, Chernov and Martin (2009) study a specification that equity dividends are $D_t = C_t^\lambda$ for some $\lambda > 0$, and cannot fit options prices in a disaster framework with constant disaster risk. They mention that a model such as the present one can work better. In their $D_t = C_t^\lambda$ framework, a high λ (about four) is necessary to get a high volatility of equity. But then, equity are immensely risky during disasters, and put prices are too high. In contrast, in the present framework, equity volatility comes from resilience volatility, and put prices can be moderate and calibrate naturally.

Proposition 3 suggests a way to extract key structural parameters of disasters from options data. Stocks with a higher put price (controlling for "normal times" volatility) should have a higher risk premium, because they have higher future expected returns. Evaluating this prediction would be most interesting. Supportive evidence comes from Bollerslev, Tauchen and Zhou (forth.). They find that when put prices are high, subsequent stock market returns are high. This is exactly what a disaster-based model predicts.

To be more quantitative, consider the "variance premium" VP_t, which is the risk-neutral expected variance, minus the expected variance (conditional on no disasters). It is easy to derive $VP_t = p_t E\left[B_{t+1}^{-\gamma}\left(1 - F_{i,t+1}\right)^2\right]$, as the jump size in a disaster is $1 - F_{i,t+1}$. Regressing returns at horizon H on the variance premium gives a mean coefficient

This model cannot account for all the patterns in the variance premium, as it is a one-factor model and the VIX index clearly shows some high-frequency transient dynamics. More elaborations are in Bollerslev, Tauchen and Zhou (forth.) and Drechsler and Yaron (2009). The disaster model does appear competitive.

4.5 Corporate Bonds

The calibration allows us to evaluate Proposition 4. The disaster risk premium is $\pi_i^D = y_i - y_G - \lambda_i$, the difference between the yield on corporate bonds and governance bonds minus the historical default rate of corporate bonds. The rare disaster model gives a macroeconomic foundation for Almeida and Philippon (2007)'s view that the corporate spread reflects the existence of bad states of the world, and for reduced-form models of credit risk.

Almeida and Philippon (2007) allow an estimate of π_i^D as the difference between this risk-adjusted annualized probability of default and the historical one.[16] For instance, it yields π_i^D to be

[16] I take Almeida and Philippon's Table III, which is the 10-year risk-neutral and historical probability, and apply

about 4.05% for a bond rated B (resp., 0.60% for a AAA bond). With $pE\left[B^{-\gamma}\right] = 19.2\%$, it means that the expected loss in a disaster is $4.05\%/0.192 = 21\%$ (resp. $0.60\%/0.192 = 3.2\%$ for a AAA bond). This is a moderate loss. We see how easily, though, a disaster model can rationalize the corporate spread.

Prediction (i) of Proposition 4 seems quite new. The intuition for it is the following. Suppose agents know that there will be hyperinflation in disasters, so that the real value of all nominal assets will be zero ($F_{\$} = 0$). Then, there is no difference in the risk premium between government bonds, AAA bonds or any nominal bond: their value will be wiped out during disasters. So, the part of their spread due to inflation risk is 0. More generally, a higher inflation risk lowers the spread between nominal bonds, because it reduces the values of all nominal bonds.

Prediction (i) provides an explanation for Krishnamurthy and Vissing-Jorgensen (2008)'s finding that when the Debt/GDP ratio is high the AAA-Treasury and the BAA-AAA spreads are low: in their 1925-2005 USA sample, regressing the AAA-Treasury and BAA-AAA spreads on the Debt/GDP ratio yields significant coefficients of resp. -1.5 and -1.2. The first AAA-Treasury can be explained by their favored interpretation of a liquidity demand for treasuries, but the BAA-AAA spread may be harder to explain via liquidity. The disaster hypothesis offers an explanation for both, hence it is complementary to the liquidity explanation. When Debt/GDP is high the temptation to default via inflation (should a risk occur) is high,[17] so $F_{\$}$ is low, thus nominal spreads are low.

Prediction (ii) of Proposition 4 allows one to think about the impact of the government Debt/GDP ratio. It is plausible that if the Debt/GDP ratio is high then if there is a disaster the government will sacrifice monetary rectitude so that J_t is high (that effect could be microfounded). This implies that when the Debt/GDP ratio (or the deficit/GDP) is high then long-term rates are high and the slope of the yield curve is steep (controlling for inflation and expectations about future inflation in normal times). In addition, in the Krishnamurthy and Vissing-Jorgensen data, regressing bond rates minus the bill rate on the Debt/GDP ratio yields a significant coefficient of 1.8, consistent with the disaster hypothesis: when Debt/GDP ratio is high, the bond risk premium is high, so the slope of the yield curve is high .

Likewise, say that an independent central bank has a more credible commitment not to increase inflation during disasters (J_t smaller). Then, real long term rates (e.g. nominal rates minus expected inflation) are lower and the yield curve is less steep. This effect works in an economy where Ricardian

the transformation $-\ln(1-x)/10$ to obtain the annualized probability of default. I also add back the AAA-Treasury spread of 0.51%, to get the actual AAA-Treasury spread. This yields a disaster premium of: 0.60% (AAA bonds), 1.11% (AA), 1.71% (A), 2.57% (BBB), 3.06% (BB), and 4.05% (B). Almeida and Philippon do not report standard errors.

[17] Catao and Terrones (2005) provide evidence for the view that high Debt/GDP leads, on average, to an increase of inflation.

equivalence holds. Higher deficits increase long term rates not because they "crowd out" investment, but instead because they increase the government's temptation to inflate away the debt if there is a disaster. In such a case there is an inflation risk premium on nominal bonds.

5 Discussion and Extensions

5.1 Epstein-Zin-Weil Preferences

For reasons that will be clear soon, I develop here the model with the preferences introduced by Epstein and Zin (EZ, 1989) and Weil (1990). Much of the previous analysis will be preserved, with the use of an enriched notion of resilience. Call ψ the intertemporal elasticity of substitution and $\theta = (1-\gamma)/(1-1/\psi)$, which is equal to 1 in the case of CRRA preferences. EZ show that the stochastic discount factor (SDF) evolves as $M_{t+1}/M_t = e^{-\rho\theta}(C_{t+1}/C_t)^{-\theta/\psi} R_{c,t+1}^{\theta-1}$, where $R_{c,t+1} = P_{c,t+1}/(P_{ct} - C_t)$ is the gross return of a consumption claim – the asset that gives a consumption C_t as a dividend, and whose price we call P_{Ct}.

The resilience of a consumption claim, $H_{ct} = p_t E\left[B_{t+1}^{1-\gamma} - 1\right]$, and is assumed to follow the LG twisted process, $\widehat{H}_{c,t+1} = \frac{1+H_{c*}}{1+H_{ct}} e^{-\phi_H} \widehat{H}_{c,t} + \varepsilon_{c,t+1}^H$ to the leading order. In what follows I consider only the leading terms; in particular, I neglect the variance terms (e.g., $var\left(\varepsilon_{c,t+1}^H\right)$ to concentrate on disaster terms). The main tool is the value of the SDF.

Theorem 3 *(SDF with EZ preferences) In the Epstein-Zin setup, the stochastic discount factor is:*

$$\frac{M_{t+1}}{M_t} = e^{-\delta}\left(1 + \frac{1-\theta}{\theta} H_{ct} + \varepsilon_{M,t+1}\right) \times \begin{cases} 1 & \text{if there is no disaster at } t+1 \\ B_{t+1}^{-\gamma} & \text{if there is a disaster at } t+1 \end{cases} \quad (33)$$

where $\delta = \rho + g_c/\psi$, $H_{ct} = p_t E\left[B_{t+1}^{1-\gamma} - 1\right]$, $\delta_c = \delta - g_c - \frac{1}{\theta} H_{c*}$ *and* $\varepsilon_{M,t+1} = \frac{\theta-1}{\theta} \frac{\varepsilon_{c,t+1}^H}{\delta_c + \phi_H}$.

The key impact of disaster is in $B_{t+1}^{-\gamma}$ as in the CRRA. However, it is now modulated by the H_{ct} term, which cancel out in the CRRA case, $\theta = 1$. This SDF causes expected returns formalized in the next Lemma.

Lemma 3 *(Expected returns with EZ preferences) Consider an asset i in the limit of small time intervals. Its normal-times expected return is:*

$$r_{i,t}^e = \delta + \frac{\theta-1}{\theta} H_{ct} - p_t E_t^D\left[B_{t+1}^{-\gamma}(1 + r_{i,t+1}) - 1\right] - cov_t^{ND}(r_{i,t+1}, \varepsilon_{M,t+1}) \quad (34)$$

where cov_t^{ND} is the covariance conditional on no disaster. The risk-free rate is: $r_f = \delta + \frac{\theta-1}{\theta} H_{ct} - p_t E_t\left[B_{t+1}^{-\gamma} - 1\right]$.

Assets receive a risk premia because of their behavior in disasters (the $E_t^D\left[B_{t+1}^{-\gamma}\left(1+r_{i,t+1}\right)-1\right]$ term), but also because of their normal-times covariance with the SDF, as shown in the last term in (34).

Next, consider a stock i. I define its *Epstein–Zin–enriched resilience* as:

$$\begin{aligned} H_{it}^{EZ} &= H_{it} + cov_t^{ND}\left(\varepsilon_{M,t+1}, \varepsilon_{i,t+1}\right) + \frac{1-\theta}{\theta} H_{ct} \\ &= p_t E_t^D\left[B_{t+1}^{-\gamma} F_{i,t+1} - 1\right] + cov_t^{ND}\left(\varepsilon_{M,t+1}, \varepsilon_{i,t+1}\right) + \frac{1-\theta}{\theta} p_t E_t^D\left[B_{t+1}^{1-\gamma} - 1\right] \end{aligned} \tag{35}$$

The first term is as in the CRRA case. The other two terms are zero in the CRRA case. The second term is a classic compensation for normal-times covariance with the stochastic discount factor. The third term is an asset-independent adjustment for the overall riskiness of the economy, which makes the riskless rate fluctuate. In the cross-section, a stock i has high resilience if it does well during disasters ($F_{i,t+1}$ is high) or if its dividends have high covariance with the SDF, so that the asset is a hedge.

I assume that the EZ-enriched resilience follows, up to second order terms, a LG process, with $H_{it}^{EZ} = H_{i*}^{EZ} + \widehat{H}_{it}^{EZ}$ and $E_t\left[\widehat{H}_{i,t+1}^{EZ}\right] = \frac{1+H_{i*}^{EZ}}{1+H_{i*}^{EZ}} e^{-\phi_H} \widehat{H}_{it}^{EZ}$. The following Proposition shows that the above results on stocks (e.g. Theorem 1, Proposition 1) follow, provided one uses the enriched notion of resilience.

Proposition 9 *(Stock price with EZ preferences) With EZ preferences, the price of a stock i is the same expression (14) as in the CRRA case, but with the EZ-enriched resilience (35):*

$$\frac{P_{it}}{D_{it}} = \frac{1}{\delta_i}\left(1 + \frac{\widehat{H}_{it}^{EZ}}{\delta_i + \phi_H}\right), \qquad \delta_i \equiv \delta - g_i - H_{i*}^{EZ} \tag{36}$$

We see that when $\widehat{H}_{ct} = 0$, we have the same predictions as in the CRRA case, up to a change in the value of δ_i.

The interesting case is $\gamma, \psi > 1$, so that $\theta < 0$ (Barro 2009. Gourio 2008b, Wachter 2009). Then, when disaster probability p_t goes up, the stock price goes down (this is true as long as $F_{it} \leq B_t$, i.e. the asset is riskier than consumption, and comes from (35)), which avoids the counterintuitive opposite prediction in the CRRA model.

In terms of calibration, the model of the aggregate stock market would calibrate as in the CRRA case, provided that the volatility of resilience is as in the CRRA case. For instance, one could hypothesize, as Wachter (2009), that all volatility comes from changes in p_t. The model has just one stochastic factor then. For instance, the risk-free rate and the P/D ratio are perfectly correlated. The formulation via resiliences allows one to have asset-specific shocks as well as economy-wide shocks, and imperfect correlations between assets. In my view, this is a plus.

The case of bonds is identical to the CRRA case under the maintained assumption that $H_{\$t}^{EZ}$ is constant. Relaxing this assumption would be easy, and would generate fluctuations in the real rate independent of the nominal factors, very analogous to the ones developed in Proposition 13.

In one case (with constant p_t and distribution of B_{t+1}), almost no parameter needs to change, except the subjective rate of time preference. This and other calibrations are discussed in the online appendix.

One conclusion is that most of the paper's derivations do not change with EZ preferences, provided one uses an enriched definition of resilience. An EZ model with stochastic probability of disaster is isomorphic to a CRRA model with stochastic recovery rate – increasing the probability of disaster in the EZ model is isomorphic to lowering the recovery rate F_{it} in the CRRA model. The joint impact is simply through resilience. Given this isomorphism, it is perhaps good to keep the CRRA model as a useful benchmark, as it is most tractable, can be solved exactly, and meshes well with traditional macroeconomics.

5.2 Other Interpretation of the Model

Some derivations on stocks and bonds do not depend finely on the disaster hypothesis. On the other hand, for some predictions about "tail assets" (e.g. options, high-grade corporate bonds) the disaster model is crucial. This is formalized in the next Proposition:

Proposition 10 *(Models generating the same stock and government bond prices as a disaster economy, but not the same options and corporate bond prices) Consider a model with stochastic discount factor* $M_{t+1}/M_t = e^{-r_f}\left(1+\varepsilon_{t+1}^M\right)$, *and a stock with dividend following* $D_{i,t+1}/D_{it} = e^{g_{iD}}\left(1+\varepsilon_{i,t+1}^D\right)$, *where all* $\varepsilon_{i,t+1}^H$*'s have expected value 0 at time t. Call* $H_{it} = E_t\left[\varepsilon_{t+1}^M \varepsilon_{i,t+1}^D\right] = H_{i*} + \widehat{H}_{i,t+1}$, *so that* $-H_{it}$ *is the risk premium on the dividend, and assume* $\widehat{H}_{i,t+1} = \frac{1+H_{i*}}{1+H_{it}} e^{-\phi_H}\widehat{H}_{it} + \varepsilon_{i,t+1}^H$ *with* $\varepsilon_{i,t+1}^H$ *uncorrelated with the innovations to* $M_{t+1}D_{i,t+1}/\left(M_t D_{it}\right)$. *Then, Theorem 1 and Proposition 1 hold, except that the equity premium is* $-H_{it}$, *and the interest rate is* r_f.

Furthermore, suppose that inflation is $I_t = I_* + \widehat{I}_t$, *and follows* $\widehat{I}_{t+1} = \frac{1-I_*}{1-I_t}\left(e^{-\phi_I}\widehat{I}_t + \varepsilon_{t+1}^I\right)$. *Call* $E_t\left[\varepsilon_{t+1}^M \varepsilon_{t+1}^I\right] = \pi_* + \pi_t$, *the inflation risk premium, and assume* $\pi_{t+1} = \frac{1-I_*}{1-I_t} e^{-\phi_J}\pi_t + \varepsilon_{t+1}^\pi$, *with* $E_t\left[\varepsilon_{t+1}^M \varepsilon_{t+1}^\pi\right] = 0$. *Also, use the notation* $\pi_* = (1-I_*)\,\kappa\left(1-e^{-\phi_I}-\kappa\right)$. *Then, Theorem 2 on bond values (with* $H_\$ = 0$*), and Propositions 2, 6-8 on bond predictability hold (except Eq. 19).*

However, such a model generically has different prices for options and corporate bonds (which are more tail-sensitive).

Proposition 10 shows that in many models stocks and bonds will behave exactly as in a disaster economy (however, options or defaultable bonds will be different), so that disaster analytics shed light on many models. On the other hand, disaster models make clearly distinctive predictions for tail-sensitive assets such as options and high-grade corporate bonds (and gold, which could be modelled similarly to a stock, but with a very high resilience). Those assets are naturally the object of scrutiny of the growing literature that examines disaster risk empirically, and to which I now turn.

5.3 A Provisional Empirical Assessment

This section provides an assessment of empirical evidence on the link between disasters and asset price movements, such as they have been worked out in the present model.

Are the movements of asset prices correlated to the movements in objective disaster risk?

Political measures of disaster risk.

A question very high on the empirical agenda is to find "objective" measures of disaster risk, ideally that do not come from asset prices. A few papers attempt to do this. Using a database of 447 major international political crises during the period 1918–2006, Berkman, Jacobsen and Lee (2009) show that high war risk leads to a fall in asset prices: returns are low when a crisis starts, and are high when it ends. Other papers measure (on shorter data sets) the impact of the probability of war on asset prices. Bittlingmayer (1998) and Frey and Kucher (2000) finds that political risk was an important factor of volatility between 1880 and World War II. Amihud and Wohl (2004), and Rigobon and Sack (2005) document the link between the probability of the second Iraq War (obtained from prediction markets) and the stock market. All in all, a growing number of studies are documenting a link between political risk and the volatility and level of asset prices, in a way consistent with the disaster hypothesis. A full structural empirical analysis has still to be carried out, probably enriched with new data, but the extant evidence is encouraging.

Disaster risk measured by tail behavior of asset prices.

Alternatively, we may detect disaster risk in asset prices. Bollerslev, Tauchen and Zhou (forth.) show that when put prices are high, future stock returns are low, like in this paper. In addition, the high price of put prices is consistent with disaster risk. Bollerslev and Todorov (2009) find large jumps in options prices that are much harder to detect than in the physical probability. In the currency markets, Farhi et al. (2009) find that when put prices on currencies are high, the return on investing in "risky" currencies (by measure of the price on their puts) yields high returns. Burnside et al. (2009) also calibrate that disaster risk might account for the violations of uncovered interest rate parity. Farhi et al. (2009), find that when a currency falls in value, its put prices increase, with a correlation of -0.4. This is also nicely consistent with the disaster view.

In conclusion, put prices are high, and they predict future returns, as in the disaster hypothesis.

Hence, the evidence, though not systematic, is supportive of the disaster hypothesis.

Finally, a recent paper by Kelly (2009) proposes another way to measure tail risk: every day, he measures the Pareto exponent of the tail of the cross–section of realized returns. He calculates the moving average of this measure. It appears to be a nice proxy for tail risk: it predicts equity returns, and loading on this measure generates a high cross-section of returns. Hence, Kelly's indirect measure may be a promising way to move forward in the measurement of tail risk.

Do high yield assets do particularly poorly during disasters?

Barro and Ursua (2009) find that indeed stocks do particularly poorly during disasters. Farhi et al. (2009) find that high-yield currencies do particularly poorly during currency market crashes, consistent with a rare-event risk premia. A systematic investigation of this issue (including corporate bonds) would be good. Ongoing work with Joachim Voth investigates Russia and Germany around 1917 and investigates Russia and Germany around 1917, and finds that high-yield stocks did do particularly badly during disasters.

Does the variation of disaster risk vary enough, compared to resilience?

In the model, we need a large enough variation of resilience. One indirect test is to compare the volatility of the needed resilience, to the dispersion of actual outcomes in the asset markets. So I define and perform the disaster counterpart of the Shiller (1982) excess volatility test. Consider indeed the asset-to-disaster dispersion ratio for a variable X that pays off during disasters:

$$DR_X \equiv \frac{\text{Dispersion of prediction of } X \text{ from asset markets}}{\text{Dispersion of realized values of } X}$$

It should be less than 1. Indeed, call V_X the standard deviation of the variable X_t. Then $DR_X \equiv \frac{V_{E[X|\mathcal{G}]}}{V_X} \leq 1$ for any information set $\mathcal{G}$.

To evaluate the dispersion of stock resiliences, I consider $X = B_{t+1}^{-\gamma}(1 + r_{i,t+1})$. As the calibration has p_t constant we have $V_H = pV_{E^D\left[B_{t+1}^{-\gamma}(1+r_{i,t+1})\right]}$. As (14) gives $V_{\ln P/D} = V_H / (\delta_i + \phi_H)$, we obtain:

$$DR_{\text{Stocks}} = \frac{\frac{\delta_i + \phi_H}{p} V_{\ln P/D}}{V_{B_{t+1}^{-\gamma}(1+r_{i,t+1})|\text{disaster}}}.$$

To evaluate this dispersion ratio, I use the Barro and Ursua (2009) data, which report series of B_{t+1} and stock market returns during disasters. Note that they use a flexible window, to circumvent a variety of econometric problems, including missing data. I find: $V_{B_{t+1}^{-\gamma}(1+r_{i,t+1})|\text{disaster}} = 5.05$. Using also $\frac{\delta_i+\phi_H}{p} V_{\ln P/D} = \frac{0.18}{0.0363} 0.33 = 1.63$, I obtain a dispersion ratio $DR_{\text{Stocks}} = 0.32$. It is less than 1, so I conclude that the stocks pass the dispersion ratio test. This is a success for the disaster hypothesis. Economically, the test means that the P/D ratio is volatile, but its is less volatile than the dispersion of (marginal-utility adjusted) actual returns of the stocks during disasters.

For inflation, I use the similar reason for the change in inflation during a disaster, $X = \Delta I_t$. As $V_\pi = pV_{B_{t+1}^{-\gamma}\Delta I_{t+1}}$, the dispersion ratio is: $DR_{\text{Inflation}} = V_\pi / \left(pV_{B_{t+1}^{-\gamma}(\Delta I_{t+1})|\text{disaster}} \right)$.

The calibration gave: $V_\pi = 2.1\%$, and the empirical value is $V_{B_{t+1}^{-\gamma}(\Delta I_{t+1})|\text{disaster}} = 6.36$, So $DR_{\text{Inflation}} = 0.09$. The dispersion ratio is less than 1, consistent with the disaster hypothesis.

I conclude that the rare disaster model passes the dispersion ratio test. There is enough dispersion in the realized outcomes during disasters to warrant the volatility of prices in samples without disasters.

6 Conclusion

This paper presents a tractable way to handle a time-varying severity of rare disasters, demonstrates its impact on stock and bond prices, and shows its implications for time-varying risk premia and asset predictability. Many finance puzzles can be understood through the lens of the variable rare disasters model. On the other hand, the model does suffer from several limitations and suggests several questions for future research.

First of all, it would be useful to empirically examine the model's joint expression of the values of stocks, bonds and options. In this paper, I have only examined their behavior separately, relying on robust stylized facts from many decades of research. The present study suggests specifications for the joined, cross-asset patterns of predictability.

It would be useful to understand how investors update their estimates of resiliences. Risk premia seem to decrease after good news for the economy (Campbell and Cochrane 1999) and for individual firms (the growth firms effect). So, it seems that updating will involve resiliences increasing after good news about the fundamental values of the economy or about individual stocks. Modeling that would lead to a link between recent events, risk premia, and future predictability. Preliminary notes suggest that this modelling is easy, given the analytics put forth in this paper.

This model is a step toward a unified framework for various puzzles in economics and finance. A companion paper (Farhi and Gabaix 2009) suggests that various puzzles in international macroeconomics (including the forward premium puzzle and the excess volatility puzzle on exchange rates) can be accounted for in an international version of the variable rare disasters framework. Furthermore, ongoing work (Gabaix 2009b, Gourio 2009) shows how to embed the rare disasters idea in a production economy in a way that does not change its business cycle properties, but changes its asset pricing properties, which lead to good empirical results. Thus, variable rare disaster modelling may bring us closer to the long-sought goal of a joint, tractable framework for macroeconomics and finance.

Appendix

6.1 Notations

The paper often uses a decomposition of a generic variable X_t as follows: $X_t = X_* + \widehat{X}_t$, where X_* is a constant part (or "typical value") and $\widehat{X}_t$ a variable part centered around 0. ε^X_{t+1} is an innovation to X, and σ_X, its standard deviation, is the volatility of variable X_t. The other notations are as follows.

B_{t+1} : recovery rate of consumption in a disaster

$\overline{B} = E\left[B_{t+1}^{-\gamma}\right]^{-1/\gamma}$: risk-adjusted average B.

β_T : slope in a predictive regression with horizon T

D_{it} : dividend of stock i

δ : "Ramsey" discount rate

δ_i : stock i's effective discount rate

$E_t^D\left[X_{t+1}\right]$ (resp. $E_t^{ND}\left[X_{t+1}\right]$): expected value conditional on a disaster (resp. on no disaster) at $t+1$.

$f_t\left(T\right)$: nominal forward rate of maturity T

$F_{\$t}$: recovery rate of a nominal dollar

F_{it} : recovery rate of stock i

g_C : growth rate of consumption

g_{iD} : growth rate of stock i's dividend

γ : coefficient of relative risk aversion

H_{it} : resilience of stock i

H_{it}^{EZ} : Epstein-Zin-enriched resilience of stock i

$H_{\$}$: resilience of a nominal dollar

I_t : inflation

I_{**} : risk-adjusted central part of inflation

J_t : jump in inflation in a disaster

κ : inflation disaster risk premium

K_T : loading on bond risk premium.

M_t : pricing kernel

μ_{it} : expected growth of the stock price, conditional on no disasters (only used for options)

Ω : generator of a LG process

p_t : probability of disaster

P_{it} : price of stock i

π_t : variable part of the bond risk premium

ϕ_X : rate of mean-reversion of variable X_t

ψ : intertemporal elasticity of substitution

ψ_X : rate of mean-reversion of variable X_t under the risk-adjusted measure

Q_t : real value of one unit of one nominal dollar

r_f : risk-free rate

r_{it}^e : stock i's expected stock return, over a one-period horizon

$r_{it \to t+T}^e$: stock i's expected stock return, over a horizon T

$r_{\$t}^e (T)$: expected return on a nominal bond of maturity T

ρ : subjective rate of time preference

t : calendar time

T : maturity (for a bond) or horizon (for a regression)

V_X : dispersion (standard deviation of the distribution) of variable X_t

y_i : yield on the debt of corporation y_i

$y_t (T)$: nominal yield of maturity T

$Z_{\$t} (T)$: price of a nominal zero-coupon bond of maturity T.

6.2 Proofs

Proof of Theorem 1 Following the general procedure for LG processes, I use (2), (3) and form:

$$\frac{M_{t+1}D_{i,t+1}}{M_t D_{it}} = e^{-\delta + g_{iD}} \left(1 + \varepsilon_{t+1}^D\right) \times \begin{cases} 1 & \text{if there is no disaster at } t+1 \\ B_{t+1}^{-\gamma} F_{i,t+1} & \text{if there is a disaster at } t+1 \end{cases}$$

As the probability of disaster at $t+1$ is p_t, and $H_{it} \equiv p_t \left(E_t \left[B_{t+1}^{-\gamma} F_{i,t+1}\right] - 1\right)$,

$$\begin{aligned} E_t \left[\frac{M_{t+1}D_{i,t+1}}{M_t D_{it}}\right] &= e^{-\delta + g_{iD}} \{ \underbrace{(1 - p_t) \cdot 1}_{\text{No disaster term}} + \underbrace{p_t \cdot E_t \left[B_{t+1}^{-\gamma} F_{i,t+1}\right]}_{\text{Disaster term}} \} \\ &= e^{-\delta + g_{iD}} (1 + H_{it}) = e^{-\delta + g_{iD}} \left(1 + H_{i*} + \widehat{H}_{it}\right) = e^{-\delta + g_{iD} + h_{i*}} \left(1 + e^{-h_{i*}} \widehat{H}_{it}\right) \\ &= e^{-\delta_i} \left(1 + e^{-h_{i*}} \widehat{H}_{it}\right) \end{aligned} \quad (37)$$

where I use the notations $h_{i*} = \ln (1 + H_{i*})$ and $\delta_i = \delta - g_{iD} - h_{i*}$. Next, as $\widehat{H}_{i,t+1}$ is independent of whether there is a disaster, and is uncorrelated with ε_{t+1}^D,

$$\begin{aligned} E_t \left[\frac{M_{t+1}D_{i,t+1}}{M_t D_{it}} \widehat{H}_{i,t+1}\right] &= E_t \left[\frac{M_{t+1}D_{i,t+1}}{M_t D_{it}}\right] E_t \left[\widehat{H}_{i,t+1}\right] = e^{-\delta + g_{iD}} (1 + H_{it}) \cdot \frac{1 + H_{i*}}{1 + H_{it}} e^{-\phi_H} \widehat{H}_{it} \\ &= e^{-\delta + g_{iD} + h_{i*} - \phi_H} \widehat{H}_{it} = e^{-\delta_i - \phi_H} \widehat{H}_{it} \end{aligned} \quad (38)$$

We see that in (5), the reason for the $1+H_{it}$ term in the denominator was to ensure that the above expression would remain linear in $\widehat{H}_{it}$.

There are two ways to conclude. The first way uses the results from LG processes: Eq. 37 and 38 ensure that $M_t D_{it}\left(1, \widehat{H}_{it}\right)$ is a LG process with generator $\begin{pmatrix} e^{-\delta_i} & e^{-\delta_i - h_{i*}} \\ 0 & e^{-\delta_i-\phi_H} \end{pmatrix}$. Eq. 57 gives the stock price (13). The second way (which is less rigorous, but does not require a knowledge of the results on LG processes) is to look for a solution of the type $P_{it} = D_{it}\left(a + b\widehat{H}_{it}\right)$ for some constants a and b. The price must satisfy: $P_{it} = D_{it} + E\left[M_{t+1}P_{i,t+1}/M_t\right]$, i.e., for all $\widehat{H}_{it}$,

$$
\begin{aligned}
a + b\widehat{H}_{it} &= 1 + E_t\left[\frac{M_{t+1}D_{i,t+1}}{M_t D_{it}}\left(a + b\widehat{H}_{i,t+1}\right)\right] = 1 + aE_t\left[\frac{M_{t+1}D_{i,t+1}}{M_t D_{it}}\right] + bE_t\left[\frac{M_{t+1}D_{i,t+1}}{M_t D_{it}}\widehat{H}_{i,t+1}\right] \\
&= 1 + ae^{-\delta_i}\left(1 + e^{-h_{i*}}\widehat{H}_{it}\right) + be^{-\delta_i - \phi}\widehat{H}_{it} = \left(1 + ae^{-\delta_i}\right) + \left(ae^{-\delta_i - h_{i*}} + be^{-\delta_i - \phi_H}\right)\widehat{H}_{it}.
\end{aligned}
$$

Solving for a and b, we get $a = 1 + ae^{-\delta_i}$, $b = ae^{-\delta_i - h_{i*}} + be^{-\delta_i-\phi_H}$, and (13).

Proof of Theorem 2 The proof is simpler when $J_* = \kappa = 0$ and this is the best case to keep in mind in a first reading. I call $\rho_I = e^{-\phi_I}$ and $\rho_J = e^{-\phi_J}$, use the inflation-adjusted (i.e., real) face value of the bond, Q_t:

$$
\frac{Q_{t+1}}{Q_t} = (1 - I_t) \times \begin{cases} 1 & \text{if there is no disaster at } t+1 \\ F_{\$,t+1} & \text{if there is a disaster at } t+1 \end{cases}
$$

and calculate the LG moments.

$$
E_t\left[\frac{M_{t+1}Q_{t+1}}{M_t Q_t}\right] = e^{-\delta}(1 - I_t)\left\{(1-p_t)\cdot 1 + p_t \cdot E_t\left[B_{t+1}^{-\gamma}F_{\$,t+1}\right]\right\} = e^{-\delta}(1 + H_\$)\left(1 - I_* - \widehat{I}_t\right).
$$

$$
\begin{aligned}
E_t\left[\frac{M_{t+1}Q_{t+1}}{M_t Q_t}\widehat{I}_{t+1}\right] &= e^{-\delta}(1-I_t)\left\{(1-p_t)E_t^{ND}\left[\widehat{I}_{t+1}\right] + p_t\cdot E_t^{D}\left[B_{t+1}^{-\gamma}F_{\$,t+1}\widehat{I}_{t+1}\right]\right\} \\
&= e^{-\delta}(1-I_t)\frac{1-I_*}{1-I_t}\left\{\left(1 - p_t + p_t E_t\left[B_{t+1}^{-\gamma}F_{\$,t+1}\right]\right)\rho_I \widehat{I}_t + p_t E_t\left[B_{t+1}^{-\gamma}F_{\$,t+1}\right]\left(J_* + \widehat{J}_t\right)\right\} \\
&= e^{-\delta}(1+H_\$)(1-I_*)\left(\rho_I \widehat{I}_t + \frac{p_t E_t\left[B_{t+1}^{-\gamma}F_{\$,t+1}\right]}{1+H_\$}\left(J_* + \widehat{J}_t\right)\right) \\
&= \Psi\left(\rho_I \widehat{I}_t + (1-I_*)\kappa(1-\rho_I-\kappa) + \pi_t\right), \qquad \Psi \equiv e^{-\delta}(1+H_\$)(1-I_*)
\end{aligned}
$$

using (8) and (9). This gives:

$$\begin{aligned} E_t\left[\frac{M_{t+1}Q_{t+1}}{M_tQ_t}\frac{\widehat{I}_{t+1}}{1-I_*}\right] &= \Psi\left(\kappa\left(1-\rho_I-\kappa\right)+\rho_I\frac{\widehat{I}_t}{1-I_*}+\frac{\pi_t}{1-I_*}\right). \\ E_t\left[\frac{M_{t+1}Q_{t+1}}{M_tQ_t}\widehat{J}_{t+1}\right] &= E_t\left[\frac{M_{t+1}Q_{t+1}}{M_tQ_t}\right]E_t\left[\widehat{J}_{t+1}\right]=\Psi\frac{1-I_*}{1-I_t}\rho_J\widehat{J}_{t+1}=\Psi\rho_J\widehat{J}_{t+1} \end{aligned}$$

so that, as $\pi_t/\left(1-I_*\right)$ is proportional to $\widehat{J}_t$ (Eq. 8),

$$E_t\left[\frac{M_{t+1}Q_{t+1}}{M_tQ_t}\frac{\pi_{t+1}}{1-I_*}\right]=\Psi\rho_J\frac{\pi_{t+1}}{1-I_*}.$$

Hence $M_tQ_t\left(1,\frac{\widehat{I}_t}{1-I_*},\frac{\pi_t}{1-I_*}\right)$ is a LG process, with generator $\Omega=\Psi\begin{pmatrix}1 & -1 & 0\\ \kappa\left(1-\rho_I-\kappa\right) & \rho_I & 1\\ 0 & 0 & \rho_\pi\end{pmatrix}$.

Eq. 58 gives the bond price, $Z_{\$t}\left(T\right)=\left(1,0,0\right)\Omega^T\left(1,\frac{\widehat{I}_t}{1-I_*},\frac{\pi_t}{1-I_*}\right)'$, which concludes the derivation of (16) when $\kappa=0$. When $\kappa\neq0$, one more step is needed. The eigenvalues of Ω are $\Psi\left\{1-\kappa,\rho_I+\kappa,\rho_\pi\right\}$. It is convenient to factorize by $1-\kappa$, hence to define: $\widetilde{\rho}_i=\left(\rho_I+\kappa\right)/\left(1-\kappa\right)$ and $\widetilde{\rho}_\pi=\rho_\pi/\left(1-\kappa\right)$, which are the discrete time analogues of the continuous time mean reversion speeds $\psi_I\equiv\phi_I-2\kappa$ and $\psi_J\equiv\phi_J-\kappa$. Calculating Ω^T (by hand or via *Mathematica*) gives the bond price:

$$Z_{\$t}\left(T\right)=\left(\Psi\left(1-\kappa\right)\right)^T\times\{1-\frac{1}{1-\kappa}\frac{1-\widetilde{\rho}_i^T}{1-\widetilde{\rho}_i}\left(\frac{\widehat{I}_t}{1-I_*}-\kappa\right)-\frac{1}{\left(1-\kappa\right)^2}\frac{\frac{1-\widetilde{\rho}_i^T}{1-\widetilde{\rho}_i}-\frac{1-\widetilde{\rho}_\pi^T}{1-\widetilde{\rho}_\pi}}{\widetilde{\rho}_i-\widetilde{\rho}_\pi}\frac{\pi_t}{1-I_*}\} \quad (39)$$

Taking the continuous time limit yields (16). The corresponding value of the yield $y_t\left(T\right)=-\left(\ln Z_{\$t}\left(T\right)\right)/T$ is:

$$\begin{aligned} y_t\left(T\right) &= \delta-H_\$+I_{**}+\frac{1-e^{-\psi_IT}}{\psi_IT}\left(I_t-I_{**}\right)+\frac{\frac{1-e^{-\psi_IT}}{\psi_I}-\frac{1-e^{-\psi_JT}}{\psi_I}}{\left(\psi_J-\psi_I\right)T}\pi_t+O\left(I_t-I_{**},\pi_t\right)^2 \quad (40)\\ &= \delta-H_\$+I_{**}+\left(1-\frac{\psi_IT}{2}+\frac{\psi_IT^2}{6}\right)\left(I_t-I_{**}\right)+\left(\frac{T}{2}-\frac{\psi_I+\psi_J}{6}T^2\right)\pi_t \\ &\quad +O\left(T^3\right)+O\left(I_t-I_{**},\pi_t\right)^2 \\ &= \delta-H_\$+I_t+\left(\kappa\left(\phi_I-\kappa\right)+\pi_t-\phi_I\left(I_t-I_*\right)\right)\frac{T}{2}. \quad (41) \end{aligned}$$

Proof of Proposition 3 We have $V_t = V_t^{ND} + V_t^D$ with:

$$\begin{aligned} V_t^{ND} &= (1-p_t)\, E_t^{ND}[e^{-\delta}(K - \frac{P_{i,t+1}}{P_{it}})^+] = (1-p_t)\, e^{-\delta} E_t[(K - e^{\mu+\sigma u_{t+1}-\sigma^2/2})^+] \\ V_t^D &= p_t E_t^D[e^{-\delta}(B_{t+1}^{-\gamma} K - \frac{P_{i,t+1}}{P_{it}})^+] = p_t e^{-\delta} E_t\left[B_{t+1}^{-\gamma}\,(K - e^{\mu} F_{i,t+1})^+\right] \end{aligned}$$

where $x^+ = \max(0,x)$. Recall that the Black-Scholes value of a put with maturity 1 is: $E_t[e^{-r}(K - e^{r+\sigma u_{t+1}-\sigma^2/2})^+] = V_{Put}^{BS}(Ke^{-r},\sigma)$. Hence, the first term is $(1-p_t)$ times:

$$e^{-\delta} E_t[(K - e^{\mu+\sigma u_{t+1}-\sigma^2/2})^+] = e^{-\delta+\mu} E_t[(Ke^{-\mu} - e^{\mu+\sigma u_{t+1}-\sigma^2/2})^+] = e^{-\delta+\mu} V_{Put}^{BS}\left(Ke^{-\mu},\sigma\right).$$

Proof of Proposition 5 Proposition 1 gives the expected returns over a short horizon T to be $r^e_{it\to t+T} = (\delta - H_{it})\,T$. Eq. 14 implies that the right-hand side of (27) is to the leading order $\ln(D/P)_t = -\ln\delta_i - \widehat{H}_{it}/(\delta_i + \phi_H)$. So the regression is to a first order, $r^e_{it\to t+T} = \left(\delta - H_{i*} - \widehat{H}_{it}\right) T = \alpha_T - \beta_T \frac{\widehat{H}_{it}}{\delta_i+\phi_H}$. Equating the $\widehat{H}_{it}$ terms, $\beta_T = (\delta_i + \phi_H)\,T$. The same reasoning gives β'_T.

Proof of Proposition 6 The Fama-Bliss regression (29) yields $\beta_T = \frac{cov\left(r^e_{\$t}(T) - r^e_{\$t}(0),\, f_t(T) - f_t(0)\right)}{var(f_t(T) - f_t(0))}$. Eq. 17 and 21 give $r^e_{\$t}(T) - r^e_{\$t}(0) = \frac{1-e^{-\psi_I T}}{\psi_I}\pi_t + O(I_t,\pi_t)^2$ and

$$f_t(T) - f_t(0) = \left(e^{-\psi_I T} - 1\right) I_t + \frac{e^{-\psi_I T} - e^{-\psi_J T}}{\psi_J - \psi_I}\pi_t + a_T + O(I_t,\pi_t)^2$$

where a_T is a constant. So to the leading order,

$$\beta_T = \frac{\frac{e^{-\psi_I T} - e^{-\psi_J T}}{\psi_J - \psi_I}\frac{1-e^{-\psi_I T}}{\psi_I} var(\pi_t)}{var\left(\left(e^{-\psi_I T} - 1\right) I_t + \frac{e^{-\psi_I T} - e^{-\psi_J T}}{\psi_J - \psi_I}\pi_t\right)}, \tag{42}$$

which implies that: $\lim_{T\to\infty}\beta_T = 0$, $\lim_{T\to 0}\beta_T = var(\pi_t)/var(\psi_I I_t + \pi_t)$, and (30).

Proof of Proposition 7 This proof is in the limit of $\sigma_I \to 0$, $I_t = 0$, $\kappa \to 0$, and $\Delta t \to 0$. Eq. 40 gives: $y_t(T) = a + b(T)\,\pi_t$, with $b(T) = \frac{\frac{1-e^{-\psi_I T}}{\psi_I} - \frac{1-e^{-\psi_J T}}{\psi_I}}{(\psi_J - \psi_I)T} = \frac{T}{2} - \frac{\psi_I + \psi_J}{6} T^2 + O(T^3)$. Hence:

$$\frac{y_{t+\Delta t}(T - \Delta t) - y_t(T)}{\Delta t} = E_t\left[dy_t(T)\right]/dt - \partial y_t(T)/\partial T = \left(-\phi_J b(T) - b'(T)\right)\pi_t$$

As $(y_t(T) - r_t)/T = b(T)\pi_t/T$, $-\beta = \frac{\phi_J b(T)+b'(T)}{b(T)/T}$, i.e.

$$\beta = \frac{-Tb'(T)}{b(T)} - \phi_J T, \tag{43}$$

so that $\beta = -1 - \frac{2\psi_j - \psi_I}{3}T + O(T^2)$ when $T \to 0$ and $\beta = -\psi_J T + o(T)$ when $T \to \infty$. The reasoning in the text of the paper comes from the fact that, for small T, $E_t[dy_t(T)]/dt = -\frac{\phi_J T}{2}\pi_t$, $-\partial y_t(T)/\partial T = \left(-\frac{1}{2} + O(T)\right)\pi_t$, so $\frac{y_{t+\Delta t}(T-\Delta t) - y_t(T)}{\Delta t} \simeq -\frac{\partial y_t(T)}{\partial T}$.

Proof of Proposition 10 With $Q_{t+1}/Q_t = 1 - I_t$, $M_t D_{it}\left(1, \widehat{H}_{it}\right)$ and $M_t Q_t\left(1, \frac{\widehat{I}_t}{1-I_*}, \frac{\pi_t}{1-I_*}\right)$ are both LG processes, with the same moments as in the disaster economy.

References

Almeida, Heitor, and Thomas Philippon, "The Risk-Adjusted Cost of Financial Distress," *Journal of Finance*, 62 (2007), 2557-2586.

Amihud, Yakov and Avi Wohl "Political News and Stock Prices: The Case of Saddam Hussein Contracts," *Journal of Banking and Finance,* 28 (2004), 1185-1200.

Backus, David, Mikhail Chernov and Ian Martin, "Disasters Implied by Equity Index Options," Working Paper, NYU, 2009.

Bansal, Ravi, and Amir Yaron, "Risks for the Long Run: A Potential Resolution of Asset Pricing Puzzles," *Journal of Finance*, 59 (2004), 1481-1509.

Bansal, Ravi, and Ivan Shaliastovich, "A Long-Run Risks Explanation of Predictability Puzzles in Bond and Currency Markets," Paper, Duke University, 2009.

Barro, Robert, "Rare Disasters, Asset Prices, and Welfare Costs," *American Economic Review* 99 (2009), 243–64.

Barro, Robert, "Rare Disasters and Asset Markets in the Twentieth Century," *Quarterly Journal of Economics*, 121 (2006), 823-866.

Barro, Robert, and José Ursua, "Macroeconomic Crises Since 1870," *Brookings Papers on Economic Activity* (2008), 255-335.

Barro, Robert, and José Ursua, "Stock Market Crashes and Depressions," NBER Working Paper 14760, 2009.

Barro, Robert, Emi Nakamura, Jón Steinsson, and José Ursua, "Crises and Recoveries in an Empirical Model of Consumption Disasters," Working Paper, Harvard, 2009.

Berkman, Henk, Ben Jacobsen and John Lee, "Time-Varying Rare Disaster Risk and Stock Market Returns" Working Paper, University of Auckland, 2009.

Binsbergen, Jules van, Michael Brandt and Ralph Koijen (2009): "On the Timing and Pricing of Cash Flows" Working Paper, Stanford University.

Bittlingmayer, George, "Output, Stock Volatility, and Political Uncertainy in a Natural Experiment: Germany 1880-1940," *Journal of Finance*, 53 (1998), 2243-2256.

Boldrin, Michele, Lawrence J. Christiano, and Jonas D. M. Fisher, "Habit Persistence, Asset Returns, and the Business Cycle," *American Economic Review*, 91 (2001), 149-166.

Bollerslev, Tim, George Tauchen, and Hao Zhou, "Expected Stock Returns and Variance Risk Premia," *Review of Financial Studies*, forth.

Brandt, Michael W., and Kevin Q. Wang, "Time-Varying Risk Aversion and Unexpected Inflation," *Journal of Monetary Economics*, 50 (2003), 1457-1498.

Brown, Stephen, William Goetzmann, and Stephen Ross, "Survival," *Journal of Finance*, 53 (1995), 853-73.

Buraschi, Andrea, and Alexei Jiltsov, "Term Structure of Interest Rates Implications of Habit Persistence," *Journal of Finance*, 62 (2007), 3009-63.

Campbell, John Y., 2003. "Consumption-Based Asset Pricing," in: G.M. Constantinides, M. Harris and R. M. Stulz (ed.), *Handbook of the Economics of Finance*, 1B (Amsterdam: Elsevier, North Holland, 2003), 803-887.

Campbell, John Y., and John Cochrane, "By Force of Habit: A Consumption-Based Explanation of Aggregate Stock Market Behavior," *Journal of Political Economy*, 107 (1999), 205-251.

Campbell, John Y., and Robert J. Shiller, "The Dividend-Price Ratio and Expectations of Future Dividends and Discount Factors," *Review of Financial Studies*, 1 (1988), 195-228.

Campbell, John Y., and Robert J. Shiller, "Yield Spreads and Interest Rate Movements: A Bird's Eye View," *Review of Economic Studies*, 58 (1991), 495-514.

Catao, Luis and Marco Terrones, "Fiscal Deficits and Inflation," *Journal of Monetary Economics*, 52 (2005), 529-554

Chen, Long, Pierre Collin-Dufresne , and Robert S. Goldstein, "On the Relation Between the Credit Spread Puzzle and the Equity Premium Puzzle" *Review of Financial Studies*, 22 (2009), 3367-3409.

Cochrane, John, "The Dog That Did Not Bark: A Defense of Return Predictability," *Review of Financial Studies*, 21 (2008), 1533-1575.

Cochrane, John, and Monika Piazzesi, "Bond Risk Premia," *American Economic Review*, 95 (2005), 138-160.

Cremers, K.J. Martijn, Joost Driessen , and Pascal Maenhout, "Explaining the Level of Credit Spreads: Option-Implied Jump Risk Premia in a Firm Value Model" *Review of Financial Studies* 21 (2008), 2209-2242

Dai, Qiang, and Kenneth J. Singleton, "Expectation Puzzles, Time-Varying Risk Premia, and Affine Models of the Term Structure," *Journal of Financial Economics*, 63 (2002), 415-441.

Daniel, Kent, and Sheridan Titman, "Evidence on the Characteristics of Cross Sectional Variation in Stock Returns," *Journal of Finance*, 52 (1997), 1-33.

Davis, James L., Eugene F. Fama, and Kenneth R. French, "Characteristics, Covariances, and Average Returns: 1929 to 1997," *Journal of Finance*, 55 (2000), 389-406.

Drechsler, Itamar, and Amir Yaron, "What's Vol GotTo Do With It," Working Paper, Wharton, 2009.

Du, Du, "General Equilibrium Pricing of options with habit formation and event risks" *Journal of Financial Economics*, forthcoming.

Duffee, Gregory, "Term Premia and Interest Rate Forecasts in Affine Models," *Journal of Finance*, 57 (2002), 405-443.

Dybvig, Philip H., Jonathan E. Ingersoll, Jr., and Stephen A. Ross, "Long Forward and Zero-Coupon Rates Can Never Fall," *Journal of Business*, 69 (1996), 1-25.

Epstein, Larry G. and Stanley E. Zin "Substitution, Risk Aversion, and the Temporal Behavior of Consumption and Asset Returns: A Theoretical Framework," *Econometrica*, 57 (1989), 937-969.

Fama, Eugene F., "The Behavior of Interest Rates," *Review of Financial Studies*, 19 (2006), 359-379.

Fama, Eugene F., and Robert R. Bliss, "The Information in Long-Maturity Forward Rates," *American Economic Review*, 77 (1987), 680-92.

Fama, Eugene F., and Kenneth R. French, "Multifactor Explanations of Asset Pricing Anomalies," *The Journal of Finance*, 51 (1996), 55-84.

Fama, Eugene F., and Kenneth R. French, "Common Risk Factors in the Returns on Stocks and Bonds," *Journal of Financial Economics*, 33 (1993), 3-56.

Farhi, Emmanuel, and Xavier Gabaix, "Rare Disasters and Exchange Rates," Working Paper, Harvard, 2009.

Farhi, Emmanuel, Samuel Fraiberger, Xavier Gabaix, Romain Ranciere, and Adrien Verdelhan, "Crash Risk in Currency Markets," NBER Working Paper 15062, 2009.

Figlewski, Stephen, "Estimating the Implied Risk Neutral Density for the U.S. Market Portfolio," in *Volatility and Time Series Econometrics: Essays in Honor of Robert F. Engle* (eds. Tim Bollerslev, Jeffrey R. Russell and Mark Watson). Oxford, UK: Oxford University Press, 2008.

Frey, Bruno and Marcel Kucher, "History as Reflected in Capital Markets: The Case of World War II," *Journal of Economic History*, 60 (2000), 468-96.

Gabaix, Xavier, "Linearity-Generating Processes: A Modelling Tool Yielding Closed Forms for Asset Prices," Paper, NYU, 2009a.

Gabaix, Xavier, "Disasterization: A Simple Way to Fix the Asset Pricing Properties of Macroeconomic Models" Paper, NYU, 2009b.

Gabaix, Xavier "Variable Rare Disasters: A Tractable Theory of Ten Puzzles in Macro-Finance," *American Economic Review Papers and Proceedings*, 98 (2008), 64-67.

Ghandi, Pryiank and Hanno Lustig, "Size Anomalies in US Bank Stock Returns: Your Tax Dollars at Work?" Working Paper, UCLA, 2009.

Ghosh, Anisha, and Christian Julliard, "Can Rare Events Explain the Equity Premium Puzzle?," Working Paper, Carnegie-Mellon University, 2008.

Gourio, Francois, "Disasters and Recoveries," *American Economic Review, Papers and Proceedings*, 98 (2008a), 68-73.

Gourio, Francois, "Time Series Predictability in the Disaster Model," *Finance Research Letters*, 5 (2008b), 191-203.

Gourio, Francois, "Disaster Risk and Business Cycles," Working Paper, Boston University, 2009.

Goyal, Amit, and Ivo Welch, "A Comprehensive Look at the Empirical Performance of Equity Premium Prediction," *Review of Financial Studies*, 21 (2008), 1455-1508.

Huang, Ming and Jing-Zhi Huang, "How Much of the Corporate-Treasury Yield Spread is Due to Credit Risk?," Working Paper, Stanford University, 2003.

Jackwerth, Jens Carsten and Mark Rubinstein, "Recovering Probability Distributions from Option Prices," *Journal of Finance*, 51 (1996), 1611-1631.

Jermann, Urban, "Asset Pricing in Production Economies," *Journal of Monetary Economics*, 41 (1998), 257-275.

Kelly, Bryan, "Risk Premia and the Conditional Tails of Stock Returns," Working Paper, NYU

Krishnamurthy, Arvind, and Annette Vissing-Jorgensen, "The Demand for Treasury Debt," Working Paper, Northwestern, 2008.

Lettau, Martin, and Stijn van Nieuwerburgh, "Reconciling the Return Predictability Evidence," *Review of Financial Studies*, 21 (2008), 1607-1652.

Lettau, Martin, and Jessica Wachter, "The Term Structures of Equity and Interest Rates," Working Paper, Wharton (2007).

Liu, Jun, Jun Pan and Tan Wang, "An Equilibrium Model of Rare Event Premia," *Review of Financial Studies*, 18 (2005), 131-164.

Longstaff, Francis, and Monika Piazzesi, "Corporate Earnings and the Equity Premium," *Journal of Financial Economics*, 74 (2004), 401-421.

Lustig, Hanno, Stijn van Nieuwerburgh, and Adrien Verdelhan, "The Wealth-Consumption Ratio" Working Paper, NYU, 2008.

Macaulay, Frederick. *Some Theoretical Problems Suggested by the Movement of Interest Rates, Bond Yields And Stock Prices in the United States Since 1856*, NBER, New York 1938.

Mehra, Rajnish, and Edward Prescott, "The Equity Premium: A Puzzle," *Journal of Monetary Economics*, 15 (1985),145–161.

Piazzesi, Monika, and Martin Schneider, "Equilibrium Yield Curves," *NBER Macroeconomics Annual*, 21 (2007), 389-442.

Rietz, Thomas, "The Equity Risk Premium: A Solution," *Journal of Monetary Economics*, 22 (1988), 117-131.

Rigobon, Roberto and Brian Sack, "The Effects of War Risk on US Financial Markets," *Journal of Banking and Finance,* 29 (2005), 1769-1789.

Santa-Clara, Pedro and Shu Yan, "Crashes, Volatility, and the Equity Premium: Lessons from S&P 500 Options," *Review of Economics and Statistics*, forth.

Shiller, Robert, "Do Stock Prices Move Too Much to Be Justified by Subsequent Changes in Dividends?" *American Economic Review*, 71 (1981), 421-436.

Uhlig, Harald, "Explaining Asset Prices with External Habits and Wage Rigidities in a DSGE Model," *American Economic Review, Papers and Proceedings*, 97 (2007), 239-43.

Veronesi, Pietro, "The Peso Problem Hypothesis and Stock Market Returns," *Journal of Economic Dynamics and Control*, 28 (2004), 707-725.

Wachter, Jessica, "A Consumption-Based Model of the Term Structure of Interest Rates," *Journal of Financial Economics*, 79 (2006), 365-399.

Wachter, Jessica, "Can Time-Varying Risk of Rare Disasters Explain Aggregate Stock Market Volatility?," NBER Working paper 14386, 2009.

Weil, Philippe, "The Equity Premium Puzzle and the Risk-Free Rate Puzzle," *Journal of Monetary Economics*, 24 (1990), 401-21.

Weil, Philippe, "Nonexpected Utility in Macroeconomics," *Quarterly Journal of Economics*, 105 (1990), 29-42.

Weitzman, Martin, "Subjective Expectations and Asset-Return Puzzles," *American Economic Review*, 97 (2007), 1102-30.