AN ESSAY IN DYNAMIC THEORY

1. THE following pages constitute a tentative and preliminary attempt to give the outline of a " dynamic " theory. Static theory consists of a classification of terms with a view to systematic thinking, together with the extraction of such knowledge about the adjustments due to a change of circumstances as is yielded by the " laws of supply and demand." It has for some time appeared to me that it ought to be possible to develop a similar classification and system of axioms to meet the situation in which certain forces are operating steadily to increase or decrease certain magnitudes in the system. The consequent " theory " would not profess to determine the course of events in detail, but should provide a framework of concepts relevant to the study of change analogous to that provided by static theory for the study of rest.

The axiomatic basis of the theory which I propose to develop consists of three propositions—namely, (1) that the level of a community's income is the most important determinant of its supply of saving; (2) that the rate of increase of its income is an important determinant of its demand for saving, and (3) that demand is equal to supply. It thus consists in a marriage of the " acceleration principle " and the " multiplier " theory, and is a development and extension of certain arguments advanced in my *Essay on the Trade Cycle*.[1]

2. Attempts to construct a dynamic theory have recently been proceeding upon another line—namely, by the study of time lags between certain adjustments. By the introduction of an appropriate lag the tendency of a system to oscillate can be established. In these studies there is some doubt as to the nature of the trend on which the oscillation is superimposed. Supposing

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[1] Especially in Ch. 2, secs. 4–5. The " Acceleration Principle " was there designated the " Relation." There is an objection to the use of the term acceleration in this connection. The study of the condition in which demand and supply are flowing at an unaltered rate has long been known as Static Theory : this implies that the equilibrium of prices and quantities resulting therefrom is regarded as analogous to a state of rest. By analogy, therefore, a steady rate of increase of demand, which is our first matter for consideration in dynamic theory, and a major effect of which is expressed by the " Relation," should be regarded as a velocity. Acceleration would be a rate of change in this.

However, the use of the expression Acceleration Principle in the sense of my relation is rapidly accelerating in current literature, and I reluctantly bow to the *force majeure* of usage.

damping measures could be introduced, to counteract the oscilla-
tion caused by the lag, would the system be stationary or advanc-
ing ?   And at what rate ?   Dynamic theory in my sense may
throw some light upon this.

Moreover it is possible, and this the following argument seeks
to establish, that the trend of growth may itself generate forces
making for oscillation.   This, if so, would not impair the im-
portance of the study of the effect of lags.   But it may be that
the attempt to explain the trade cycle by *exclusive* reference to
them is an unnecessary *tour de force*.   The study of the operation
of the forces maintaining a trend of increase and the study of
lags should go together.

3. The significance of what follows should not be judged solely
by reference to the validity or convenience of the particular
equations set forth.   It involves something wider : a method of
thinking, a way of approach to certain problems.   It is necessary
to " think dynamically."   The static system of equations is set
forth not only for its own beauty, but also to enable the economist
to train his mind upon special problems when they arise.   For
instance, an economist may pose to himself the question, What
would be the effect on the system of an increase of exports or of a
labour-saving invention ?   By reference to the static equations,
he then proceeds to work out the new equilibrium position sup-
posing the new higher level of exports to be maintained in
perpetuity or the labour-saving invention to be incorporated in the
productive technique once for all.

But let the question be : Suppose the level of exports begins
and continues to increase steadily, or suppose its rate of increase
to increase, or suppose labour-saving inventions begin to be made
in a steady or growing stream ; then the static method will not
suffice.   The static theorist may hope to reduce this supposed
steady increase to a succession of steps up, each having the same
effect.   But if the following argument is correct, the effect on the
moving equilibrium of advance may often be in the opposite
direction to the effect on the static equilibrium produced by each
of the steps considered singly.   A new method of approach—
indeed, a mental revolution—is needed.

Once the mind is accustomed to thinking in terms of trends of
increase, the old static formulation of problems seems stale, flat
and unprofitable.   This is not to deny to static theory its own
appropriate sphere.   It will become apparent which kind of prob-
lem belongs to each branch of study.

4. I now propose to proceed directly to the Fundamental

Equation, constituting the marriage of the acceleration principle and the multiplier theory. This probably gives too much importance to the acceleration principle, and the necessary modification is introduced subsequently.

Let G stand for the geometric rate of growth of income or output in the system, the increment being expressed as a fraction of its existing level. G will vary directly with the time interval chosen —e.g., 1 per cent. per annum = $\frac{1}{12}$ per cent. per month. Let G_w stand for the warranted rate of growth. The warranted rate of growth is taken to be that rate of growth which, if it occurs, will leave all parties satisfied that they have produced neither more nor less than the right amount. Or, to state the matter otherwise, it will put them into a frame of mind which will cause them to give such orders as will maintain the same rate of growth. I use the unprofessional term warranted instead of equilibrium, or moving equilibrium, because, although every point on the path of output described by G_w is an equilibrium point in the sense that producers, if they remain on it, will be satisfied, and be induced to keep the same rate of growth in being, the equilibrium is, for reasons to be explained, a highly unstable one.

If x_0 is output in period 0 and x_1 output in period 1, $G = \dfrac{x_1 - x_0}{x_0}$. Since we suppose the period to be short, x_0 or x_1 may alternatively stand in the denominator.

x_0 and x_1 are compounded of all individual outputs. I neglect questions of weighting. Even in a condition of growth, which generally speaking is steady, it is not to be supposed that all the component individuals are expanding at the same rate. Thus even in the most ideal circumstances conceivable, G, the actual rate of growth, would diverge from time to time from G_w, the warranted rate of growth, for random or seasonal causes.

Let s stand for the fraction of income which individuals and corporate bodies choose to save. s is total saving divided by x_0 or x_1. This may be expected to vary, with the size of income, the phase of the trade cycle, institutional changes, etc.

Let C stand for the value of the capital goods required for the production of a unit increment of output. The unit of value used to measure this magnitude is the value of the unit increment of output. Thus, if it is proposed in month 1 to raise the output of shoes, so that in month 1 and all subsequent months output is one pair higher than in month 0, and the machine required to do this—neglecting all other capital that may be required—has a value 48 times the value of a pair of shoes, C per month = 48.

The value of C is inversely proportional to the period chosen. C per annum $= 4$ in this case.[1] The value of C depends on the state of technology and the nature of the goods constituting the increment of output. It may be expected to vary as income grows and in different phases of the trade cycle; it may be somewhat dependent on the rate of interest.

Now, it is probably the case that in any period not the whole of the new capital is destined to look after the increment of output of consumers' goods. There may be long-range plans of capital development or a transformation of the method of producing the pre-existent level of output. These facts will be allowed for in due course. For the moment let it be assumed that all new capital goods are required for the sake of the increment of output of consumers' goods accruing.

Reserving proof for the next paragraph, we may now write the Fundamental Equation in its simplest form [2] :—

$$G_w = \frac{s}{C} \qquad . \qquad . \qquad . \qquad . \qquad . \qquad . \qquad 1$$

It should be noticed that the warranted rate of growth of the system appears here as an unknown term, the value of which is determined by certain " fundamental conditions "—namely, the propensity to save and the state of technology, etc. Those who define dynamic as having a cross-reference to two points of time may not regard this equation as dynamic; that particular definition of dynamic has its own interest and field of reference. I prefer to define dynamic as referring to propositions in which a rate of growth appears as an unknown variable. This equation is clearly more fundamental than those expressing lags of adjustment.

5. The proof is as follows. Let C_p stand for the value of the increment of capital stock in the period divided by the increment of total output. C_p is the value of the increment of capital per

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[1] If a month is the unit, the number of shoes added per period is 1, if a year 144. The value of G per annum is 12 times as great as that of G per month, since the numerator of G per annum is 144 times as great and the denominator 12 times as great as the numerator and denominator respectively of G per month. The number of machines added per month is $1 \equiv 48$ shoes $\equiv 48$ units of increment of output. C per month $= 48$. The number of machines added per year is $12 \equiv 48 \times 12$ shoes. Thus the value in shoes of the annual increment of capital required to produce an annual increment of 144 shoes is 48×12 units. Therefore C per annum $= \dfrac{48 \times 12}{144} = 4 = \dfrac{1}{12}$ of C per month.

[2] Since the value of G_w varies directly and that of C inversely with the unit period chosen, and the value of s is independent of the unit, the validity of the equation is independent of the unit period chosen.

unit increment of output actually produced. Circulating and fixed capital are lumped together.

$$G = \frac{s}{C_p} \quad . \qquad . \qquad . \qquad . \qquad . \qquad 1(a)$$

is a truism, depending on the proposition that actual saving in a period (excess of the income in that period over consumption) is equal to the addition to the capital stock. Total saving is equal to sx_0. The addition to the capital stock is equal to $C_p(x_1 - x_0)$. This follows from the definition of C_p. And so,

$$sx_0 = C_p(x_1 - x_0)$$
$$\therefore \frac{s}{C_p} = \frac{x_1 - x_0}{x_0} = G$$

G is the rate of increase in total output which actually occurs; C_p is the increment in the stock of capital divided by the increment in total output which actually occurs. If the value of the increment of stock of capital per unit increment of output which actually occurs, C_p, is equal to C, the amount of capital per unit increment of output required by technological and other conditions (including the state of confidence, the rate of interest, etc.) then clearly the increase which actually occurs is equal to the increase which is justified by the circumstances. This means that, since C_p includes all goods (circulating and fixed capital), and is in fact production minus consumption per unit increment of output during the period, the sum of decisions to produce, to which G gives expression, are on balance justified—i.e., if $C = C_p$, then $G = G_w$, and (from $1(a)$ above)

$$G_w = \frac{s}{C}$$

This is the fundamental equation, stated in paragraph 4, which determines the warranted rate of growth. To give numerical values to these symbols, which may be fairly representative of modern conditions : if 10 per cent. of income were saved and the capital coefficient per annum (C) were equal to 4, the warranted rate of growth would be $2\frac{1}{2}$ per cent. per annum.

It may be well to emphasise at this point that no distinction is drawn in this theory between capital goods and consumption goods. In measuring the increment of capital, the two are taken together ; the increment consists of total production less total consumption. Some trade-cycle theorists concern themselves with a possible lack of balance between these two categories ; no doubt that has its importance. The theory here considered is more fundamental or simple ; it is logically prior to the considerations regarding lack

of balance, and grasp of it is required as a preliminary to the study of them.

6. To use terminology recently employed by distinguished authorities, C_p is an *ex post* quantity. I am not clear if C should be regarded as its corresponding *ex ante*. C is rather that addition to capital goods in any period, which producers regard as ideally suited to the output which they are undertaking in that period. For convenience the term *ex ante* when employed in this article will be used in this sense.

The truism stated above, 1(a), gives expression to Mr. Keynes' proposition that saving is necessarily equal to investment—that is, to *ex post* investment. Saving is not necessarily equal to *ex ante* investment in this sense, since unwanted accretions or depletions of stocks may occur, or equipment may be found to have been produced in excess of, or short of, requirements.

If *ex post* investment is less than *ex ante* investment, this means that there has been an undesired reduction of stocks or insufficient provision of productive equipment, and there will be a stimulus to further expansion of output; conversely if *ex post* investment exceeds *ex ante* investment. If *ex post* investment is less than *ex ante* investment, saving is less than *ex ante* investment. In his *Treatise on Money* Mr. Keynes formulated a proposition, which has been widely felt to be enlightening, though experience has led him subsequently to condemn the definitions employed as more likely to be misconstrued than helpful. He said that if investment exceeded saving, the system would be stimulated to expand, and conversely. If for the definitions on which that proposition was based, we *substitute* the definition of *ex ante* investment given above, it is true that if *ex ante* investment exceeds saving, the system will be stimulated, and conversely. This truth may account for the feeling of satisfaction which Mr. Keynes' proposition originally evoked and the reluctance to abandon it at his behest. In many connections we are more interested in *ex ante* than in *ex post* investment, the latter including as it does unwanted accretions of stocks. Mr. Keynes' proposition of the *Treatise* may still be a useful aid to thinking, if we substitute for " Investment " in it *ex ante* investment as defined above.

7. Two minor points may be considered before we proceed with the main argument.

(i) It may be felt that there is something unreal in this analysis, since the increase in capital which producers will regard as right in period 1 is in the real world related not to the increase

of total output in period 1, but to prospective increases in subsequent periods. This objection may be divided into two parts. (a) In view of the fact that much of the outlay of capital is connected with long-range planning, it may be held that the fundamental equation gives too much weight to the short-period effect of the acceleration principle. This objection is freely admitted and allowed for in the subsequent modification of the equation. (b) It may further be objected that even in the sphere in which the acceleration principle holds there must be some lag between the increased provision of equipment (and stocks?) and the increased flow of output which they are designed to support. There may be some force in this. But the point is deliberately neglected in this part of the argument, along with all questions of lags. The study of these lags is of undoubted importance, but a division of labour in analysis is indispensable, and in this case the neglect is necessary in order to get the clearest possible view of the forces determining the trend and its influence as such. Moreover, the lag referred to in this sub-heading (b) may properly be regarded as unimportant, since, in the event of a *steady* advance (G) being maintained, the difference between $x_1 - x_0$ and $x_2 - x_1$ will be of the second order of small quantities. In other words, it matters not whether we regard the increment of capital as required to support the increment of total output in the same period or in the one immediately succeeding it.

8. (ii) In the demonstration given above (paras. 6 and 7) reference was made to the distinction between the *ex post* and the *ex ante* increase of capital goods. No reference was made to the distinction between *ex post* and *ex ante* saving.[1] Suppose that G is not equal to G_w might not the discrepancy show itself on the other side of the equation, not in any divergence of C_p from C, but in *ex post* saving not being equal to *ex ante* saving?

I have no very clear view as to possible causes likely to operate in a systematic way to distort *ex post* from *ex ante* saving, or of the probable importance of such distortions. It is said, for instance, that in a time of rising prices, fixed-income classes will not adapt their modes of life simultaneously, and so may save less than they would be disposed to do had they clearly foreseen the impending rise. *Per contra* variable-income classes may not foresee their own rise of income, and so spend less than they would have been disposed to do.

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[1] Be it noted that *ex ante* is here used of saving in a sense analogous to that defined in the expression *ex ante* investment; it is the saving which savers would choose to make in any period, were they able to adapt expenditure simultaneously with the changing circumstances of the period.

This question of the possible divergence of *ex post* from *ex ante* saving must be kept entirely distinct from that of the variations in *s* in the different phases of the trade cycle, which not only are admitted, but also play a part in the argument. *s* may vary because the level of income or of profit is abnormally swollen or depressed.

The neglect of these possible divergences has no importance for the argument, since they will have the same effect on growth as the divergences of C_p from C for which they may serve as substitute. Thus if G exceeds G_w, the right-hand side of the equation must exceed s/C. If the whole of this effect is found in C_p it will be *less* than C, and this is a stimulus to expansion.[1] Firms finding themselves short of stock or equipment will increase their orders. If, on the other hand, the whole of this effect is found in a divergence of *ex post s* from *ex ante s*, *ex post s* will be *greater* than *ex ante s*. Savers will find that they have saved more than they would have done had they foreseen their level of income or the level of prices correctly. Consequently they will be stimulated to expand purchases, and orders for goods will consequently be increased. Throughout the following pages the reader, whenever he finds a reference to the excess or deficiency of C_p compared with C, may substitute, if he prefers it, a supposed deficiency or excess of *ex post* saving compared with *ex ante* saving, without affecting the course of the argument.

9. We now come to a point of major importance, constituting the difference between the dynamic equilibrium (warranted rate of growth) and the static equilibrium. Normally the latter is stable and the former unstable. This gives a *prima facie* reason for regarding the dynamic analysis as a necessary propædeutic to trade-cycle study.

Some recent writers have been disposed to urge that the static equilibrium is not so stable as is sometimes claimed. Suppose that an increased output of a commodity, constituting a departure from equilibrium, is tried, so that its supply stands at a point at which the supply curve is above the demand curve. It is argued that, instead of a relapse at once occurring, reducing supply to the point of intersection of the supply and demand curves—this showing the stability of the old equilibrium—the upshot depends on how all parties now proceed. It is suggested that there may be a tendency to waltz round the point of intersection or, more broadly, that in the backward adjustment there may be

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[1] The reader who is surprised that an excess of G over G_w is stimulating will find the explanation in the next paragraph.

wide repercussions disturbing the whole system. It is even held that the whole question of the stability of the static equilibrium, in the sense of the tendency of a relapse to it when a random departure occurs, is itself a dynamic problem, which cannot be looked after by the system of static equations. I have the impression that this type of criticism exaggerates the importance of this problem, and constitutes to some extent a failure to see the wood for the trees, and that on its own ground the theory of static equilibrium is well able to hold its own.

But when we look at the dynamic equilibrium, new vistas are opened. The line of output traced by the warranted rate of growth is a moving equilibrium, in the sense that it represents the one level of output at which producers will feel in the upshot that they have done the right thing, and which will induce them to continue in the same line of advance. Stock in hand and equipment available will be exactly at the level which they would wish to have them. Of course what applies to the system in general may not apply to each individual separately. But if one feels he has over-produced or over-ordered, this will be counterbalanced by an opposite experience of an equal importance in some other part of the field.

But now suppose that there is a departure from the warranted rate of growth. Suppose an excessive output, so that G exceeds G_w. The consequence will be that C_p, the actual increase of capital goods per unit increment of output, falls below C, that which is desired. There will be, in fact, an undue depletion of stock or shortage of equipment, and the system will be stimulated to further expansion. G, instead of returning to G_w, will move farther from it in an upward direction, and the farther it diverges, the greater the stimulus to expansion will be. Similarly, if G falls below G_w, there will be a redundance of capital goods, and a depressing influence will be exerted; this will cause a further divergence and a still stronger depressing influence; and so on. Thus in the dynamic field we have a condition opposite to that which holds in the static field. A departure from equilibrium, instead of being self-righting, will be self-aggravating. G_w represents a moving equilibrium, but a highly unstable one. Of interest this for trade-cycle analysis !

Suppose an increase in the propensity to save, which means that the values of s are increased for all levels of income. This necessarily involves, *ceteris paribus*, a higher rate of warranted growth. But if the actual growth was previously equal to the warranted growth, the immediate effect is to raise the warranted

rate above the actual rate. This state of affairs sets up a depressing influence which will drag the actual rate progressively farther below the warranted rate. In this as in other cases, the movement of a dynamic determinant has an opposite effect on the warranted path of growth to that which it has on its actual path. How different from the order of events in static theory !

The reader may have some difficulty in the expression " stimulus to expansion." What is the significance of this, in view of the fact that some growth is assumed as a basic condition ? It must be remembered that the value of G depends on aggregates x_0 and x_1. These are sums of numerous quantities for which individuals are responsible. It must be supposed that at all times some individuals are jogging on at a steady level, others are risking an increase of orders or output, others are willy-nilly curtailing. G is the resultant of their separate enterprises. Some are in any event likely to be disappointed. If G is equal to G_w, it is to be supposed that the general level of enterprise undertaken in period 0, including in sum a certain increase over that in the preceding period, is found to be satisfactory. Those running short of stock balance those with surpluses. This justifies further action on similar lines, though the individuals increasing orders for stock in trade or planning new equipment in period 1 may not be identical in person with those doing so in period 0. If an expansive force is in operation, more individuals, or individuals having greater weight, will be induced by their trading position to venture increases than did so in the preceding period. Conversely if a depressing force is in operation.

The dynamic theory so far stated may be summed up in two propositions. (i) A unique warranted line of growth is determined jointly by the propensity to save and the quantity of capital required by technological and other considerations per unit increment of total output. Only if producers keep to this line will they find that on balance their production in each period has been neither excessive nor deficient. (ii) On either side of this line is a " field " in which centrifugal forces operate, the magnitude of which varies directly as the distance of any point in it from the warranted line. Departure from the warranted line sets up an inducement to depart farther from it. The moving equilibrium of advance is thus a highly unstable one.

The essential point here may be further explained by reference to the expressions over-production and under-production. The distinction between particular over-production and general over-production is well known. In the event of particular over-

production, there will normally be a tendency to reduce production of the particular line, and so equilibrium will be restored. We may define general over-production as a condition in which a majority of producers, or producers representing in sum the major part of production, find they have produced or ordered too much, in the sense that they or the distributors of their goods find themselves in possession of an unwanted volume of stocks or equipment. By reference to the fundamental equation it appears that this state of things can only occur when the actual growth has been *below* the warranted growth—*i.e.*, a condition of general over-production is the consequence of producers in sum producing too little. The only way in which this state of affairs could have been avoided would have been by producers in sum producing more than they did. Over-production is the consequence of production below the warranted level. Conversely, if producers find that they are continually running short of stocks and equipment, this means that they are producing above the warranted level.

But the condition of over-production, or, as we should perhaps call it, apparent over-production, will lead to a curtailment of production or orders, or a reduction in the rate of increase on balance, and consequently, so long as the fundamental conditions governing the warranted rate are unchanged, to a larger gap between actual and warranted growth, and so to an intensification of the evils which the contraction was intended to cure.

It must be noted that a rate of growth lying on either side of the warranted rate is regarded here as unwarranted. If the actual rate exceeds the warranted rate, producers on balance will not feel that they have produced or ordered too much; on the contrary, they will be running short of stocks and/or equipment. Thus they will not feel that they have produced the warranted amount plus something; on the contrary, they will feel that everything which they have produced has been warranted, and that they might warrantably have produced something more. None the less, we define their production as unwarrantably large, meaning by that that they have produced in excess of the unique amount which would leave them on balance satisfied with what they had done and prepared to go forward in the next period on similar lines.

10. The foregoing demonstration of the inherent instability of the moving equilibrium, or warranted line of advance, depends on the assumption that the values of s and C are independent of the value of G. This is formally correct. The analysis relates to a single point of time. s is regarded as likely to vary with a

change in the size of income, but a change in the rate of growth at a given point of time has no effect on its size. C may also be expected to vary with the size of income, e.g., owing to the occurrence of surplus capital capacity from time to time, but the same argument for regarding it as independent of the rate of growth at a particular point of time applies.

It may be objected, however, that this method of analysis is too strict to be realistic, since the discovery that output is excessive or deficient, and the consequent emergence of a depressing or stimulating force, takes some time, and in the interval required for a reaction to be produced an appreciable change in s or C may have occurred.

Consider this with reference to an experimental increase in G above a warranted level. According to the theory of instability, any such experiment will be apparently over-justified, stocks or equipment running short in consequence of it. Is it possible that if resulting changes in the values of s or C are taken into account, this doctrine will have to be modified?

In order to justify modifying the doctrine, it would be necessary to show that, in consequence of the experimental increase, s was substantially increased or C reduced. It is unlikely that C would be reduced. The capital coefficient may often stand below the level appropriate to the technological conditions of the age, owing to the existence of surplus equipment. If this were so, the higher rate of output consequent upon the experimental increase would tend to raise C. A smaller proportion of firms would come to find their capacity redundant, and a larger proportion would have to support a greater turnover by ordering extra equipment.

With saving the case is different. An expansion of activity might increase the proportion of income saved. What increase of saving is required for a modification of the instability theory?

This can be shown simply. Let x_e be an experimental increase of output above the warranted level. Let s_m stand for the fraction of the consequential income saved. The instability principle requires that

$$Cx_e > s_m x_e$$

i.e., that
$$s_m < C$$
$$< \frac{s}{G_w}$$

This condition needs interpretation. Since C and G_w do not both appear in the equation, it is necessary to define the period by which G_w is measured. This should be done by reference to the reaction time mentioned above—namely, the time required for

an undue accretion or depletion of capital goods to exert its in-
fluence upon the flow of orders.   If this reaction time is six months,
then G_w must be measured as growth per six months.

Thus the instability condition requires that the fraction of
marginal income saved shall not be more than the fraction of total
income saved multiplied by the total income and divided by the
increment of warranted income per six months.   Thus if the
warranted growth is $2\frac{1}{2}$ per cent. per annum, or $1\frac{1}{4}$ per cent. per
six months, the instability principle requires that the fraction of
marginal income saved must be less than eighty times the fraction
of average income saved.   Supposing that the high figure of 50
per cent. is taken as the fraction of marginal income saved, the
fraction of total income saved must be greater than five-eighths of
1 per cent.   Thus for any normal warranted rate of growth and
level of saving, the instability principle seems quite secure.

The force of this argument, however, is somewhat weakened
when long-range capital outlay is taken into account.   It will
then appear that the attainment of a neutral or stable equilibrium
of advance may not be altogether improbable in certain phases of
the cycle.

11. It should be noticed that the instability theory makes the
empirical verification of the acceleration principle more arduous.
For it leads to the expectation that in the upward phase of the
cycle the actual rate will tend to run above the warranted rate,
and the accretion of capital to be less than that required by the
acceleration principle; and conversely in the downward phase.
Thus a finding that the volume of investment fluctuates less than
is required by direct computation from the acceleration principle
is consistent with the theory here set forth, in which, none the
less, the acceleration principle is presented as a leading dynamic
determinant.

12. It is now expedient to introduce further terms into our
equation to reduce the influence of the acceleration principle.
Some outlays of capital have no direct relation to the current
increase of output.   They may be related to a prospective long-
period increase of activity, and be but slightly influenced, if at all,
by the current increase of trade.   Or they may be induced by
new inventions calculated to cheapen production or change
consumers' modes of spending their income, so that they are not
related to increments of output, but are designed to revolutionise
the methods for producing some portion of already existing output
or to substitute one line of goods for another in the consumers'
budget.   There are doubtless numerous factors, including the

state of confidence and the rate of interest affecting the volume of such outlay. It may suffice for the purpose in hand to divide it into two parts.

One part, K, is conceived to be quite independent both of the current level of income and its current rate of growth. The other, expressed as a fraction of income, k, is conceived to vary with the current level of income, as distinct from its rate of growth. This seems a reasonable assumption. Long-period anticipations are bound to be influenced by the present state of prosperity or adversity : even public authorities are apt to reduce the volume of public works in a slump. Companies may relate their expenditure on long-range plans to the current state of their profit account.

Having regard to the principle that the total increase of capital is equal to the total saving in the period, our fundamental equation may be modified as follows :—

$$G_w = \frac{s - k - \dfrac{K}{x}}{C} \qquad . \qquad . \qquad . \qquad . \ (2)\,[1]$$

$$\therefore \frac{s - k - \dfrac{K}{x_0}}{C_p} = \frac{x_1 - x_0}{x_0} = G$$

$$\therefore G_w = \frac{s - k - \dfrac{K}{x_0}}{C}$$

It must be noticed that C and C_p now stand not for the total increase of capital (desired and actual, respectively) per unit increment of output, but only for the net increase of capital after the capital represented by k and K has been subtracted.

It may be noticed that the larger the volume of outlay which will be sustained independently of the current rate of growth, the *smaller* is the warranted rate of growth. A larger part of savings being absorbed in such outlay, there will be a smaller part to be looked after by the acceleration principle.

13. In the following pages the expression long-range capital outlay will be used for the magnitude denoted by $xk + K$. This must not be supposed to cover all investment in durable fixed equipment; for much of that is related to, and directly governed by, the current output of consumption goods. It refers only to that part of the output of fixed equipment the production of which is not governed by the current demand for consumption goods.

If long-range capital outlay were large by comparison with that required to support the current increase in turnover of consumable

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[1] $sx_0 = C_p(x_1 - x_0) + kx_0 + K$

goods, the peculiar conditions defined in § 10 for the invalidity of the instability principle might in certain circumstances be realised. For the fraction of total income saved *and* devoted to the finance of the increase of current output might be very small compared with the fraction of marginal income saved. It is not, however, to be supposed that it would normally be small enough to invalidate the instability principle. For, with normal growth at $2\frac{1}{2}$ per cent., saving at 10 per cent., marginal saving at 50 per cent. and the reaction time 6 months, this would mean that fifteen-sixteenths of capital would normally be devoted to long-range capital outlay and only one-sixteenth would be directly associated with the current increase of output (cf. § 10). But such a situation might well arise in certain phases of the trade cycle, especially when capital capacity was redundant and saving low. In that case a stable equilibrium of advance might for a time be achieved.

14. To complete the picture, foreign trade must be taken into account. It is reasonable to measure exports, including invisible exports and the earnings of foreign investments, in absolute terms. The value of income which may be earned in this way may be conceived to be independent both of the level of activity at home and of its growth (though in so far as the trade cycle is worldwide, its value will be *de facto* related to income). Let E stand for this value. Imports, on the other hand, are better taken as a fraction, i, of the current level of income. We then have, by parity of reasoning,

$$G_w = \frac{s + i - k - \dfrac{K}{x} - \dfrac{E}{x}}{C} \qquad . \qquad . \qquad . \ (3) \ [1]$$

i need not be equal to $\dfrac{E}{x}$; the difference represents an international movement of capital. The influence of the various magnitudes on the warranted rate of growth is shown by the equation.

15. The fundamental dynamic equation has been used to demonstrate the inherent tendency of the system to instability. Space forbids an application of this method of analysis to the successive phases of the trade cycle. In the course of it the values expressed by the symbols on the right-hand side of the equation undergo considerable change. As actual growth departs

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[1] The principle now is that saving plus income expended on imports must be equal to the increase of capital in the country plus income derived from abroad. This is deducible from the fact that income derived from the sale of home made goods to consumers at home is equal to the income devoted to their purchase. Thus :—

$$sx_0 + ix_0 = C_p(x_1 - x_0) + kx_0 + K + E$$

upwards or downwards from the warranted level, the warranted rate itself moves,[1] and may chase the actual rate in either direction. The maximum rates of advance or recession may be expected to occur at the moment when the chase is successful.

For the convenience of the reader who may be tempted to experiment with this tool, it must be observed that C is always positive. Being the total quantity of capital required in connection with increments (or decrements) of current output divided by the increment (or decrement) of that output, when the latter is negative the former is negative also, and the coefficient remains positive. C_p, on the other hand, may be negative; it is not negative whenever there is a depletion of capital goods, but only when the amount of capital goods outstanding is moving in the opposite direction to the level of total output.

The formula is not well adapted to dealing with the case of zero growth. But that matter is quite simple. Zero growth is only warranted when the amount of saving is equal to the amount required for long-range capital outlay. If the amount of saving exceeds this, there will be a tendency for output to decline, and conversely.

It may be well to make one point with regard to a downward departure from the warranted position of sufficient importance to outlive one reaction time and bring the system within the field where the centrifugal forces have substantial strength. The downward lapse will then continue until the warranted rate, determined by the values on the right-hand side of the equation, itself moves down. This will happen when the numerator falls or the denominator rises. But in a phase of declining rate of growth the capital coefficient is not in general likely to rise. And so long as there is still some positive growth, albeit at a declining rate, the fraction of income saved is not likely to fall. Therefore, once the rate of growth is driven downwards from the warranted level, the warranted level is not itself likely to fall, or the downward movement therefore to be checked until the rate of growth becomes negative and the level of income recedes. Now, if the actual rate is standing below the warranted rate, the centrifugal force will continue to operate, driving the actual rate progressively downwards, unless or until the warranted rate itself falls to a level as low as the actual rate. But, since the actual rate is now negative, this cannot happen until the numerator of the right-

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[1] This idea is analogous to that propounded by Mr. D. H. Robertson that the " natural " rate of interest may be expected to vary in the different phases of the trade cycle. Cf. ECONOMIC JOURNAL, December 1934.

hand side of the equation becomes negative—that is, until saving falls below the level required for long-range capital outlay:

16. Alongside the concept of warranted rate of growth we may introduce another, to be called the natural rate of growth. This is the maximum rate of growth allowed by the increase of population, accumulation of capital, technological improvement and the work/ leisure preference schedule, supposing that there is always full employment in some sense.

There is no inherent tendency for these two rates to coincide. Indeed, there is no unique warranted rate; the value of warranted rate depends upon the phase of the trade cycle and the level of activity.

Consideration may be given to that warranted rate which would obtain in conditions of full employment; this may be regarded as the warranted rate " proper " to the economy. *Prima facie* it might be supposed healthier to have the " proper " warranted rate above than below the natural rate. But this is very doubtful.

The system cannot advance more quickly than the natural rate allows. If the proper warranted rate is above this, there will be a chronic tendency to depression; the depressions drag down the warranted rate below its proper level, and so keep its average value over a term of years down to the natural rate. But this reduction of the warranted rate is only achieved by having chronic unemployment.

The warranted rate is dragged down by depression; it may be twisted upwards by an inflation of prices and profit. If the proper rate is below the natural rate, the average value of the warranted rate may be sustained above its proper level over a term of years by a succession of profit booms.

Thus each state of affairs has its appropriate evils. There is much to be said for the view that it is better that the proper warranted rate should be lower rather than higher than the natural rate.

17. In order fully to grasp the dynamic principle, it is necessary to bear in mind that changes in fundamental conditions have opposite effects on the actual rate and the warranted rate. An increased amount of long-range capital outlay, an increase in the capital coefficient, an increase in the propensity to consume, and an increase in the active balance on international account, or a decline in the passive balance, are all properly thought to have a stimulating effect on the system. But they all tend, as may

readily be seen from the equation, to reduce the warranted rate. This paradox may be readily explained.

Suppose that one of these stimulants begins to operate when the actual rate is equal to the warranted rate. By depressing the warranted rate, it drags that down below the actual rate, and so automatically brings the actual rate into the field of centrifugal forces, driving it away from the warranted rate—that is, in this case, upwards. Thus the stimulant causes the system to expand.

It must not be inferred that these stimulants are only of temporary benefit. For it may be healthy for an economy to have its proper warranted rate reduced. This is likely to be so when its proper warranted rate is tending to be above the natural rate.[1] The long-run value of the stimulant can only be assessed if it is known whether, in its absence, the proper warranted rate is running above or below the natural rate.

It is often felt that a high propensity to save should warrant a great increase in the output of wealth, and this induces an extreme aversion to accept Mr. Keynes' view that excessive saving in the modern age is hostile to prosperity. The feeling is justified to the extent that higher propensity to save does, in fact, *warrant* a higher rate of growth. Trouble arises if the rate of growth which it warrants is greater than that which the increase of population and the increase of technical capacity render permanently possible. And the fundamental paradox is that the more ambitious the rate *warranted* is, the greater the probability that the actual output will from time to time, and even persistently fall below that which the productive capacity of the population would allow.

18. Policy in this field is usually appraised by reference to its power to combat tendencies to oscillation. Our demonstration of the inherent instability of the dynamic equilibrium confirms the importance of this. But there are two points to be noticed in this connection. 1. The nature of the measures suitable for combating the tendency to oscillate may depend on whether the natural rate is above or below the proper warranted rate. 2. In addition to dealing with the tendency to oscillation when it occurs, it may be desirable to have a long-range policy designed to

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[1] This may be the most fundamental rational explanation of the common view that it is dangerous for an old country to be a large importer of capital. For this involves a high warranted rate of growth, and it is dangerous to have a high warranted rate when the natural rate is low. *Per contra* for a young country, whose natural rate is high, it is considered healthy and proper to have a large import of capital.

influence the relation between the proper warranted rate of growth and the natural rate.

If, in the absence of interference, the proper warranted rate is substantially above the natural rate, the difficulties may be too great to be dealt with by a mere anti-cycle policy. In the first place, there is the probability of a slump occurring before full employment is reached, since during the revival the warranted rate may be dangerously near the actual rate, and liable at any time to overpass it, thus generating depression. Secondly, there is an acute problem if the actual rate reaches the ceiling of full employment and is depressed to the natural rate, and therefore below the warranted rate. An attempt may then be made to drag down the warranted rate below its normal level by increasing public works (K). But the difficulty of the proper warranted rate being above the natural rate will be chronic, and this means that only by keeping in being a large and growing volume of public works can the slump be prevented. In fine, the anti-cycle policy has to be converted into a permanent policy for keeping down the proper warranted rate.

19. The ideal policy would be to manipulate the proper warranted rate so that it should be equal to the natural rate. If this could be achieved—but in fact only a rough approximation would be possible—an anti-cycle policy would none the less be an indispensable supplement. For the warranted rate is bound to be disturbed by the varying incidence of inventions and fluctuations in the foreign account. An anti-cycle policy would be necessary to combat the run-away forces which come into being as soon as a substantial change occurs in the warranted rate.

20. A low rate of interest makes for a low warranted rate of increase, by encouraging high values of K and C and, possibly also, by having a depressing influence on s. Since the effects of changes in the rate of interest are probably slow-working, it may be wise to use the rate of interest as a long-range weapon for reducing the warranted rate of growth, and to reserve *suitable* public works for use against the cycle. It is not suggested, however, that a low rate of interest has sufficient power of its own to keep down the warranted rate without the assistance of a programme of public works to be kept permanently in operation.

If permanent public works activity and a low long-term rate availed to bring the proper warranted rate into line with the natural rate, variations in the short-term rate of interest might come into their own again as an ancillary method of dealing with oscillations.

21. This essay has only touched in the most tentative way on a small fraction of the problems, theoretical and practical, of which the enunciation of a dynamic theory suggests the formulation. In the last paragraph it was implicitly hinted that our present situation is one of a relatively high proper warranted rate. The evidence for this comes from inside and outside the dynamic theory itself. According to the dynamic theory, the tendency of a system to relapse into depression before full employment is reached in the boom suggests that its proper warranted rate exceeds its natural rate. Outside evidence includes the known decline in the growth of population, which involves a decline in the natural rate. More controversial points are the tendency of a more wealthy population to save a larger fraction of its income (high value of s involves high warranted rate), and the tendency of modern progress to depress rather than elevate the value of C (low value of C involves high warranted rate).

The main object of this article, however, is to present a tool of analysis, not to diagnose present conditions.

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