Marginal Productivity and the Macro-Economic Theories of Distribution

COMMENT ON SAMUELSON AND MODIGLIANI

Professors Samuelson and Modigliani have written a long critical essay on macro-economic theories of distribution which demonstrates, not only the splendid analytical powers of the two authors, but also the intellectual sterility engendered by the methods of Neo-classical Economics. The assumption of Profit Maximization under conditions of Universal Perfect Competition involves, as a logical step (given the postulate of substitute relationships between factors), the assumption of production functions which are linear-homogeneous and " well behaved " (with isoquants asymptotic to the axes). In addition, it has also been found necessary to assume either that capital is completely " malleable ", or else that capital-labour intensities are identical in all industries in all circumstances—so that *real* capital can be uniquely measured in value (money) terms—and that there is no technical progress, except of the " Harrod neutral " type which falls like manna from Heaven. Given sufficient refinement of analysis no doubt many other such " assumptions " may have to be added. (One obvious candidate which has not been incorporated yet in neo-classical models is the absence of " Sraffa effects "—though it may be difficult to formulate the necessary conditions explicitly). There is no room here for increasing returns, learning by doing, oligopolistic competition, uncertainty, obsolescence and other such troublesome things which mar the world as we know it. Markets operate in such a way that " competition will *enforce* [their italics] *at all times* [my italics] equality of factor prices to factor marginal productivities " (p. 271) and even if marginal productivities did not exist (in the " fixed coefficient case " on pp. 287-289) " markets " would still operate in such a way as to punish immediately a factor in excess supply, be it Capital or Labour, with a zero price.

I would not wish to deny that these " abstractions " are necessary to make the system logically consistent, given the basic assumptions concerning how markets behave and how entrepreneurs behave (profit maximization *combined with* universal perfect competition). But one must not fall into the error of supposing that assertions about reality can be derived from *a priori* assumptions. Whether well-behaved homogeneous-and-linear production functions exist or not is a question of fact. They cannot be presumed to exist as a consequence of some basic postulate, such as " profit maximization under competitive conditions ". If adequate empirical observation *established* the existence of production functions of constant returns to scale; if entrepreneurs can be *shown* to be confronted by infinitely elastic demand and supply curves; if the progress in technical knowledge could be *shown* to be of the kind which affected the productivity of all resources equally, and which proceeded at some autonomous rate in time, independently of entre-preneurial decisions concerning production and investment, the situation would be different. But all that we can tell from empirical observation is that output per unit of labour increases with the passage of time—to an extent that varies significantly between different industries and different countries—whilst output per unit of capital shows no systematic trend, upwards or downwards. We know that in a majority of cases, if not all, there is a positive association between the rate of increase of output-per-man and the rate of increase of output. But it is not possible to isolate the element of " autonomous " technological

change from elements which are induced by, or associated with, changes in output or in investment.

However, for Professors Samuelson and Modigliani, assumptions like homogeneous-linear and " well-behaved ", production functions, or autonomous " Harrod-neutral " technical change, are not meant, I suppose, to be descriptions of reality (though in many places they argue as if they were)—they are " abstractions " which are intended merely as intermediate stages in the process of analysis. It is the hallmark of the neo-classical economist to believe that, however severe the abstractions from which he is forced to start, he will " win through " by the end of the day—bit by bit, if he only carries the analysis far enough, the scaffolding can be removed, leaving the basic structure intact. In fact, these props are *never* removed; the removal of any one of a number of them—as for example, allowing for increasing returns or learning-by-doing—is sufficient to cause the whole structure to collapse like a pack of cards.[1] It is high time that the brilliant minds of M.I.T. were set to evolve a system of non-Euclidean economics which starts from a non-perfect, non-profit maximizing economy where such abstractions are initially un-necessary. (Of course the neo-classical purist would argue, again on *a priori* grounds, that in the long run the non-profit maximizers will fall by the wayside—profit-maximization will emerge by a Darwinian process of selection. This *may* be true in a static world with perfect foresight, in which profits can *only* be made through the relentless pursuit of the principle of substitution. But in a world of imperfect foresight and changing technology, the Darwinian process may favour the successful innovator who operates on hunches rather than the *homo oeconomicus* of the more pedestrian type, the careful equator of marginal substitution ratios.)

These general observations will, I think, be helpful in appreciating the particular comments that follow.

(1) Capitalists and Workers

Samuelson and Modigliani assume that any macro-economic theory which makes use of the notion of differences in savings propensities between profits and wages requires an identifiable class of hereditary barons—a class of capitalists " with permanent member-ship "—distinguished by a high savings propensity and of a " permanent " class of workers distinguished by a low savings propensity. I cannot of course speak for Dr Pasinetti, but as far as my own ideas are concerned, I have always regarded the high savings propensity out of profits as something which attaches to the nature of business income, and not to the wealth (or other peculiarities) of the individuals who own propery. It is the enterprise, not the particular body of individuals owning it at any one time, which finds it necessary, in a dynamic world of increasing returns, to plough back a proportion of the profits earned as a kind of " prior charge " on earnings in order to ensure the survival of the enterprise in the long run. This is because: (i) continued expansion cannot be ensured in an uncertain world, and in the long run, unless *some proportion* of the finance required for expansion comes from internal sources; (ii) the competitive strength of any one enterprise, in a world of increasing returns, varies with the enterprise's share of the market—it declines with any decrease in that share, and improves with an increasing share; hence (iii) in a world of expanding markets, continued expansion (by the individual firm) is necessary merely to maintain the competitive strength of the enterprise. Hence the high savings propensity attaches to profits as such, not to capitalists as such.

In the early days of industrial capitalism when the ownership and management of

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[1] This is occasionally admitted—cf. e.g. footnote 7 which says that " the assumption of constant returns to scale is essential both to Pasinetti's and our own analysis, for otherwise the concept of a golden age steady state becomes self-contradictory ". If the specification of a " golden age steady state " includes perfect competition, profit maximization, *et al.*, this is obviously true. But if by a " golden-age steady state " no more is meant than a steady rate of growth with a constant rate of profit, the proposition is not true.

businesses were united in the same person (as in the case of the early ironmasters of England, or, in more recent times, of Henry Ford) a high propensity to plough back business profits inevitably entailed a high propensity to save out of individual income. The capitalists of the nineteenth century, as Keynes once said, " were allowed to call the best part of the cake theirs, and were theoretically free to consume it, on the tacit under-lying condition that they consumed very little of it in practice ".[1] But nowadays businesses are to a large extent owned by rentier-capitalists (shareholders) whose personal savings-propensity need bear no relation to the savings propensity of the enterprises which they own. They are free to consume, in addition to their dividend income, as much of their capital (or their capital gains) as they like; in so far as they do so, this goes to offset the net acquisition of business assets by the " workers ": it does *not* reduce, but on the contrary, enhances, the difference in savings propensities between business income and personal income. (For reasons explained in the Appendix, the shareholders' consumption out of capital gains cannot be treated as a reduction of s_c; it has exactly the same effect as a reduction of s_w, since it causes a reduction of the net savings of persons that is available to finance business investment).

(2) The " Anti-Pasinetti Theorem "

The foregoing remarks are sufficient, I think, to refute the authors' contention that provided the savings propensity of workers is high enough, the " capitalists " (distinguished by their high savings propensity) will be gradually eliminated so that, in a golden-age equilibrium, only one " savings propensity " is left. For this purpose they consider a situation in which the basic " Pasinetti inequality " (i.e. that the share of investment in total income is higher than the share of savings in wages, or in total personal income) does not hold as regards the " equilibrium " level of investment. They then proceed to demonstrate that the " capitalists " will be gradually " squeezed out ". However, the end of it all is not a violent revolution, à la Marx, but the cosy world of Harrod, Domar and Solow, where there is only a single savings propensity applicable to the economy—where in other words,

$$sY = s_w Y.$$

The simple answer to all this is that, if the basic Kaldor-Pasinetti inequality is not satisfied, no Keynesian macro-economic distribution theory could survive for an instant, let alone in Golden Age equilibrium. If the " equilibrium " level of investment was less than the workers' savings, it is impossible to contemplate that investment should play the active role, and savings the passive role; for if we postulated that investment decisions were autonomous, either the full employment assumption would break down, or profits would have to be negative; and in either case it is clearly inconceivable that profits should be determined by the need to generate sufficient savings to finance investment. Moreover if we assume that profits are determined quite independently of this relation—either by marginal productivities in the Wonderland of Perfect Competition, etc., or by, say, Kalecki's " degree of monopoly "—we need further to assume a purely non-Keynesian system where there is necessarily just enough investment to finance full employment savings—where, in other words, savings *govern* investment, not the other way round.[2] It is easy to refute

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[1] *The Economic Consequences of the Peace*, p. 17.
[2] On the subject of " full employment " the two halves of the Samuelson-Modigliani paper take up wholly contradictory positions. In the first half it is automatically (and continually) ensured by the marginal productivity equations. But in the second half, dealing with macro-economic theories, they become highly sceptical of its ever occurring, and pour scorn on people like Marshall, Pigou or myself for suggesting that unemployment has been small or trendless. (Compare, for example, the statement on p. 277 that " as long as the production function is well behaved, failure of (8) or even (7) to hold *cannot interfere* with full employment " (my italics) with footnote 3, p. 294, which says that if you believe in a mechan-ism making for full employment, " You can—as the Duke of Wellington once said—believe in anything "). Is there nothing, literally nothing, in a capitalist system which makes for full employment equilibrium—outside the cosy world of neo-classical theory?

Pasinetti by postulating conditions in which the Pasinetti model could not possibly work, and where therefore something else must take its place—whatever that something else may be. Professors Samuelson and Modigliani assume, as a matter of course, that it must be Walras. In disproving Pasinetti they conjure up a Walrasian world in all its purity—a world in which all savings get invested somehow, without disturbing full employment: because any excess of savings over its equilibrium level induces a corresponding excess of investment over its equilibrium level. It is a world in which excess savings in search of investment necessarily depress the rate of interest (r) to whatever level required to induce the necessary addition to investment, which means that, given a sufficient fall in r, a value of k/y can always be found (this is where " well-behaved " production functions come in) to make $nk/y = s_w$.

The validity of the Pasinetti inequality, $s_w < I/Y$—or, in the Samuelson-Modigliani way of looking at it, $s_w < \alpha(k)s_c$—is a matter of fact, just like perfect competition, constant returns to scale, *et al.* are matters of fact. Empirical investigation can disprove the former, just as it can disprove the latter. The question is, does it, and if so, which?

(3) A Side Glance to Reality

There is just one occasion where the two authors find it necessary to discuss the realistic values of the variables in order to justify their concern with the " anti-Pasinetti " case where " capitalists " are saved out of existence by the " workers ". It occurs in footnote 1, p. 274, which is adduced in support of the statement that values of

$\dfrac{P}{Y} = \alpha(k^*) = \tfrac{1}{4}$, $s_c = \tfrac{1}{5}$ and $s_w = 0\cdot05$, which (in their view) rule out the Pasinetti theorem,

" are econometrically reasonable for a mixed economy like the U.S., U.K. or Western Europe ".[1] In this demonstration however, they make no less than four slips:

(i) The " corporate savings propensity of $\tfrac{1}{5}$ " relates to savings out of net profits, after capital consumption allowances. The share of corporate profits of $\tfrac{1}{4}$ relates to the share of *gross* profits in the GNP. The empirical " savings propensity " out of gross profits after tax is not $\tfrac{1}{5}$, but $0\cdot7$ for the United States and the United Kingdom, and somewhere around $0\cdot8$ for Germany and Japan. (From the point of view of the mechanics of a Keynesian model, it is *gross* savings out of *gross* profits, and *gross* investment, that are relevant, not *net* savings and *net* investment; indeed in a more realistic " vintage " model it is not even possible to say *ab initio* what the latter are.)

(ii) The value $s_w = \tfrac{1}{12}$ is probably a realistic one for the net savings of wage and salary earners (i.e. net of dis-savings of retired workers); it is not however an indication of the savings available for the acquisition of business capital or for lending to the business sector, since a large part of it goes to finance personal investment in consumer durables. The only consumer durable which is statistically relevant in this connection is residential housing (since furniture, cars, etc. are statistically reckoned in consumption, not in investment, in official national income accounts) and the latter must be deducted before the value of " s_w " relevant to the model can be estimated.

The net acquisition of financial assets by the personal sector—which is the measure

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[1] Incidentally, the very demonstration that these figures are inconsistent with the Pasinetti theorem is based on a piece of circular reasoning—it assumes what it intends to prove. For it supposes that the share of profits is determined by the technical coefficients of the production function, irrespective of anything else, in particular the investment coefficient, I/Y, which is here supposed to be governed by the savings generated by a predetermined profits share, dictated by the production function—whereas the whole dispute between Keynesian and non-Keynesian theories is whether investment determines savings, or *vice versa*. (I admit that they do not quite go the whole hog and postulate a world under a full Cobb-Douglas dictatorship where the share of profit is a technologically given constant, irrespective of anything else.)

of the personal saving available for lending to other sectors—appears to be very much smaller, as the following estimates show: [1]

In the United Kingdom personal saving as a percentage of personal disposal income appears to be lower than in the United States. But because investment in dwellings by the personal sector and investment by unincorporated businesses is relatively less important in the United Kingdom, net acquisition of financial assets appears to be somewhat higher.

Hence, whether we take the U.S. or the U.K. figures, the value of s_w—in the only sense in which this is relevant to the Pasinetti theorem or the anti-Pasinetti theorem—is more like $\frac{1}{50}$ or $\frac{1}{100}$ than $\frac{1}{12}$.

(iii) In the above figures the " increase in life assurance and pension funds " is probably a good indication of the net savings of wage and salary earners (after deduction of the savings that go to repay mortgages on houses), since these are so largely contractual in character. On the other hand, the net diminution in " other financial assets " is probably a good measure of the net consumption of property-owners out of capital or capital gains. Since net spending in excess of dividend-income involves a net sale of securities, which goes to offset at least a part of the net demand for securities originating from wage and salary earners' savings, it should be evident that spending out of capital gains is indistinguishable in its effects from a reduction in workers' savings. The fact that some of the capital gains are spent is thus in no sense a criticism of the " realism of the strict Pasinetti assumptions ". There is nothing in the model which requires s_w to be positive—the model works just as well when it is negative, with the business sector being a net lender, not a

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[1] The figures are derived from the official national income accounts for the United States and the United Kingdom and are, as far as possible, on a comparable basis. The source of the figures for the United States is the *Survey of Current Business*, May 1966; for the United Kingdom, *Financial Statistics*, June 1966 and *Economic Trends*, April 1966.

Personal disposable income, as defined here, is reckoned before deducting capital consumption. Gross personal saving includes any capital transfers received *less* any capital transfers paid. For both countries independent estimates of identified acquisitions of financial assets *less* liabilities exceed the residual figures obtained from the official national income accounts; it is impossible to say how far the discrepancy is due to an underestimate of personal savings in the national accounts (and an overestimate of corporate etc. savings) and how far to under-recording of financial transactions between the personal sector and the others. For the U.S. estimates based on identified financial transactions may be deduced from the May 1966 issue of the *Federal Reserve* Bulletin:

	Percentages of personal disposable income Average 1961-1965
Increase in life assurance and pensions funds	3·4
Increase in other financial assets	−0·5
Net acquisition of financial assets by personal sector	2·9

For both countries the figures of personal saving and of net acquisitions relate to the personal sector as a whole, including unincorporated enterprises. Since the income of these enterprises is small in relation to total of personal income (14 to 15 per cent in the United States and 9 to 10 per cent in the United Kingdom) no great error can result from assuming that the saving (covering both capital consumption and other additions to reserves) of unincorporated enterprises is roughly balanced by their investment. (I am indebted to Mr L. S. Berman of the Central Statistical Office for these estimates.)

	Percentage of personal disposable income (average 1960 to 1965)	
	United Kingdom	United States
Personal saving	6·9	10·3
less investment in dwellings	−2·8	−4·7
less investment in fixed assets and stocks by unincorporated businesses	−1·8	−4·5
equals net acquisition of financial assets by the personal sector	2·3	1·1
of which		
increase in life assurance and pension funds	4·7	2·6
increase in other financial assets (net)	−2·4	−1·5

net borrower, to the personal sector.[1]   Despite the laborious attempt to " generalise " the model to all kinds of cases—Pasinetti and anti-Pasinetti—Samuelson and Modigliani ignore the " super-Pasinetti " case of $s_w < 0$ or the special " arch-Pasinetti " case of $s_w = 0$.

(iv) With a " realistic " value of s_w of, say, 0·01-0·03, and a " realistic " value of $s_c = 0·7$, one would have thought that the Kaldor-Pasinetti type of model is safe enough on empirical grounds—even allowing for " Kuh-Meyer " effects, the validity of which is in any case highly questionable.[2]   However, if one goes in for " realism ", surely one cannot ignore the existence of a third class, which is more likely to overspend its income than either " workers " or " capitalists ", namely the Government.   The " net acquisition of financial assets by the personal sector " need not in fact finance business investment: it often goes to finance (probably only a part of) the net borrowing of the public sector. To find out whether " the realistic value of the variables " threatens the Pasinetti model in an anti-Pasinetti direction, one should pose another question: how much of business investment is in fact financed out of personal savings?   This information is found by looking at the net acquisition of financial assets of the (private) corporate sector; in the U.S. this item has been positive for three out of the last six years, and in the U.K. it has been positive for five out of the six years (even after adjustment for net long-term invest-ment abroad).   In such years the corporate sector is a net lender to the rest of the economy, and not a net borrower.   This comes to the same, from the point of view of the mechanics of the Keynesian-type distribution theory, as assuming a negative value of s_w; in its effects on profits it makes no difference whether the net dis-saving is due to the activities of the personal sector or of the Government.

(4) The " Generalized " Neo-Classical Theory

In section X of their paper the two authors discover that their results do not, after all, depend on " marginal productivity notions of the Clark-Wicksteed-Solow-Meade type ".   *All* that they require is the postulate that the rate of profit, or interest, should be a single-valued function, ϕ, of the capital/labour ratio (K/L), with $\phi' < 0$.   No reason whatever is adduced to show why this assumption is any less restrictive than the whole bag of tricks specified in equations (1) and (2).   The assumption of a functional relation between the rate of profit and the capital/labour ratio is *implied* in the assumption under-lying equations (1) and (2); but without them it is purely arbitrary.   Nor is any attempt made to support the validity (or plausibility) of such an assumption empirically.   The K/L ratio, unlike the K/Y ratio, shows the widest of variations between the different countries—it is perhaps twenty times as high in the U.S. as in India—whilst the rate of profit is often to be found to be higher in countries with a relatively high K/L ratio than with a low K/L ratio.   (Of course they might have argued, in the manner of the Stanford inquiry, that in terms of " corrected " labour units—corrected for Harrod-neutral differ-ences in " efficiency "—capital/labour ratios are everywhere the same!)

(5) Fixed Coefficients

Finally, they assert that macro-economic distribution theories " fare best " under the assumption of fixed coefficients " for under variable coefficients one would have *no need*

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[1] Cf. *Appendix* below, on " A Neo-Pasinetti Theorem ".
[2] The correlation found by Kuh and Meyer between corporate savings and corporate investment does not in any way prove that corporate investment is confined by, or governed by, corporate savings: they may both reflect a common factor, the rate of return on capital employed.   A company's retention ratio is clearly influenced by its long-run capital requirements: the raison d'etre of the plough back is to preserve a company's liquidity and thus to prevent its long-run expension from being hampered by financial embarrassments.   But there is always a " liquidity cushion " (in Anglo-Saxon countries, often a pretty large cushion) between the current cash inflow of savings and the current cash outflow on capital expenditure; the purpose of retention is to protect the cushion, rather than to finance expenditure on a day-to-day basis.

[my italics] for a genuinely alternative theory of distribution "—why search for something new when you have such a satisfying explanation already?

It is at this point that they should have taken another sideglance into reality and adduced some empirical evidence to show that the most important " predictions " of the marginal productivity theory—diminishing productivity to labour in the short period, constant returns to scale to both labour and capital in the long period—are valid, as a matter of fact. However they could not have done this, because:

(i) all empirical studies concerning the short period relationship between output and employment (at least in manufacturing activities) show the elasticity of the former with regard to changes in the latter to be greater, not less, than unity (" Okun's Law " makes it 3) which implies of course that the short-period marginal product of labour exceeds the average product. Since profits are non-negative, the proposition that the price of each factor is " at all times " equal to its marginal product cannot possibly hold. Another way of putting the same point is that in order to get diminishing short-period marginal productivity for labour (or increasing marginal labour cost) firms must operate plant near full (or maximum) capacity. Experience shows that even in times when production is limited by labour shortages, plant is not (normally) fully utilized.

(ii) All empirical studies concerning the relationship of productivity and production (again, for manufacturing activities) reveal the existence of (long-run) increasing returns, both on account of the economies of large-scale production, and of the subdivision of processes (and of industries) with an increase in the scale of activities. Again the proposition that factor prices are equal to their marginal products cannot hold, because the marginal products do not " add up " to the total product.

However, the authors believe they have demonstrated (by *a priori* reasoning) that there is no room for anything other than marginal productivity if there are possibilities of factor substitution. Hence " recognizing the empirical oddity of the postulate " they proceed to explore the case of fixed coefficients, and find that here at last the Kaldor-Pasinetti proposition has some " definite meaning " as an explanation of relative shares. However, they hasten to add that " whatever value the above model has as an exercise, its economic relevance is in our view very dubious " because an economic system of this kind would be subject to the wildest instability: if the capital/labour ratio wasn't exactly right, either profits would be zero, or wages would be zero (according to whether capital were just a little too much or a little too little); and there is no guarantee that a system fluctuating between the extremes of zero profits and zero wages would tend to generate just the right amount of capital to be consistent with positive values for both.

But what kind of an " economic system " are they " observing? " Is it Ruritania, Solovia, Cloud-Cuckooland, or the U.S.A. and other members of the Group of Ten? If " excess " labour caused wages to fall to zero, how could there ever be unemployment—of the Keynesian or any other kind? And if " excess " capital caused profits to fall to zero, how could there ever be profits, considering that the employment-capacity of the prevailing capital equipment, in any advanced community, is always larger and often very considerably larger, than what is needed to secure full employment for the existing labour force? Indeed how could capacity ever be under-utilized and still earn a profit?

The whole antithesis that *either* marginal productivity " must explain " pricing and distribution *or else* there must be fixed coefficients, is neo-classical circular reasoning carried *in extremis*. The proposition is true but only in the abstract Wonderland of perfect competition, profit maximization plus the etceteras specified at the beginning. Their appeal however to the need to reconcile " the relatively smooth functioning of behaviour and of share imputation in *observed* economic systems " is an appeal, not to the Wonderland, but to the Real World. Can't they see that it is *possible* for a market economy to be " competitive " without satisfying the neo-classical equations? Can't they imagine a world in which marginal productivities are *not* equal to factor prices, and are not in any

definite relationship to factor prices—a world, for example, in which, with the approach of labour scarcity, the share of wages is falling, not rising, despite the fact that the marginal productivity of labour is constant or rising and Capital (in the relevant sense) is redundant in relation to Labour? Unless they make a more imaginative effort to reconcile their theoretical framework with the known facts of experience, their economic theory is bound to remain a barren exercise. One is reminded of what Clapham wrote 44 years ago: " I have a fear lest a theory of value which should prove permanently unable to state of what particular and individual values some of its more important conclusions were true might in the long run be neglected by mankind." [1]

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APPENDIX

A Neo-Pasinetti Theorem [2]

Dr Pasinetti has shown that on certain conditions the rate of profit, in a true long-run Golden Age equilibrium, will be independent of the rate of savings of " workers ", because the additional consumption generated out of the workers' property income will offset their savings out of wage-income. The difficulty with this proposition (apart from the fact that it is " very long run ") is that it assumes that workers spend the same fraction of their income, irrespective of whether it accrues to them as property income or wages. In a world where enterprises are organised as corporations, and property-income takes the form of dividends, this would imply (as Professors Samuelson and Modigliani point out) overspending their dividend income by the exact fraction required to make their consumption equal to $(1-s_w)P_w$, irrespective of the division of profits between corporate retentions and dividends.

Moreover once we allow spending in excess of divided income, there is no reason to confine such spending to workers. " Capitalists " also spend some part of their capital gains (or even their capital, in the absence of such gains), and as Professor Modigliani has reminded us, the limited length of human life must add to such temptations.

Hence at any time there must be capitalists (or shareholders) who over-spend their current (dividend) income, (and the same must be true of course of retired workers who consume over the years of retirement their accumulated savings) just as there are active workers who save a certain fraction of their income for retirement. Just as net saving out of income sets up a demand for securities, net dis-saving out of income (= net consumption out of capital or capital gains) sets up a supply of securities. There is also a net supply of new securities issued by the corporate sector. Since, in the securities' market prices will tend to a level at which the total (non-speculative) supply and demand for securities are equal, there must be some mechanism to ensure that the spending out of capital (or capital gains) just balances the savings out of income *less* any new securities issued by corporations.

Let us divide the community into wage and salary earners (W) who save (through the intermediaries of pension funds and insurance companies) some fraction of their income during their working life and consume it in retirement; so long as the population is rising and income per head is rising, the savings of the working population must exceed the dis-savings of the retired population by an amount which can be expressed as some fraction (s_w) of current wage-and-salary income. (I am assuming also that s_w is net of personal investment in consumer durables, i.e. in housing.)

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[1] *Economic Journal*, December 1922, p. 561.
[2] I should like to acknowledge the benefits from discussions with F. H. Hahn, L. Pasinetti and J. A. Mirrlees.

Let us further suppose that the shareholders' net consumption out of capital (i.e. their consumption in excess of their dividend income) is some fraction (c) of their capital gains G.

And finally, let us suppose that corporations (having decided on retaining a fraction, s_c, of their profits) decide in addition to issue new securities equal to some fraction, i, ($|\,i\,|<1$), of their current investment expenditure, gK (where K = capital, g = the growth rate).

Equilibrium in the securities market then requires that

$$s_wW = cG+igK.$$

For such an equilibrium to exist, at least one of these items must be responsive to changes in the market prices of securities. Such an item is cG, since G is nothing else than the change in the market value of securities. This varies not only with the rise in dividends and earnings per share, but also with the " valuation ratio " (v)—i.e. the relation of the market value of shares to the capital employed by the corporations (or the " book value " of assets). In other words, if the market value of securities is pN (where N = number of shares, p = price per share) and given a constant valuation ratio

$$G = N\Delta p = v\Delta K-p\Delta N \qquad \qquad \text{...(1)}$$

(i.e. the increase in the corporations' assets *multiplied* by the valuation ratio *less* the value of new securities issued). Since

$$\Delta K = gK$$

$$p\Delta N = igK \qquad \qquad \text{...(2)}$$

$$G = vgK-igK$$

Hence [1]
$$s_wW = c(vgK-igK) = igK \qquad \qquad \text{...(3)}$$

There is in addition the savings = investment equation

$$s_wW-c(vgK-igK)+s_cP = gK. \qquad \qquad \text{...(4)}$$

Since $W = Y-P$, $P = \rho K$ (P = profits, ρ = the rate of profit), the above can be written

$$s_wY-s_w\rho K-cvgK+cigK = igK \qquad \qquad \text{...(3a)}$$

and
$$s_wY+(s_c-s_w)\rho K-cvgK+cigK = gK. \qquad \qquad \text{...(4a)}$$

After rearranging the terms and dividing through by gK we get

$$\frac{s_w}{g}\frac{Y}{K} - \frac{s_w\rho}{g} - cv+ci = i \qquad \qquad \text{...(3b)}$$

$$\frac{s_w}{g}\frac{Y}{K} + \frac{(s_c-s_w)\rho}{g} - cv+ci = 1. \qquad \qquad \text{...(4b)}$$

Solving for v and ρ we get

$$v = \frac{1}{c}\left[\frac{s_w}{g}\frac{Y}{k} - \frac{s_w}{s_c}(1-i)-i(1-c)\right] \qquad \qquad \text{...(5)}$$

and
$$\rho = \frac{g(1-i)}{s_c}. \qquad \qquad \text{...(6)}$$

The interpretation of these equations is as follows. Given the savings-coefficients and the capital-gains-consumption coefficient, there will be a certain valuation ratio

[1] We subsume in the definition of c a constant share of assets owned by shareholders, for reasons explained below.

Y

which will secure just enough savings by the personal sector to take up the new securities issued by corporations. Hence the *net* savings of the personal sector (available for investment by the business sector) will depend, not only on the savings propensities of individuals, but on the policies of the corporations towards new issues. In the absence of new issues the level of security prices will be established at the point at which the purchases of securities by the savers will be just balanced by the sale of securities of the dis-savers, making the net savings of the personal sector zero. The issue of new securities by corporations will depress security prices (i.e. the valuation ratio) just enough to reduce the sale of securities by the dis-savers sufficiently to induce the net savings required to take up the new issues. If i were negative and the corporations were net *purchasers* of securities from the personal sector (which they could be through the redemption of past securities, or purchasing shares from the personal sector for the acquisition of subsidiaries) the valuation ratio would be driven up to the point at which net personal savings would be negative to the extent necessary to match the sale of securities to the corporate sector.[1]

In a state of Golden Age equilibrium (given a constant g, and a constant K/Y, however determined), v will be constant, with a value that can be $\gtrless 1$, depending on the values of s_c, s_w, c and i. (All that one can assert is that, given the Pasinetti inequality, $gK > s_w Y$, $v < 1$ when $c = (1 - s_w)$, $i = 0$; with $i > 0$ this will be true a fortiori.[2]

The rate of profit in a Golden Age equilibrium (as given by equation (6)) will depend only on g, s_c and i, and will then be independent of the " personal " savings propensities, s_w and c. In this way it is similar to the Pasinetti theorem in that the rate of profit will be independent of s_w (and also of c) but it is reached by a different route; it will hold in any steady growth state, and not only in a " long-run " Golden Age; it does not postulate a class of hereditary capitalists with a special high-saving propensity. In the special case $i = 0$, it reduces to the simple Pasinetti formula, $\rho = g/s_c$.

The assumption that corporations issue securities which are a constant fraction, i, of their investment, irrespective of anything else (in particular, irrespective of v) is of course arbitrary. It is possible to conceive of numerous other assumptions to characterise the corporations' collective behaviour with regard to the issue of new securities. For example it would be possible to assume that the corporations' issue of new securities will depend on the *ex-post* difference between their savings $(s_c P)$ and their investment (gK) and that such *ex-post* differences are only " recognized " at the end of certain intervals of time (of the accounting periods); any intervening difference being met by a depletion

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[1] The above equations assume that savings out of dividends are zero; cG is intended as the net excess of shareholders' consumption over dividend income. It would be possible to assume that there is only a single savings propensity for the household sector which applies equally to wages, dividends and capital gains. If we denote this by s_h, equations (3) and (4) above become

$$s_c P + s_h (W + (1 - s_c)P) - (1 - s_h)gK(v - i) = gK$$

and

$$s_h (W + (1 - s_c)P) - (1 - s_h)gK(v - i) = igK.$$

From this we obtain

$$\rho = \frac{g(1 - i)}{s_c}$$

as before and

$$v = 1 - \frac{1 - s_h \dfrac{Y}{gK}}{1 - s_h}$$

in place of equation (5) above. This implies that $v < 1$, when $s_h Y < gK$ (since $Y > gK$, in all cases).

[2] Assuming $g - s_w Y/K > 0$, it follows from (5) that $v < 1$ provided that

$$\frac{s_c - s_w}{cs_c}(1 - i) + i \leqq 1. \qquad \qquad \dots(i)$$

When $c = 1 - s_w$; (i) will hold if

$$\frac{s_c - s_w}{(1 - s_w)s_c} \leqq 0$$

or

$$s_c - s_w \leqq s_c - s_w s c. \qquad \qquad \dots(ii)$$

Since $s_c < 1$, (ii) must hold.

(or accretion) of their cash reserves. In other words, they issue securities periodically, to make good any past depletion of their liquid reserves; in the converse case, they respond *ex-post* to an accretion of cash reserves by redeeming securities (such as debenture issues or preference shares). Assuming that these accounting intervals are long enough, v and s will establish themselves at values corresponding to $i = 0$; this means that net personal savings will be zero, and the *ex-post* difference between $s_c P$ and gK will be zero: *no* new securities will, in fact, be issued (or redeemed) because there will be no occasion to. This kind of behaviour will thus lead to the simple Pasinetti formula $p = g/s_c$.

Has this " neo-Pasinetti theorem " any very-long-run " Pasinetti " or " anti-Pasinetti " solution? So far we have not taken any account of the change in the distribution of assets between " workers " (i.e. pension funds) and " capitalists "—indeed we assumed it to be constant. However since the capitalists are *selling* shares (if $c>0$) and the pension funds are buying them, one could suppose that the *share* of total assets in the hands of capitalists would diminish continually, whereas the share of assets in the hands of the the workers' funds would increase continually until, at some distant day, the capitalists have no shares left; the pension funds and insurance companies would own them all !

But this view ignores that the ranks of the capitalist class are constantly renewed by the sons and daughters of the new Captains of Industry, replacing the grandsons and granddaughters of the older Captains who gradually dissipate their inheritance through living beyond their dividend income. It is reasonable to assume that the value of the shares of the newly formed and growing companies grows at a higher rate than the average, whilst those of older companies (which decline in relative importance) grow at a lower rate. This means that the rate of capital appreciation of the shares in the hands of the capitalist group as a whole, for the reasons given above, is greater than the rate of appreciation of the assets in the hands of pension funds, etc. Given the difference in the rates of appreciation of the two funds of securities—and this depends on the rate at which new corporations emerge and replace older ones—I think it can be shown that there will be, for any given constellation of the value of the parameters, a long run equilibrium distribution of the assets between capitalists and pension funds which will remain constant. But it would require further investigation that goes beyond the limits of this Appendix to demonstrate this formally.