REPLY TO PASINETTI AND ROBINSON [1]

1. We must begin by recording our dismay that our long paper should end up appearing to Dr Pasinetti as primarily apologetics for a specific theory—the neo-classical theory of marginal productivity. We trust other readers will conclude otherwise.

It is true that we have succeeded in working out exhaustively, for the general neo-classical one-sector technology, the dynamic and statical implications of the Pasinetti hypothesis about different rates of saving out of pure-profit income and other incomes; thus, what Solow did a decade ago for the Harrod-Domar model, we have done definitively for the Pasinetti model, freeing our analysis from the earlier Cobb-Douglas assumptions of Meade and deriving stability conditions for the golden-age equilibria that are more definite than one ordinarily encounters in a model of this complexity. Just as Solow's earlier work called for subsequent extension to two-or-more-sector canonical models—by Uzawa, Solow, Inada, Findlay, Drandakis, Burmeister, and many others—so does our one-sector model of Pasinetti saving invite subsequent extensions to such technologies.

No apologies are needed then for the detail of our investigation. Furthermore, even those more critical of the degree of empirical relevance of the neoclassical notions than we are, might be expected to welcome our elucidation of just how neoclassical notions fit in with Pasinetti's earlier analysis, a task which he had only lightly touched upon (and in a set of equations blemished by a mathematical error). " Know thy enemy " is good advice for anyone, for to understand the full implications of a theory is to begin to understand its limitations.

2. None the less the major motivation for undertaking the paper was something quite different. It stemmed from our original perception that the Pasinetti golden-age equilibrium, instead of being *the general one*, had to be recognised as but one of two golden-age equilibria, being matched so-to-speak by what we called the Dual or Anti-Pasinetti equilibrium. (More neutral terminology would be Primal and Dual equilibria, a distinction that has essentially naught to do with the somewhat unfortunate earlier Meade terminology of neo-Keynesian and neo-classical.)

As is the usual case for duality relations, there is complete symmetry between the Primal and Dual equilibria. *Neither is more general than the other.* This symmetry Dr. Pasinetti once more denies. He continues to regard his golden-age equilibrium as the more general one, being in some special sense relevant independently of marginal productivity assumptions, of so-called well-behaved functional relationships between profit rates and capital-output ratios and so forth.

Let no reader misunderstand. We claim that the truth is made up of its two halves. The Dual golden-age regime is every bit as general as the Primal Pasinetti golden age: its existence has nothing to do with well-behaved marginal productivities or with one-directional functional relationships between interest rates, capital-output ratios, or other magnitudes.

Readers who have followed these discussions—read the 1962 Pasinetti article, the 1963 Meade paper and 1964 Pasinetti reply, the 1965 Meade-Hahn paper and the resulting 1966 interchange between Meade and Pasinetti, and our present paper—will, we think, sense which way the wind is blowing. But there does remain one constructive task for this reply, namely to demonstrate that the symmetry of generality between the Dual and Primal regimes does definitely hold for any *multiple-blue-print technology* of the kind that Professor Joan Robinson and MIT programmers think useful to analyze. This should

[1] Unfortunately, Kaldor's paper reached us too late for comment.

establish once and for all that symmetry does not depend on any simple diminishing returns assumptions of neo-classical type. The important thing is for the reader to understand the reasoning upon which the symmetry of the two regimes is based.

In providing this constructive demonstration, we shall also be able to isolate that *special* technological case, which if one believed it to be realistic, would provide considerable justification for concentrating on the Pasinetti regime to the exclusion of the Dual on the ground that the Dual regime is in the nature of a knife-edge solution. (And, dually, we shall be able to isolate that special technological case which makes the Pasinetti regime a rare and fortuitous occurrence of knife-edge type.)

Here we shall be examining only the question of *existence* of golden-age equilibrium, omitting completely all discussions of stability and instability and all consideration of the dynamic transient behaviour of the system when not in a golden age.

3. For simplicity assume a single consumption good (either simple or a composite market basket). Assume a finite number, large or small, of different blue-print pages, each corresponding to a different activity or capital process, calling different machines alpha, beta, gamma, ... in the usual Robinson fashion. As is well known, each profit or interest rate excludes many pages of the blue prints as not being competitively viable, leaving one or more sets of activities that can be viable at the given golden-age profit rate. We stress that nothing " well-behaved " is assumed about the technology other than that the factor-price frontier relating the real wage and the profit rate must be downward sloping. This frontier may have changing curvatures and along it there may be reswitching effects of the Cohen-Sraffa type, shifts toward lower capital-output ratios as the profit rate falls, and any kind of Wicksell effects. No singular equal-factor-intensity assumptions are made that might validate any surrogate capital concepts. In short, we shall demonstrate the symmetry of the two regimes where nothing faintly neo-classical is assumed about the technology.

Now assume a natural rate of growth of labour that is positive, say $n = 0{\cdot}04$ per year (for simplicity of exposition ignoring Harrod-neutral technical change). What golden-age configurations can prevail for this natural rate of growth, n, at each profit rate r? Clearly only competitively viable activities, and with a mix among the consumption and viable-machine sectors that provides for balanced growth at rate n of *all* physical inputs and stocks. Thus, for fixed n and each r, there will emerge an admissible configuration of processes and price ratios (it being understood that essential joint products like mutton and wool, which require demand conditions, are ruled out): hence, and this is what matters here, there will be for given n an admissible set of capital values and ratios of aggregate capital value to value of output, which could be plotted against the profit rates. It is slightly more convenient to work with the reciprocal of the aggregate capital-output ratio, which is the so-called average product of capital, A or $A(r;\ n)$: this is a percentage per annum, but is now quite divorced from any physical capital of jelly or surrogate type, being merely the ratio of value of total market stocks of capital to value of total output (both being expressed in *any* common *numeraire* unit).

4. Fig. 1 plots, for fixed positive n, all the various average products of capital (i.e. reciprocals of the capital-output ratios) that go with each rate of interest. The AZ locus thus indicates the $A(r;\ n)$ functional correspondence for a general blue-print technology. We have made it as " pathological " and " ill-behaved " as we could think of: for example, the locus is not single-valued, having instead vertical stretches inevitably at switch points, like RS where one finds blends of the equally-viable R and S techniques; moreover, between switch points we have not required the locus to be a rising one—as in the neo-classical parables where a lower interest rate had always to be associated with a higher capital-output ratio—but instead show the system moving to the lower capital-output ratio of R from that of S as the interest rate falls; and furthermore, on each side of the RS vertical segment, we show the curve as a falling one as a result of perfectly admissible

Wicksell effects; nor finally do we exclude any number of reswitching effects of the Cohen-*curiosum* type. (All this is in contrast with the Solow-Clark one-sector model involving neo-classical surrogate capital and smooth productivities, in which the AZ locus would be a simple ever-rising one.[1]) Furthermore, a change in n will shift the whole AZ locus, since the new weighting of the consumption and investment sectors would change capital-output ratios and factor shares in the absence of any special " equal factor-intensity " assumptions.

In Fig. 1 the 45° line represents points where the interest rate and average product of capital are equal, leaving no return at all for labour: thus it is the locus of capital

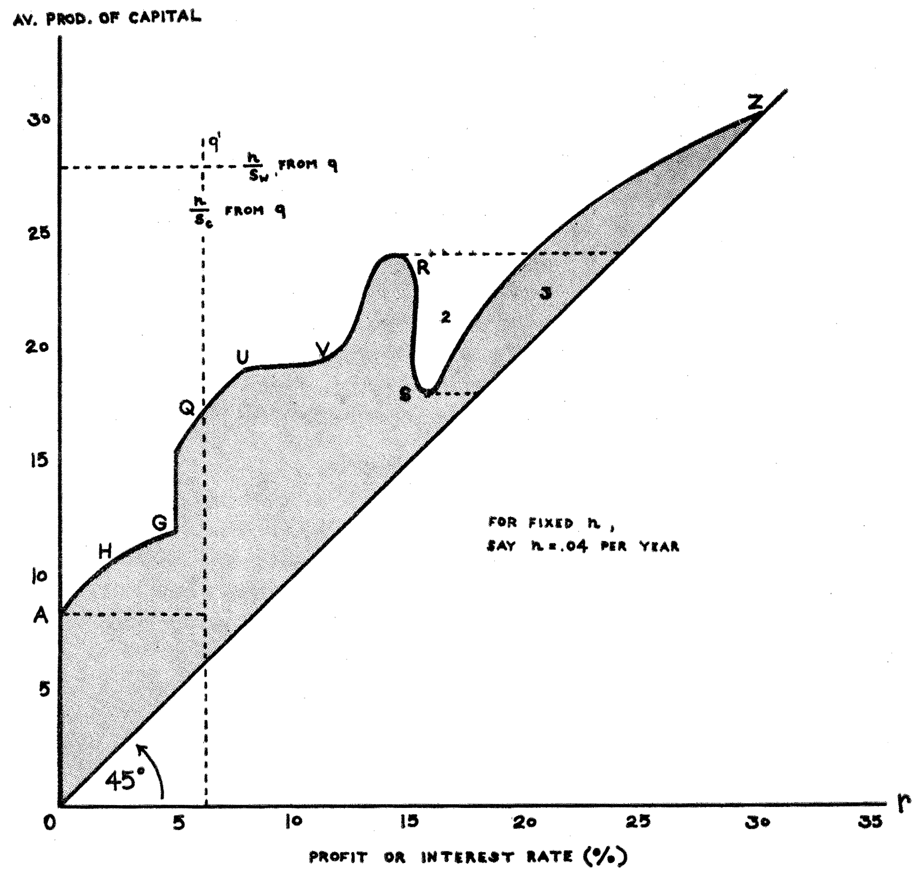

FIGURE 1

AZ is the locus of Average Products corresponding to each profit rate, where n has been fixed. At any point, like Q, the reciprocal of the slope of a ray from the origin to it measures capital's relative share, α_K: since labour's share cannot be negative, only the area above the 45° line is relevant. At A where $r = 0$, labour gets all; at the point of maximum interest rate Z, capital gets all. Vertical segments, like RS represent switch points; other segments where the locus slopes downward represent so-called perverse Wicksell effects and other admissible phenomena not found in one-sector well-behaved neoclassical models.

[1] In these neoclassical cases, $r = f'(k)$, $f''<0$. If the elasticity of substitution, σ, is always 1, the Cobb-Douglas case, AZ is a straight line through the origin with slope greater than one; for the CES function with σ a constant below unity, it is a convex curve going out from the origin and arching above the 45° line until it hits that line at the maximum r; if σ is a constant above unity, AZ begins at a point on the 45° line, accelerating above the line toward infinity like a parabola; for $\sigma = \infty$, AZ is a vertical curve above the 45° line; for $\sigma = 0$, AZ is a horizontal curve from the vertical axis to the 45° line; if $f'(k)$ is asymptotic to both axes, with σ passing through unity, AZ rises out from the origin but not necessarily with fixed curvature.

share equal to unity, or $\alpha_k = 1$. At any point on the locus, capital's share α_k is read off as the reciprocal of the slope of a ray out from the origin to the point on AZ, by virtue of the arithmetical relations,

$$\alpha_k = r/A = (\text{profit/capital})/(\text{output/capital}) = \text{profit/output}.$$

Since wages can never be negative, we never have use for the part of the diagram below the 45° line.[1]

Thus far all has been technology and competitive cost minimization. Fig. 2 now introduces saving behaviour of the Pasinetti type. On the horizontal axis is plotted s_c, the fraction of profit permanently saved by those who earn only profit; on the vertical axis is s_w, the fraction of income of whatever sort permanently saved by those who work *and* own capital from previous accumulation. From any point on Fig. 2, like q with its specified (s_c, s_w) values, and with n given, our task is to go back to Fig. 1 and find the corresponding golden-age equilibrium point or points. It is our contention that this can be done for a general blue-print technology; and when it is done, we find that there are two symmetrical regions, the one marked Dual and the one marked Pasinetti in Fig. 2.[2]

Let us review the requirement for golden-age equilibrium that *any* technical model must satisfy once n and (s_c, s_w) are specified. An (r, A) point on AZ of Fig. 1 can be a golden-age solution (r^∞, A^∞) if and only if it satisfies the following four conditions:

(i) It must be on the AZ locus $A(r;\ n)$, or $A^\infty = A(r^\infty, n)$, to be competitively viable

(ii) $r^\infty \leqq n/s_c$, so that K_c does not grow faster than L

(iii) $A^\infty \leqq n/s_w$, so that K_w does not grow faster than L

(iv) At least one of (ii) and (iii) must hold as an equality, or $(r^\infty - n/s_c)(A^\infty - n/s_w) = 0$, so that K grows as fast as L.

Note that (i)-(iv) hold for *any* technology.

There are two regimes satisfying (i)-(iv), Pasinetti and Dual.

Pasinetti:

$$r^\infty = r^* = n/s_c$$

$$A^\infty = A(r^*;\ n) < n/s_w, \text{ or } s_w < \frac{r^\infty}{A^\infty} s_c = \alpha(r^*;\ n)s_c.$$

Dual:

$$A^\infty = A^{**} = n/s_w$$

$$r^\infty = r^{**} < n/s_c, \text{ or } s_w > \frac{r^\infty}{A^\infty} s_c = \alpha(r^{**};\ n)s_c.$$

Of course, there is the borderline knife-edge where

$$r^\infty = r^* = r^{**} = n/s_c \text{ and } A^\infty = A^{**} = A^* = n/s_w$$

and we are simultaneously in both regimes, but with $(k_c/k)^\infty = 0$.

After this review, we can relate Figs. 2 and 1. To demonstrate all this, first concentrate on a point like q in Fig. 2, which represents high rentier thrift relative to worker

[1] The vertical axis below A is shown as part of the boundary in Fig. 1, which is debatable in that it assumes that capital goods representing excess capacity can be accumulated viably at a zero interest rate (perhaps being costlessly stored). If capital goods are subject to exponential depreciation independently of use and storage methods—say at a rate of 5 per cent per year—AZ would have to be extrapolated into the second quadrant of negative interest rates, with a vertical line down to the axis stemming from the $-0{\cdot}05$ terminus. If money is costlessly storable and prices are not rising at a 5 per cent or greater annual rate, it might be impossible to effectuate such a negative real interest rate. When we come to discuss regions of possible " oversaving ", issues of effective demand and liquidity traps will depend upon which of the various assumptions concerning A's position is made.

[2] When the mapping is done properly, one recognises that the shaded area in Fig. 1 corresponds in a certain definite sense to the shaded Dual area in Fig. 2, and likewise for the unshaded Pasinetti regions.

thrift. In Fig. 1, Q will turn out to be the only equilibrium corresponding to q of Fig. 2, and will represent Pasinetti equilibrium. Why? Because on Fig. 1 we locate q's vertical n/s_c line and its horizontal n/s_w line: these intersect in the q' image-point on Fig. 1 corresponding to q of Fig. 2. Our equilibrium conditions above tell us we must end up either due south of the image-point q' or due west of it; and only Q on AZ, south of q', can be found to meet these conditions. Because we are on the n/s_c vertical, we are assuredly in

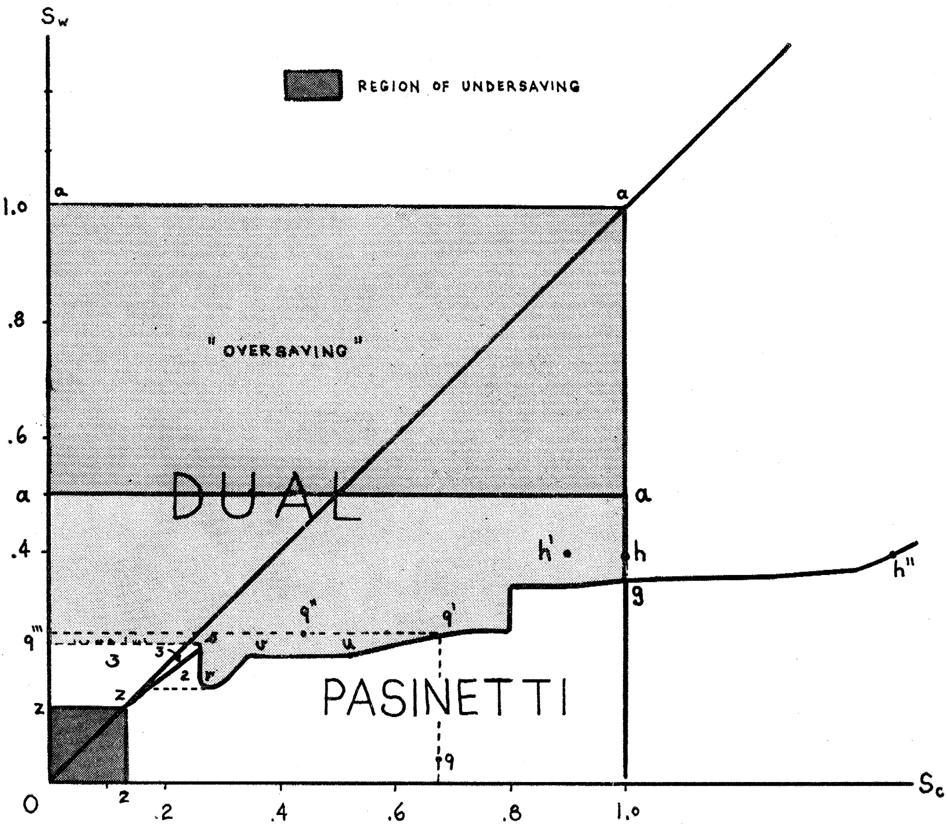

FIGURE 2

The boundary *zsrvuq'gh''* divides the whole region of saving coefficients into the Dual equilibrium region and the Pasinetti or primal equilibrium region. This boundary provides our $s_w \gtreqless \alpha_x^* s_c$ criterion, and is given by an exact mapping of Fig. 1's AZ locus into Fig. 2's variables, after taking into account the division into n of the saving coefficients. Directions are reversed because of this reciprocation, up becoming down and left right, but vertical and horizontal segments and direction reversals correspond exactly. If saving coefficients are so low that capital cannot widen to keep up with labour growth, we are in the undersaving region of the southwest. (If s_w becomes so large as to produce excess capacity and zero profit rate, we are in the " oversaving " region of the north with possible effective demand and liquidity-trap problems. The small regions marked with 3's and 2's represent positions of multiple equilibria, resulting from the waviness of AZ.)

Pasinetti equilibrium. The n/s_w horizontal line of our original q would intersect the AZ locus at an irrelevantly higher interest rate, and hence the dual equilibrium just cannot occur for the (s_c, s_w) values given by Fig. 2's q.

What is true for q is true for any other point on Fig. 2, on the vertical line through q, up to the boundary point q'. For all such points we are on the same vertical n/s_c going through Q, with the competing horizontal n/s_w irrelevantly higher than Q. And now we recognize why q' does indeed represent the boundary point on the border between the Primal and Dual equilibria. By definition, q' is that point which has as its image point a point on the AZ locus itself (namely Q): both its vertical n/s_c and horizontal n/s_w lines are fully relevant and hence both Dual and Pasinetti equilibrium criteria are simultaneously satisfied.

Now we can show how a Dual equilibrium point is determined. Consider in Fig. 2 the point q'', which is due west of the boundary point q'. If we plot its n/s_w horizontal, that will of course go through Q in Fig. 1 because its q'' is at q's latitude. But because its s_c is smaller than that of q', its vertical n/s_c will be east (east, not west) of Q, corresponding to a still higher interest rate. Now it is the vertical which is irrelevant, and so q'' does go over into Q—*but now with a Dual equilibrium.* Similarly every point due west of q' gives rise to the same Dual equilibrium at Q, as the reader should verify.[1]

It should be clear that the *zsrvuq'gh;* boundary in Fig. 2 corresponds in precise detail to the AZ locus in Fig. 1, the only difference being that the reciprocals of the variables are taken and scale changes are made corresponding to the growth rate n. If we had plotted the capital-output ratio and the reciprocal of r (i.e. the " number of years purchase ") in Fig. 1, the diagrams would have been exactly the same except for scale factor n. If both figures had been plotted on double-log paper, one locus would simply be the upside-down reversed image of the other. The equation of Fig. 2's boundary is simply given in terms of 1's $A(r;\ n)$, in accordance with our general criterion:

$$s_w = \alpha_K(r;\ n)s_c = \frac{r}{A(r;\ n)}\, s_c = \alpha_K(n/s_c;\ n)s_c.$$

This last expression was called $\alpha_K^* s_c$ in our paper.

We must warn against a possible, but quite unnecessary confusion. In Fig. 2 one can be at *any* point whatsoever; in Fig. 1, one is at equilibrium *only on* the AZ locus. When you are at any point in the Pasinetti region of Fig. 1, s_w is *deduced* to be less than the golden-age I/Y ratio. When one is at any point in the Dual region, s_w is deduced to be *equal* (yes, equal) to the golden-age I/Y ratio. Dr. Pasinetti has misunderstood the mathematics of our paper and of this reply if he thinks that s_w is kept from ever exceeding the golden-age I/Y ratio by some neo-classical regularity assumption on our part. Here we make no such regularity assumption, and yet s_w greater than golden-age I/Y is quite impossible. It is quite impossible from the nature of golden-age equilibrium under Pasinetti's own saving hypothesis. Remember that the workers save from two sources —from wages as well as from the same profits source that rentiers save from; and if workers saved a larger fraction than the whole community saves, that would imply the ultimate (relative) extinction of the rentier class and a self-contradiction in the form of the whole's saving more than itself saves!

There remains a need to explain the square region of undersaving *Ozzz* in Fig. 2. For very small saving propensities, relative to the need to widen capital to keep up with labour growth, the system will be unable to come into balanced growth, instead approaching the maximum interest rate as shown at Z. This may happen with either regime dominant depending upon the relative sizes of s_c and s_w. (We have discussed such matters in our Appendix.)

Perhaps more interesting is the *aaaa* region of " oversaving " in Fig. 2. If there is a minimum capital-output ratio beyond which capital goods become redundant, too high

[1] The reader can test his comprehension by examining the golden-rule golden age configuration shown at G in Fig. 1. Here the interest and growth rates are equal, $n = r$ (both being equal to 4 per cent per annum) In Fig. 2 the point g, or any point west of it or below it, will give rise to this configuration of maximum *per capita* consumption. But note that only in the Dual equilibrium can one attain golden ages with interest rates lower than it. This is because 100 per cent at most can be saved out of profit income and that will be insufficient to maintain capitals in a regime of still lower interest, as at H (and h or h'). If population growth is expected to fall in the distant future—or if society will want to disinvest capital in the future, perhaps to provide consumption for a transiently higher population—it may be rational for society to plan to be *now* at a point like H, showing that a socialist society might well often want to save much more than the income on the *property* it owns, preferring to use its taxing power on the labour of the people to finance greater capital formation.

saving on the part of workers can lead to a Dual equilibrium at A where the rate of interest has been bid down to zero. Needless to say it is somewhat paradoxical if people should persist in wanting to save a positive fraction of their incomes when there is no useful purpose served by further capital goods of any kind. Still, as Keynes emphasized in 1936, in a monetary economy where people might try to save or hoard against some future need, there could arise " liquidity-trap " and other effective-demand problems. It is noteworthy that the liquidity-trap-at-zero-r problem arises only in the Dual equilibrium. This is because, in the Pasinetti equilibrium, reduction of profits to zero will automatically take care of any saving out of profits. (However, if one thinks it realistic to fear " liquidity-profit traps " at higher-than-zero rates of return, because of risk aversions and other factors, then the Pasinetti regime can also run into effective demand troubles.) We put " oversaving " between inverted commas because we realize that various devices to tax money to create a negative interest rate or various assumptions excluding money or any hoardable asset might do away with some of these effective demand problems. However, this is not the place to grapple with such issues.

We relegate to a footnote the elucidation of the areas in the two figures marked with 2's and 3's, simply remarking here that reversals of direction in the $A(r)$ locus inevitably bring in the possibility of multiple Dual equilibria.[1]

In concluding this taxonomic discussion, note that in the neoclassical case with a monotonic AZ going out forever from the origin, the " oversaving " region of Fig. 2 shrinks away to nothing and the undersaving region to the point at the origin, leaving us with simple Pasinetti and Dual regions with a well-behaved boundary between them.

5. By now we think the reader will agree with us that we have shown the complete symmetry between the Primal and Dual equilibria. In general, neither is more general than the other.

There are, however, special cases in which one becomes more restricted than the other. Because Professor Meade has already touched upon elements of this in connection with his 1966 one-sector discussion, we shall be very brief.

Suppose the AZ curve is always a strict horizontal line. That is a constant capital-output ratio with a vengeance. (And it can strictly occur only if there are completely fixed coefficients in both the consumption and other sectors and if in addition there are strong " equal-intensity " or " organic composition of capital " assumptions made. Otherwise a shift in n would shift and twist the AZ locus, permitting it to be horizontal for at most one n and then only because of fortuitous and singular Wicksell effects. If AZ is to be " approximately constant ", we must be " approximately near to " the fixed coefficient case just described.)

In this special case, as Fig. 3*a* shows, the Dual equilibrium region is completely swallowed up by the regions of undersaving and " oversaving ". Only the Pasinetti region is free of these pathologies. So after all, detailed analysis does demonstrate what we had suspected all along—that the primacy of the Pasinetti equilibrium to the exclusion of the Dual equilibrium is not a *general* feature of these systems but rather is true only in

[1] Any point in the region of Fig. 2 marked with 3's has its horizontal n/s_w line intersecting AZ in three places, while at the same time its vertical n/s_c line is irrelevantly off to the east of all of them. Hence any of the three points represents Dual golden-age equilibrium. It would take us beyond our present task to discuss their local and global stabilities. (Note that horizontal segments UV and vu give rise to an infinity of alternative Dual equilibria.) Any point in the region marked with 2's has its horizontal n/s_w line with the above triplet property; but now its vertical n/s_c lies west of one of the intersections and thus invalidates it, leaving us with two valid Dual equilibria and one valid Pasinetti equilibrium due south of the point in question. These multiple Dual equilibria, with varying interest rates show that our heuristic rule that " the system tries to find the lowest interest rate " does not really represent true teleology. The system doesn't really care if it ends up at a Dual equilibrium with a higher interest rate rather than with a lower one.

Multiple equilibria for the Pasinetti case occur only on the vertical loci, RS and sr. Of course the multiplicity there does not affect the uniqueness of the profit rate, which is everywhere the same on any such vertical; but it does affect the observable capital-output ratios, which are in indifferent equilibrium along segments of such a vertical stretch.

the special case which involves a fixed capital output ratio or borders on it. Those whose studies of modern technology suggest to them that such constancy is realistic are welcome to the hypothesis. Our reading of experience and engineering points quite otherwise. But in any case our earlier discussion of the fixed-coefficient model, far from being an exercise in polemics, should be welcomed by those who believe in that model's relevance. In any case, let no one claim that from the general laws of arithmetic and logic comes any primacy of one of the equilibria as against its Dual. (Remember the Primal is Dual to the Dual, as usual.)

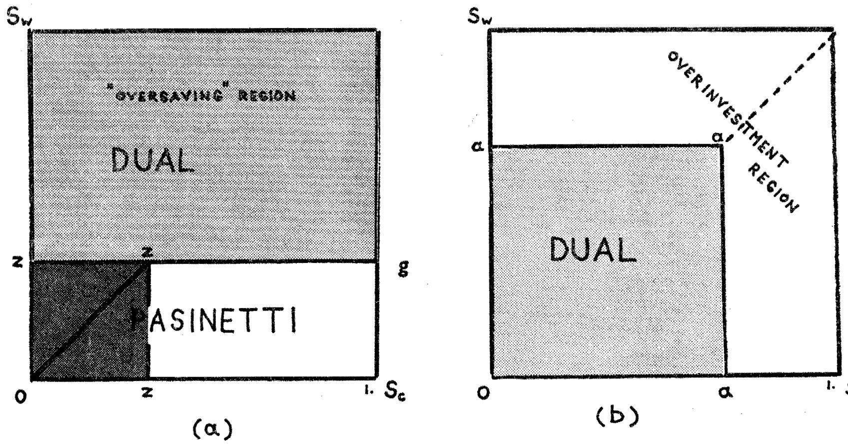

FIGURE 3

These show the complete symmetry of the singular special cases in which one of the two equilibria turns out to be more restricted than the other. In 3*a* we have the fixed coefficient case where the only viable Dual regime that does not involve an " oversaving " danger is along the knife-edge *zg*. This special case leaves the most favourable room for an alternative theory of distribution of macroeconomic type. In 3*b* we have the case of unlimited possibilities to use capital at a minimal interest rate, with no diminishing returns to accumulation possible. Now the only viable Pasinetti equilibrium shrinks down to the knife-edge *aa*; to its left there are an infinity of viable Dual equilibria, but above and to its right the saving propensities will be so great as to lead forever to investment that causes capital to grow faster than labour, permitting no finite golden age to occur.

To clinch the fact that there is symmetry even for the special cases where one equilibrium is more restricted than the other, examine Fig. 3*b*. This represents a technology, perhaps of robots, where the golden-age profit rate cannot be other than at one positive number. Now the Pasinetti region shrinks into the vertical razor's edge shown as *aa*, since only one n/s_c level will happen to match the technologically given profit rate. Any s_c below this critical level will give rise to a Dual equilibrium in which s_c is irrelevant. For any s_c (or for that matter s_w) above this critical level, the growth rate of capital goods will forever exceed that of labour, giving rise to no finite golden-age equilibrium at all. We have labelled this a region of overinvestment rather than " oversaving " since, as our Appendix had suggested, in such a case there need not be effective demand problems in virtue of the fact that interest remains positive.[1]

6. Our qualitative thesis has now been substantiated against Dr Pasinetti's objections. In general, both regimes occupy a considerable area in the crucial Fig. 2 domain. Their relative importance cannot, of course, be gauged by measuring the areas they occupy: had that been the case, the Dual regime would be given a spurious look of importance in Fig. 2. However, a look at Fig. 1 provides a corrective to such a spurious conclusion. Since reciprocation makes one area large where it had previously been small, in Fig. 1

[1] Incidentally, if AZ in Fig. 1 had begun at a point A that fell up on the 45° line itself, as in the case where σ is a constant above unity, we should already have encountered an overinvestment rather than oversaving region in the Dual domain, and might well have encountered an overinvestment region in the Pasinetti domain. However, Fig. 1 restricts itself to finite blue-print activities.

the Dual area looks smaller than that of the Primal. And if we had used double-log charts both areas would be infinite in size! Obviously the economic relevance of the two kinds of equilibrium would have to depend upon the *empirical* probability density with which it is reasonable in a modern mixed economy to expect the saving propensities in Fig. 2 to fall. To Dr Pasinetti's critical remarks on our econometric estimates of these quantities we now briefly turn. (But in doing so, we first gather some important fruit from the present qualitative exercise. We now know that the observable capital-output ratios and factor shares can be summarized completely on the *AZ* locus of Fig. 1; and that every such observable point can belong to a point on the boundary in Fig. 2 or to an *infinity* of points in either region which share its latitude or longitude. Hence, without *independent* measurement of at least one of s_c or s_w, one cannot in general hope to infer, even in principle, which of the equilibrium regions a given economy does truly belong to!)

7. And this leads us to comment briefly on Pasinetti's criticism of the values of α and s_c—namely 0·25 and 0·20, respectively—which we have used to illustrate why, in our judgment (s_c, s_w) is most unlikely to fall in the region of Fig. 2 which corresponds to a Pasinetti solution. We must first acknowledge that our footnote reference to the above numbers as " econometrically reasonable " was rather unfortunate since this evaluation really meant to apply only to the estimate of α. As we have made it abundantly clear in the concluding section, we do not believe that there exists an identifiable class of capitalists in Pasinetti's definition. i.e. a class of permanent pure profit receivers whose present members' forefathers and heirs derive, have derived and will derive their entire income exclusively out of capital. In view of this disbelief it would make little sense to speak of a " reasonable " estimate of their propensity to save. What we meant to convey is that, even if we set aside our qualms and just to play the game were willing to identify the capitalists with today's rentiers, we would find it hard to believe that the propensity to save of this group—a quite varied lot of hereditary aristocrats, playboys and retired households—would exceed some 20 per cent. When this figure is combined with the much more solid estimate of α it implies, that for Pasinetti's theorem to hold the saving propensity of the rest of the community, s_w should be no more than 5 per cent. This we regard as an unplausibly low figure when it is remembered that the " workers " to which s_w applies are again a quite mixed lot including labourers, professionals, self-employed entrepreneurs, civil servants, business executives of all ranks, and so on. In short, " the workers " are in fact a fairly representative cross-section of the " active population " and therefore their propensity to save should not be appreciably different from the overall average propensity—possibly a shade lower because of the exclusion of some very rich heirs, possibly a shade higher because of the exclusion of retired households and small rentiers.

Pasinetti criticizes these values of α and s_c on the ground that they " imply an investment to income ratio of 5 per cent per year . . . less than half those observed in U.S., U.K., and western Europe ". But in deriving this supposed " implication " Pasinetti is simply taking as already proven the very proposition that is under debate, to wit, that the economies from which we draw these estimates of α, n and I/Y are in fact in Pasinetti golden age equilibrium! If we are not in such a golden age equilibrium that α and s_c imply nothing about I/Y, and conversely; and if we are in (or close to) a Dual golden age, then the only implication we can draw is: $\alpha s_c < I/Y$. With I/Y of the order of 12 to 16 per cent and α of the order of 0·25, this means that s_c must be less than $\frac{1}{2}$ to $\frac{2}{3}$, perfectly consistent with our conviction that s_c is unlikely to exceed significantly some 20 per cent. In fact we submit that Pasinetti's own figures provide striking support for our contention. For, given the above values of I/Y and α, to suppose that we are in Pasinetti golden age equilibrium would imply a value of s_c of between $\frac{1}{2}$ and $\frac{2}{3}$, which, to paraphrase Pasinetti, seems to us far beyond the saving propensity of the rentier class either at present or in the foreseeable future.

8. The above discussion paves the way for a consideration of the Comment by

Professor Joan Robinson. As she says, we did put the rabbit into the hat in full view of the audience before drawing it out again. i.e. our logical theorems do follow correctly from our axiomatic conditions, a fact for self-congratulation not apology. Her further implication—that our logical proofs of stability and existence are so transparently obvious as to involve a trivial waste of time—reveals more what she considers tiresome than an objective finding. Actually, in her (incomplete) summary of our analysis there is naught for us to quarrel with or to give comfort to Dr Pasinetti's critique. What is useful in her comment is the reminder that the one-sector leets model does have special properties that must not be extrapolated to more general models. In this Reply, we have returned good for good—showing in Figs. 1 and 2 what happens to existence problems in a general blue-print technology. This same discussion throws light on the problem she once discussed under the heading of Harrod's Knife Edge.[1] Her two limiting cases

$$(s_c, s_w) = (s_c, 0) \text{ and } (s_c, s_w) = (s, s)$$

are represented in Fig. 2 by the horizontal axis and the 45° line: the Dual and Pasinetti regions of Fig. 2 show the variety of patterns to be expected (including undersaving and oversaving possibilities) and show that the Dual (or Harrod-type) equilibrium can be subject to an embarrassment of riches in the form of multiple rather than non-existent solutions.[2]

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[1] J. Robinson, *Collected Essays*, Vol. III, pp. 52-55.

[2] For a 2-sector model with homogeneous capital, the system satisfies equations of the form

$$\dot{k} = T(k, 1;\ c) - nk$$
$$\dot{k}_c = [s_c \partial T(k, 1;\ c)/\partial k - n]k_c$$
$$\dot{k} = s_w[k\partial T(k, 1;\ c)/\partial k + \partial T(k, 1;\ c)/\partial L] + (s_c - s_w)[k_c \partial T(k, 1;\ c)/\partial k].$$

Equate the right-hand sides of the first and last relations and repeat the first two relations to get the 3-variable system of the form

$$\dot{k} = f^1(k, k_c;\ c;\ n, s_c, s_w)$$
$$\dot{k}_c = f^2(k, k_c;\ c;\ n, s_c, s_w)$$
$$0 = f^3(k, k_c;\ c;\ n, s_c, s_w).$$

In the neighbourhood of a golden-age stationary solution (k^∞, k_c^∞), the last equation can generally be solved for c, leading after substitution to an autonomous differential equation system

$$\dot{k} = F^1(k, k_c;\ n, s_c, s_w)$$
$$\dot{k}_c = F^2(k, k_c;\ n, s_c, s_w)$$

whose (k^∞, k_c^∞) point (or points) can have its stability tested by the usual methods, revealing no doubt points possibly unstable because of Wicksell effects.