Master APE - Economics of Inequalities

Final Exam, February 29, 2013 - 15h00-17h00. No document allowed.

Exercise 1: Income Inequalities (5 points)

In 1953, Kuznets suggested that long-run inequalities follow an inverted U-shape over the path of development.

1) Does Kuznets' hypothesis accord well with the data in developed economies? Why? (1 point)

Answer No: rising top income shares in anglo-saxon countries; and slightly rising or flat top shares in continental Europe and Japan.

2) What is the share of pre-tax income earned by the top 1% in the U.S. today? How does this share compare to its level in the 1920s? (1 point)

Answer 20%, same as in the 1920s.

3) Discuss the role that institutions may have played in the evolution of inequalities in rich countries since the 1970s. (1.5 point)

Answer Unions, social norms, taxes, education policy, governance, etc. Main problem = difficult to identify.

4) Discuss the role that market forces may have played in the evolution of inequalities in rich countries since the 1970s. (1.5 point)

Answer skill-biased technical change, global competition for skills, superstars, etc. Pb: why surge of top income shares in anglo-saxon countries only?

Exercise 2: Capital Inequalities in the Dynastic Model (5 points)

Consider a discrete-time closed economy economy with a continuum [0; 1] of infinite-horizon dynasties that maximize utility function:

$$U_t = \sum_{t \geqslant 0} \frac{U(c_t)}{(1+\theta)^t}$$

where c is consumption, θ is the rate of time preference, $U'(c) > 0$ and $U''(c) < 0$. Output is given by a standard production function $f(k_t)$ where k_t is the average capital stock of the economy at period t. Markets are fully competitive.

Assume a two-points distribution of the capital stock: dynasties can either own a large capital stock k_t^A or a low capital stock k_t^B. The proportion of high-capital dynasties is exogenous and equal to λ (and the proportion of low-capital dynasties equal to $1-\lambda$).

1) What is the steady-state interest rate r^* equal to? (1 point)

Answer $r^* = \theta$: this is a direct consequence of the first-order condition of each dynasty's maximization problem (Euler equation).

2) Express the steady-state average capital stock k^* as a function of f and θ (1 point)

Answer Perfect markets imply that k^* must be such that $f'(k^*) = r^* = \theta$.

3) Express the average capital stock in the economy k_t as a function of λ, k_t^A and k_t^B. (1 point)

Answer $k_t = \lambda k_t^A + (1-\lambda) k_t^B$.

4) Characterize the set of possible steady-state distributions of capital (k^A, k^B) in this model. (1 point)

Answer Any distribution of capital (k^A, k^B) such as the average capital stock in the economy is equal to k^* is a steady-state.

5) Contrast steady-state capital inequality in the dynastic model with steady-state capital inequality in the pure lifecycle model in which capital accumulation is entirely driven by life-cycle motives (saving for retirement), there is no uncertainty, and no economic and population growth. (1 point)

Answer In the pure dynastic model, capital accumulation = class war. In the pure lifecycle model, capital accumulation = age war.

Exercise 3: Wealth-Income Ratios in Open Economies (10 points)

We consider an open economy in which we assume away government assets and liabilities. At time t, national wealth W_t is thus equal to the domestic capital stock K_t plus the net foreign asset position W_{Ft} which can be positive or negative: $W_t = K_t + W_{Ft}$.

Domestic income Y_{pt} (i.e., the output of domestic production activities) is given by a standard Cobb-Douglas net-of-depreciation production function $Y_{pt} = F(K_t, H_t) = K_t^\alpha H_t^{1-\alpha}$ where K_t is the domestic capital stock, $H_t = L_t e^{gt}$ is the quantity of efficient labor, g is the exogenous rate of productivity growth, and L_t is the quantity of raw labor.

There is a unique, constant world rate of return r on capital. National income Y_t is equal to the sum of domestic income Y_{pt} and net foreign asset income rW_{Ft}:

$Y_t = Y_{pt} + rW_{Ft}$.

We define:

- $\beta_{Kt} = K_t/Y_{pt}$ the domestic capital / domestic income ratio.
- $\beta_{Ft} = W_{Ft}/Y_{pt}$ the foreign wealth/ domestic income ratio.
- $\beta_t = W_t/Y_t$ the national wealth / national income ratio.

The goal of the exercise is to compute the steady-state national wealth / national income ratio β^*. We first start with two empirical questions.

1) What are typical values for β_t today in developed economies? (1 point)

Answer 400-600%.

2) Looking at the historical record of rich countries, how large can β_{Ft} be? (1 point)

Answer As large as 100-150% (France, beginning of twentieth century) and even 200% (UK, beginning of twentieth century).

3) In the model described above, explain why β_{Kt} is permanently equal to $\beta_K^* = \alpha/r$ (1 point).

Answer At any point in time we must have that the rate of return on capital r is equal to the flow of domestic capital income divided by the stock of domestic capital K_t. But with Cobb-Douglas production the flow of domestic capital income is equal to αY_{pt}, and so for all t we have $r = \alpha Y_{pt}/K_t = \alpha/\beta_{Kt}$, hence $\beta_{Kt} = \alpha/r$.

4) We define $Y_{Lt} = (1-\alpha)Y_{pt}$ the flow of labor income in the economy, and $Y_{Kt} = rW_t = \alpha Y_{pt} + rW_{Ft}$ the flow of capital income (domestic capital income plus net foreign asset income). We define s_L the net saving rate on labor income, and s_K the net saving rate on capital income. Show that the increase in foreign wealth $dW_{Ft}/dt = \dot{W}_{Ft}$ is equal to (1 point):

$$\dot{W}_{Ft} = s_L Y_L + s_K Y_K - g\alpha Y_{pt}/r$$

Answer $W_F = W - K$
$\dot{W} = s_L Y_L + s_K Y_K$ and $\dot{K} = gK = g\alpha Y_{pt}/r$

5) We define s as $s = (1-\alpha)s_L + \alpha s_K$. By differentiating $\beta_{Ft} = W_{Ft}/Y_{pt}$, show that β_{Ft} obeys the following dynamic equation (1 point):

$$\dot{\beta}_{Ft} = (s - g\alpha/r) - (g - s_K r)\beta_{Ft}$$

Answer: This follows directly from applying the chain rule to $\beta_{Ft} = W_{Ft}/Y_{pt}$, then using the result of question 4, and re-arranging.

6) If $s_K r > g$, what happens to β_{Ft} as $t \to +\infty$? Interpret (1 point).

Answer: The dynamic equation admits no stable steady-state: β_{Ft} tends towards infinity. In the long run domestic output becomes negligible as compared to foreign asset income.

7) We note $r^* = \alpha g/s$ (r^* would be the steady-state rate of return on capital if the economy was closed). Using the equation in question 5, compute the steady-state β_F^* when $s_K r < g$, as a function of s, r, r^*, g, and s_K (1 point).

Answer: Setting $\dot{\beta}_{Ft} = 0$ and re-arranging we find:

$$\beta_F^* = \frac{s(1 - r^*/r)}{g - s_K r}$$

8) When is β_F^* positive? Negative? Interpret. (1 point)

Answer: $\beta_F^* > 0$ iff $r > r^*$, i.e if the world rate of return is higher than the closed-economy rate of return.

9) Show that the national wealth / national income ratio β_t is always equal to (1 point):

$$\beta_t = \frac{\beta_{Kt} + \beta_{Ft}}{1 + r\beta_{Ft}}$$

Answer: This is simply a consequence of the definition of these ratios.

10) Use the results of questions 3, 7, and 9 to compute β^*, as a function of s_L, s_K, g, and r (1 point).

Answer:

$$\beta^* = \frac{s_L}{g - r(s_K - s_L)}$$