On the Long-Run Evolution of Inheritance: France 1820-2050

Data Appendix

Part 1

Thomas Piketty

Paris School of Economics *

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* Professor of Economics at the Paris School of Economics (PSE) & Directeur d'études at the Ecole des Hautes Etudes en Sciences Sociales (EHESS)

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Appendix A: National Accounts Data

The first key data source used in this research is national income and wealth accounts. The main conceptual and methodological issues regarding national accounts and the way we use them, in particular in order to compute the economic inheritance flow, are discussed in the working paper (see sections 3.1 and 3.2). In this appendix we provide the complete series used in this research, as well as additional details about sources, methodology and concepts.

In section A1 we describe our general series on national income Y_t and private wealth W_t in France. In section A2 we describe how we used these series in order to compute the economic inheritance flow series B_t. In section A3 we provide supplementary series on the structure of national income Y_t (including decomposition by production sector, factor income, taxes and savings, etc.). In section A4 we provide supplementary series on the structure of private wealth W_t (including decomposition by types of assets, etc.). Because of the incompleteness of available private wealth series, especially regarding the 1914-1969 period, we have to construct our own annual series, which we do by estimating an accumulation equation for private wealth in France, using savings flows from national accounts; full details on this method and resulting series are provided in section A5. Finally, in section A6 we provide supplementary series on price indexes in France, which we use at various points in the previous tables.

A.1. General national accounts series for France: Y_t and W_t (Tables A1-A2)

Our national income series Y_t and private wealth series W_t are reported on Table A1 (annual series) and Table A2 (decennial averages). Here we describe how these tables were constructed.

Col. (1) of Tables A1-A2: National income Y_t in current prices

Our basic series for national income Y_t are reported on col. (1), expressed in current billions currency, by which we mean current billions euros for the 1949-2009 period and current billions old francs for the 1820-1948 period.[1]

[1] The old franc was replaced by the new franc on January 1st 1960 (1 new franc = 100 old francs), and the new franc was replaced by the euro on January 1st 2002 (1 euro = 6.55957 new francs). In order to convert

National income Y_t is defined according to the standard international definition: it is equal to gross domestic product minus capital depreciation plus net foreign factor income.[2]

We use the official Insee series for the 1949-2009 period,[3] and the Villa (1994) retrospective series for the 1896-1949 period, with minor adjustments so as to ensure continuity in 1948-1949.[4] The various subcomponents of Y_t are given in section A3 below.

There also exists annual series for French national income covering the 1820-1896 period. But we do not feel that the year-to-year variations depicted in these series are fully reliable. In addition we do not really need annual series for our purposes. Therefore prior to 1896 we only provide decennial averages estimates of national income. I.e. the value of 11.3 billions old francs reported for 1820 on col.(1) of Table A2 corresponds to an estimated arithmetic average of national income Y_t over the years 1820-1829, the value of 13.5 billions old francs reported for 1830 corresponds to an estimated arithmetic average over the years 1830-1839, and so on.[5] We computed the average 1820-1929 to 1890-1899 estimates by using the annual series provided by Bourguignon and Lévy-Leboyer (1985), anchored to the 1900-1909 and 1910-1913 values obtained from our annual series.[6] If we were to use alternative series due to other authors such as Toutain (1997),

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1949-2001 current currency values into what we call current euros, we simply divided 1960-2001 new francs values by 6.55957, and 1949-1959 old francs values by 655.957. Current prices national accounts series released by Insee adopt the same monetary convention.

[2] See section A3 below for the corresponding equations and decompositions, using standard ESA 1995 definitions. All raw series and computations are provided in the appendix excel file.

[3] We downloaded the complete set of recently released Insee retrospective 1949-2008 national income accounts series on www.insee.fr on 15/09/2009. We used Insee tables 3.101 to 3.601 and the "tableaux économiques d'ensemble". It is preferable to use these Insee tables rather than the series released by international data collectors such as Oecd, Eurostat or the Imf, because Insee tables are more detailed and cover longer time spans. The estimates for 2007-2008 are likely to be slightly revised by Insee in the near future. For 2009-2010 we upgraded the 2008 values using the latest growth projection figures available (we assumed a nominal growth rate equal to -2.0% for 2009 and 0.0% for 2010). Official French national accounts have been established and released by Insee since 1949, and currently follow the "Base 2000" (B2000) methodology (all retrospective series were recently retropolated using B2000 concepts), which is the French version of ESA 1995 (European System of Accounts) and SNA 1993 (UN System of National Accounts). In what follows we often refer to the ESA 1995 classification codes (the ESA 1995 manual is available on-line: http://circa.europa.eu/irc/dsis/nfaccount/info/data/ESA95/en/esa95en.htm).

[4] See section A3 below for more details on the way we used the Villa (1994) series. For additional details on alternative historical national accounts series in France, see Piketty (2001, pp.693-720).

[5] All decennial averages reported on Table A2 (and on subsequent decennial tables) were computed in this way, with the exception of the 1910 decennial average, which corresponds to an arithmetic average over years 1910-1913 rather than 1910-1919 (the earlier and later parts of the 1910-1919 decade are so different that it does not make much sense to compute a decennial average; in addition it is useful to have a 1910-1913, pre-World War 1 reference point).

[6] The raw averages computed from the series provided by Bourguignon and Lévy-Leboyer (1985, pp.318-322) are slightly below our final series: they get an average national income of 38.3 billions old francs for 1910-1913 (while we get 42.7 billions) and 26.6 billions for 1900-1909 (while we get 33.9 billions). The Villa

we would obtain similar decennial averages and overall profiles of national income growth over the course of the 19th century.[7]

Col. (2) of Tables A1-A2: Aggregate private wealth W_t in current prices

Our basic aggregate series for private wealth W_t are reported on col. (2), again expressed in current billions currency, as defined above.

Private wealth W_t is defined as the market value of all tangible assets (in particular real estate assets) and financial assets owned by private individuals (i.e. households), minus their financial liabilities. Private wealth W_t is estimated at asset market prices prevailing on January 1st of each year.

We use the official Insee-Banque de France series for the 1970-2009 period. I.e. for years 1970-2009, the value of private wealth W_t reported on Table A1 is simply equal to the net worth of the personal (household) sector balance sheet published by Insee for the corresponding year.[8] Complete breakdowns of these W_t 1970-2009 series by asset categories, as well as net worth series for the government and corporate sectors, are provided in section A4 below (see Tables A13-A16).

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series are more sophisticated and more comparable to modern national accounts than the BLL series, so anchoring the entire BLL series to the Villa values for 1900-1909 and 1910-1913 appears to be the most reasonable option.

[7] Using the gross domestic product series provided by Toutain (1997, pp.54-57), again anchored to the Villa values for 1900-1909 and 1910-1913, we get an average value of 10.2 billions in 1820-1829 (vs 11.3 billions using the BLL series) and 26.2 billions in 1870-1879 (vs 28.6 billions using the BLL series); by construction, both series yield 42.7 billions in 1910-1913. The maximum gap between the Toutain and BLL series (which are based upon very different raw statistical material and methodologies) is less than 10%, which is very small over a one-century-long period, and negligible for our purposes.

[8] We downloaded the complete set of recently released Insee retrospective 1979-2009 national wealth accounts series (i.e. balance sheets) on www.insee.fr on 15/09/2009. We used Insee tables 5.407 to 5.415. As far as financial assets and liabilities are concerned, these tables are identical to those released by Banque de France (who is the primary producer of French financial accounts), except that the latter include more types of financial assets; on the other hand the advantage of Insee tables is that they also include tangible assets. It is preferable to use these Insee-Banque de France tables rather than the financial accounts released by international data collectors such as Oecd, Eurostat or the Imf, again because the former are more detailed and cover longer time spans. Insee-Banque de France balance sheets are estimated at market prices prevailing on December 31st of each year; so our January 1st 2009 estimates are in fact December 31st 2008 estimates, etc., and our January 1st 1979 estimates are in fact December 31st 1978 estimates. The estimates for 2008-2009 are likely to be slightly revised by Insee in the near future. For 2010 we assumed that asset prices between January 1st 2009 and January 1st 2010 declined as much as between January 1st 2008 and January 1st 2009 (i.e. -5.4% on average); see section A5 below. Currently available Insee-Banque de France retrospective wealth accounts series cover only the 1979-2009 period, so for the 1970-1978 subperiod we had to use series that were previously published by Insee using less sophisticated concepts and methodology (see "25 ans de comptes de patrimoines (1969-1993)", *INSEE Résultats* n°348 (Economie générale n°98), december 1994); in order to ensure continuity, these 1970-1978 series were anchored to the 1979 values; more details are given in section A4 below.

Prior to 1970, there exists no official estimate of aggregate private wealth in France, so we had to use various non-official estimates and to compute our own series. As we explain in the working paper (see section 3.2), non-official private wealth estimates are plentiful and relatively reliable for the 1820-1913 period, so we simply used the best decennial averages available in the historical literature.[9] The period 1914-1969 is the most problematic: we only have (relatively) reliable private wealth estimates for 1925 and 1954, and we computed our own annual private wealth W_t series by estimating an accumulation equation for private wealth (see section A5 below).

Col. (3)-(4) of Tables A1-A2: Aggregate Y_t and W_t in 2009 consumer prices

Col. (3) and (4) of Tables A1 and A2 were obtained by multiplying col.(1) and (2) by P_{2009}/P_t, where P_t is the consumer price index (CPI) reported on col.(1) of Table A20. This is done for illustrative purposes only. Over such long time periods, we are not sure that constant price series are really meaningful.

Col. (5)-(12) of Tables A1-A2: Per capita & per adult Y_t and W_t

Col.(5) to (12) of Tables A1 and A2 were obtained by dividing col. (1) to (4) by total population or adult population (col. (1) and (7) of Table C1). We find that per adult national income y_t rose from 602 francs in the 1820s to 1,637 francs in 1910-1913 and 36,197 euros in 2008, while per adult private wealth w_t rose from 3,302 francs in the 1820s to 10,713 francs in 1910-1913 and 203,696 euros in 2008 (see Table A2, col. (7) and (8)). Expressed in 2009 consumer prices (whatever it means), per adult national income y_t rose from 2,991 euros in the 1820s to 5291 euros in 1910-1913 and 36,342 euros in 2008, while per adult private wealth w_t rose from 16,413 euros in the 1820s to 34,626 euros in 1910-1913 and 204,511 euros in 2008 (see Table A2, col. (11) and (12)).

Col. (13) of Tables A1-A2: Wealth-income ratio $\beta_t = W_t/Y_t$

Col. (13) of Tables A1 and A2 was obtained by dividing col. (2) by col. (1). By construction, it is also equal to col. (4) divided by col. (3), or col. (6) by col. (5), etc. We find that the ratio

[9] The exact references are given in section A5 below.

β_t between aggregate private wealth W_t and national income Y_t was equal to 549% in the 1820s, 654% in 1910-1913, and 563% in 2008.

Col. (14)-(16) of Tables A1-A2: Disposable income ratios

We choose to use national income rather than (personal) disposable income as the income denominator when we compute wealth-income ratios. However it is useful to have in mind what the results of the computations would be if one were to use disposable income as denominator. On col. (14) of Tables A1-A2 we report the ratio between disposable income and national income; this ratio was equal to 95% in the 1820s and in 1910-1913 and to 70% in 2008.[10] Col. (15) of Tables A1-A2 was obtained by multiplying col.(11) by col. (14); we find that per adult disposable income y_{dt} (expressed in 2009 consumer prices) rose from 2,842 euros in the 1820s to 5,005 euros in 1910-1913 and 25,281 euros in 2008. Col. (16) of Tables A1-A2 was obtained by dividing col. (13) by col. (14); we find that the ratio between aggregate private wealth and disposable income was 578% in the 1820s, 692% in 1910-1913 and 809% in 2008.

A.2. Computation of the economic inheritance flow series B_t (Tables A3-A4)

Our economic inheritance flow series B_t and related ratios are reported on Table A3 (annual series) and Table A4 (decennial averages). Here we describe how these tables were constructed.

Col. (1) to (6) of Tables A3-A4: $b_{yt} = \mu_t^* m_t \ \beta_t$ and $b_{wt} = \mu_t^* m_t$

As we explain in the working paper (see section 3.1), the basic accounting equation relating the aggregate economic inheritance flow B_t and aggregate private wealth W_t is the following:

$$B_t = \mu_t^* m_t \ W_t \qquad \text{(A.1)}$$

Where μ_t^* is the gift-corrected ratio between average wealth of (adult) decedents and average wealth of the (adult) living, and is estimated from available data on the age profile

[10] Col. (14) of Tables A1 and A2 is borrowed from col.(1) of Table A10. See section A3 below.

of wealth (see Appendix B2); and m_t is the (adult) mortality rate and comes from standard demographic data (see Appendix C1).

Alternatively, equation (A.1) can also be expressed in terms of inheritance-income and inheritance-wealth aggregate ratios:

$$b_{yt} = B_t/Y_t = \mu_t^* \, m_t \, W_t/Y_t = \mu_t^* \, m_t \, \beta_t \qquad (A.2)$$

$$b_{wt} = B_t/W_t = \mu_t^* \, m_t \qquad (A.3)$$

The computations reported on Tables A3-A4 follow directly from the mechanical application of these formulas. On col. (1) of Tables A3-A4 we report the aggregate wealth-income ratio β_t, which we borrow from col. (13) of Tables A1-A2. On col. (2) of Tables A3-A4 we report the adult mortality rate m_t, which we borrow from col. (11) of Table C1. On col. (3) of Tables A3-A4 we report the gift-corrected μ_t^* ratio, which we borrow from col. (12) of Table B5. Col. (4) of Tables A3-A4 was then obtained by multiplying col. (1), (2) and (3) (i.e. $b_{yt} = \mu_t^* \, m_t \, \beta_t$). We find that the aggregate economic inheritance flow-national income ratio b_{yt} was equal to 20.3% in the 1820s, 22.7% in 1910-1913, and 14.5% in 2008. Col. (5) of Tables A3-A4 was obtained by multiplying col. (2) and (3) (i.e. $b_{wt} = \mu_t^* \, m_t$). We find that the aggregate economic inheritance flow-private wealth ratio b_{wt} was equal to 3.7% in the 1820s, 3.5% in 1910-1913, and 2.6% in 2008. We also report on col. (6) of Tables A3-A4 the estate multiplier ratio $e_t = W_t/B_t = 1/b_{wt}$: col.(6) is simply equal to one divided by col. (5). We find that according to our economic inheritance flow computations, aggregate private wealth was equal to 27.0 years of inheritance flow in the 1820s, 28.9 years in 1910-1913 and 38.7 years in 2008.

<u>Col. (7) to (9) of Tables A3-A4: $B_t = \mu_t^* \, m_t \, W_t$</u>

We also report the results of our economic inheritance flow computations expressed in billions currency and not only in ratios. On col. (7) of Tables A3-A4 we report our aggregate private wealth series W_t, which we borrow from col. (2) of Tables A1-A2. Col. (8) of Tables A3-A4 was then obtained by multiplying col. (2), (3) and (7) (i.e. $B_t = \mu_t^* \, m_t \, W_t$). We find that the aggregate economic inheritance flow was equal to 2.3 billions francs in the 1920s, 9.6 billions francs in 1910-1913, and 246.7 billions euros in 2008. For the purpose of comparison, we also report the ratio B_t/B_t^f between our economic inheritance flow and our fiscal inheritance flow series: col. (9) of Tables A3-A4 was obtained by dividing col. (8) of Tables A3-A4 by col. (10) of Tables B1-B2; we find a ratio of 105% in the 1820s, 111% in 1910-1913, and 115% in 2008.

Col. (10) to (12) of Tables A3-A4: $b_t = \mu_t^* \beta_t \ y_t$

We also report our economic inheritance flow estimates expressed in per capita terms. If we note b_t average per decedent inheritance, and y_t average per adult national income, then the equations above can also be written as follows:

$$b_t = \mu_t^* \beta_t \ y_t \qquad \text{(A.4)}$$

On col. (10) of Tables A3-A4 we report per adult income y_t, which we borrow from col. (7) of Tables A1-A2. Col. (11) of Tables A3-A4 was obtained by multiplying col. (1), (3) and (10) (i.e. $b_t = \mu_t^* \beta_t \ y_t$). On col. (12) of Tables A3-A4 we report the ratio b_t/y_t. We find that according to our economic inheritance flow computations average inheritance was equal to 5,497 francs in 1820s (i.e. 9.1 years of average income), 17,406 francs in 1910-1913 (i.e. 10.6 years of average income), and 453,344 euros in 2008 (i.e. 12.5 years of average income).

Col. (13) to (16) of Tables A3-A4: fiscal flow ratios

For comparison purposes we also report on Tables A3-A4 our fiscal inheritance flow estimates. On col. (13) of Tables A3-A4, we report ratios B_t^f/Y_t between fiscal inheritance flow and national income, which we borrow from col. (12) of Tables B1-B2. On col. (14) of Tables A3-A4, we report ratios B_t^f/W_t between fiscal inheritance flow and private wealth, which we borrow from col. (13) of Tables B1-B2. We also report the fiscal estate multiplier $e_t^f = W_t/B_t^f$ on col. (15) of Tables A3-A4 (equal to one divided by col. (14)) and the fiscal b_t^f/y_t ratio on col. (16) of Tables A3-A4 (equal to col. (13) divided by the mortality rate, i.e. by col. (2)). We find that according to fiscal data average inheritance was equal to 8.5 years of average income in the 1820s, 9.5 years in 1910-1913 and 10.9 in 2008.

A.3. Supplementary series on the structure of national income Y_t (Tables A5-12)

Detailed annual series on the structure of national income Y_t in France over the 1896-2008 period (including decomposition by institutional production sector, factor income, taxes and savings, etc.) are reported on Tables A5 to A11. Prior to 1896, available series are more rudimentary. The only series that we can provide for the entire 1820-2008 period are decennial-averages estimates of capital and labor shares, rates of return, aggregate tax rates and savings rates; these summary macro variables are reported on Table A12. These series are useful in order to better understand how the general structure of income and wealth has evolved in France over the past two centuries. They also play important specific roles at various points in this research. In particular, we need saving rates series for estimating the private wealth accumulation equation (see Appendix A5 below), and we need both saving rates series and rates of return series for simulating the dynamics of the age-wealth profile (see Appendix D). The computation of average macroeconomic rates of return to private wealth requires detailed series on factor income and taxes. Rates of return play a critical role in this research. So we try to explain carefully how Tables A5 to A12 were constructed.

Table A5: National income vs gross domestic product (1896-2008)

On Table A5 we report the most basic decomposition of national income Y_t :

$$Y_t = Y_{pt} + FY_t \qquad (A.5)$$
$$Y_{pt} = GDP_t - KD_t \qquad (A.6)$$

With: Y_t = national income (i.e. net national product)
Y_{pt} = net domestic product
FY_t = net foreign factor income
GDP_t = gross domestic product
KD_t = capital depreciation

On col. (1)-(3) and (10)-(11) of Table A5, we report values of Y_t, Y_{pt}, FY_t, GDP_t and KD_t expressed in billions current currency. On col. (4)-(9) and (12)-(13) of Table A5, as well as on all columns of Tables A6-A10, we report values expressed as fractions of national income Y_t (or other aggregates). All series reported on Tables A5-A10 come directly from

Insee official series for the 1949-2008 period, and from the Villa (1994) series for 1896-1948 period, with minor adjustments which we describe as they come.[11]

Col. (4)-(8) of Table A5 show that changes in net foreign factor income FY_t are almost entirely due to changes in net foreign capital income FY_{Kt} (net foreign labor income FY_{Lt} seems to have always been relatively small).[12] Most importantly, they show that net foreign capital income made up approximately 4% of national income at the eve of World War 1, then fell abruptly during war years (due to foreign assets repudiation and inflation), and never recovered: from the 1920s up until 2008, it has generally been about 0%-1% of national income (this is consistent with the fact that the net foreign asset position of France seems to have been relatively small throughout this period; see section A4 below). However gross flows have risen enormously in recent decades (due to financial globalization): in 1978, gross capital income inflow and outflow were around 1% of national income; in 2008, both were around 10% of national income.

On col. (9) we report the value of net foreign tax and transfers, which we note FT_t.[13] According to standard international definitions, this should added to national income Y_t in order to compute so-called “national disposable income”. Note that FT_t has actually been negative since the 1950s up to 2008 (around -1% of national income), due mostly to the remittances of immigrant workers.[14]

[11] For 1949-2008 we used Insee tables 3.101 to 3.601 and the "tableaux économiques d'ensemble" (downloaded from www.insee.fr on 15/09/2009). For 1896-1948 we used Villa’s long.xls data base (downloaded from www.cepii.fr on 15/10/1998; these series are identical to those published in Villa (1994, pp.84-153), and have not been updated since then). All raw Insee and Villa series expressed in billions current currency are provided in the excel file AppendixTables(NationalAccountsData).xls (see Table A0). The file also includes the formulas used to construct all other tables.

[12] Net foreign capital income is equal to gross capital income inflow (capital income received by French residents on their foreign financial assets) minus gross capital income outflow (capital income received by foreign residents on their French financial assets), while net foreign labor income is equal to gross labor income inflow (labor income received by French residents while working abroad) minus labor income outflow (labor income received by foreign residents while working in France). In pre 1949 series we only observe net foreign capital income (not the gross flows), and foreign labor income was not recorded at all (given post 1949 values we set it to 0% for national income computations).

[13] I.e. gross inflow of taxes and unilateral transfers flowing from the rest of the world to French residents, minus gross outflow of taxes and unilateral transfers flowing from French residents to the rest of the world.

[14] In contrast, according to the Villa series, FT_t was positive and fairly large (2% to 6% of national income) during the 1920s, which (partly) reflects German transfer payments. Note that we included all tax flows in FT_t , including production taxes (D2 in ESA 1995 classification). According to ESA 1995 definitions, net foreign production taxes should actually be included in the primary income account (together with net foreign factor income FY_t), rather in the secondary income account (which should only include net foreign direct taxes, in addition to net foreign transfers); i.e. they should be included in the computation of national income Y_t (and not only in the computation of national disposable income). However pre 1949 series are not sufficiently detailed to properly isolate net foreign production taxes (in the current sense) within FT_t; so it made more sense to adopt a simplified definition of national income and to omit this (small) term throughout the 1896-2008 period; in addition, the conceptual difference between foreign flows of production taxes vs other taxes is somewhat obscure.

Col. (12)-(13) of Table A5 show that capital depreciation seems to have been relatively stable around 9%-11% of gross domestic product between 1900 and the 1970s, and then gradually rose during the past three decades, up to about 13%-14% today. Of course capital depreciation estimates are notoriously fragile, and some of the short-run variations reported on Table A5 might partly be due to measurement limitations (rather than to real changes in the age structure and depreciation rates of capital inputs). Given that we are mostly interested in long run evolutions, we feel that these data limitations are not really relevant for our purposes.[15]

Table A6: decomposition by institutional production sectors (1896-2008)

Net domestic product Y_{pt} can be further decomposed into the net product (net-of-capital-depreciation, net-of-production-taxes value-added) of the various institutional production sectors used in national accounts:

$$Y_{pt} = Y_{ht} + Y_{set} + Y_{ct} + Y_{gt} + T_{pt} \qquad \text{(A.7)}$$

With:

Y_{ht} = net product of the housing sector[16]

Y_{set} = net product of the self-employment sector[17]

Y_{ct} = net product of the corporate sector (non-financial + financial)[18]

[15] Currently available Insee retrospective capital depreciation series cover only the 1978-2008 period (except for the government sector, where they start in 1949). For the 1896-1977 period we used the capital depreciation series provided by Villa (1994). Detailed depreciation series (broken down at the institutional production sector level) are given in the excel file.

[16] Following standard national accounts practice, Y_{ht} is defined as the net-of-depreciation rental value of the housing fixed assets owned by households (including imputed rent). Note that this is (slightly) smaller than the total value of housing services produced in the economy, because a (small) fraction of the housing capital stock is owned by corporations and by the government.

[17] We define Y_{set} as the net product of the household sector minus Y_{ht}. Note that Y_{set} is (slightly) bigger than the net product of unincorporated businesses, since it also includes the (wage) labor income of domestic wage earners (i.e. wage earners directly employed by households in order to produce domestic services). It also includes the (wage) labor income of wage-earners employed by unincorporated businesses. This explains why the share of Y_{set} in national income is typically bigger than the share of self-employed in total employment (more on this below). The national accounts tables reported in Piketty (2001, pp.693-720) display lower estimates of Y_{set} than those reported in the more consistent series presented here. This is because Piketty (2001) used pre-1970 national accounts series based upon older concepts and definitions: the wage bill paid by unincorporated businesses (which was non negligible during the first half of the 20th century, both in the rural and urban economy) was in effect attributed to the corporate sector in these older series, thereby resulting in an upward bias in the estimate of the corporate labor share (see below).

[18] In the same way as for wealth accounts (see below), and in order to simplify notations and tables, we include in the corporate sector both non-financial corporations and financial corporations. Separate series for non-financial and financial corporations are provided in the excel file.

Y_{gt} = net product of the government sector (incl. the non-profit sector)[19]

T_{pt} = production taxes (incl. value-added taxes) [20]

As one can see from Table A6, the sectoral structure of national income has changed in important ways in France during the 1896-2008 period. First, the implicit average production tax rate, which we define as production taxes divided by factor-price national income (i.e. T_{pt} divided by Y_t - T_{pt}), was about 7%-8% prior to World War 1, then rose during the interwar and postwar period, and stabilized around 17%-18% since the 1950s up to 2008.[21]

Next, the share of the housing sector in (factor-price) national income has gone through a U-shaped pattern over the past 100 years: it was about 8% prior to World War 1, then fell abruptly to 3% in 1920, recovered during the interwar, fell again during World War 2, with a nadir at only 2% in 1945, and then gradually recovered during the past 60 years, up to 8%-9% in the 1990s-2000s.[22] These large historical variations seem to reflect (at least in part) the evolution of rent control policies.[23]

Next, the share of the government sector (whose contribution to net product in existing national accounts is simply measured by the wage bill of the government sector)[24] rose dramatically. It was only 2%-3% of (factor price) national income prior to World War 1, then rose to 5%-6% during the interwar, 12%-13% in the postwar period, before (apparently) stabilizing around 19%-20% in the 1990s-2000s.

[19] In the same way as for wealth accounts (see below), all government levels are included (central and local government, social security administrations, as well as the non-profit sector). It is somewhat arbitrary to include the non-profit sector into the government sector (it could as well be included in the personal or corporate sectors). However this simplifies notations and tables. In any case the non-profit sector has always been relatively small in France (about 1% of national income).

[20] This includes all "production taxes" in the national accounts sense (D2 in ESA 1995 classification), i.e. the sum of "product taxes" strictly speaking (D21 in ESA 1995 classification: this includes value-added taxes, excise duties, import taxes and various consumption taxes) and "other production taxes" (D29 in ESA 1995 classification: this includes a number of property taxes and non-social-contributions payroll taxes, see below), net of subsidies (D3 in ESA 1995 classification).

[21] This can be compared to the general VAT rate, which is currently 19.6% in France (the reduced rate is 5.5%). However one must keep in mind that VAT revenues strictly speaking make only about half of total production taxes revenues (in 2008,136.8 billions € out of 256.5 billions €). Note also that "factor price national income" is merely an accounting concept, and certainly does not imply that production taxes are entirely shifted to prices: first, some of the VAT itself is probably shifted to factor income, depending on sectoral supply and demand elasticities; next, some of the other taxes included in D2 ESA 1995 classification (e.g. a number of business and personal property taxes – "taxe professionelle", "taxe foncière", etc. – and non-social-contributions payroll taxes – "taxe sur les salaries", "versement transport", etc.) are closer to factor income taxes.

[22] See Figure A7.

[23] Available indexes of housing rent for France and Paris, divided by CPI, follow almost exactly the same pattern over the 20th century. See e.g. Piketty (2001, pp.89-91, graphs 1-9 and 1-10).

[24] The residual profit share of the government sector was included in production taxes (see below).

Next, the share of the self-employment sector declined even more dramatically. It was about 50% at the eve of World War 1, about 40% in the aftermath of World War 2, and gradually declined to little more than 10% in the 2000s. At the same time, the share of the corporate sector gradually rose from about 30% of (factor price) national income around 1900 to about 60% during the 1990s-2000s.

Finally, note that the long run evolution of the relative shares of the government, self-employment and corporate sectors (which are the three production sectors using labor input) is broadly consistent with the corresponding evolution of the employment structure of France.[25]

Table A7: profits & wages in the corporate sector (1896-2008)

On Table A7 we report the standard decomposition of corporate value-added into wages and profits. That is, we break down net corporate product Y_{ct} into a labor income component Y_{Lct} and a capital income component Y_{Kct} :

$$Y_{ct} = Y_{Lct} + Y_{Kct} \qquad (A.8)$$

With: Y_{Lct} = total wage bill of the corporate sector (incl. social contributions)
$Y_{Kct} = Y_{ct} - Y_{Lct}$ = net corporate profits

One can then define the corporate capital share $\alpha_{ct} = Y_{Kct}/Y_{ct}$ and the labor share $1- \alpha_{ct} = Y_{Lct}/Y_{ct}$ in net corporate product. We choose to focus upon net-of-depreciation functional shares, first because they are more meaningful from an economic viewpoint, and next because this is what we need in order to compute average rates of return on private wealth (see below). For the purpose of comparison with other studies, we also report on Table 6 series for the gross profit share in gross corporate product $(Y_{Kct}+KD_{ct})/(Y_{ct}+KD_{ct})$

[25] I.e. over the past century public employment share rose from 2%-3% to about 20% of total employment, while self-employment share declined from about 50% to less than 10% of self-employment (see Piketty (2001, p.51, graph 1-4)). The self-employment sector output share $Y_{set}/(Y_{gt}+Y_{set}+Y_{ct})$ was actually even larger than 50% around 1900 (it was as high as 65%-70%), which can be accounted for by the fact that Y_{set} also includes the wages of wage-earners directly employed by households and unincorporated businesses (see above). Also the boundaries between unincorporated and corporate businesses in early 20th century national accounts series are somewhat fragile (e.g. at that time many not-so-small manufacturing businesses were still unincorporated), so one would need to collect additional data in order to push further this kind of analysis.

(where KD_{ct} denotes capital depreciation of the corporate sector) and the corresponding labor share in gross corporate product $Y_{Lct}/(Y_{ct}+KD_{ct})$.[26] Gross functional shares are often used in policy discussions and typically deliver labor shares around two thirds and capital shares around one third.

Note that pre-1949 factor income data is definitely of lower quality than that used in post-1949 Insee series, and one should be cautious when interpreting pre-1949 variations and levels of labor and capital shares.

During the 1949-2008 period, labor and capital shares in France appear to display the standard two-thirds-one-third pattern. During the 1950s-1960s, the gross profit share is relatively stable around 30%-32% of gross corporate product; during the 1990s-2000s, the gross profit share is relatively stable around 32%-34% of gross domestic product (see Table A7, col. (8)). Two caveats are in order, however. First, there are important medium term variations. One observes large U-shaped fluctuations during the 1970s-1980s: the gross profit share suddenly falls from 32% in 1973-1974 to 25% in 1981-1982,[27] and then returns to 33% in 1986-1987.[28] Next, if one looks at net profit shares in net corporate product, then all capital shares are reduced substantially (typically by about 10 points), which makes the medium term variations look even bigger. The net profit share was about 20%-22% of net corporate product in France during the 1950s-1960s, then fell to as little as 12% in the late 1970s-early 1980s, and was again about 20%-22% during the 1990s-2000s (see Table A7, col.(2)). This is fairly different from the standard two-thirds-one-third textbook pattern.

Time variations in the way profits are used are also significant. Over the 1949-2008 period, corporate income taxes were relatively stable around 5% of net corporate product (typically between a quarter and a third of net profits), and distributed profits (dividend and interest payments) were relatively stable around 10% of net corporate product;[29] retained earnings on the other hand were highly volatile and absorbed most of the time variations in

[26] Note that because we put aside all production taxes T_{pt}, our corporate capital and labor shares series always sum up to 100%, which makes evolutions easier to interpret. The price to pay for this simplification is that we are implicitly assuming that that the component of production taxes T_{pt} that is not shifted to prices is shifted proportionally to labor and capital factor income, which seems acceptable as a first approximation, but which strictly speaking might not be true.

[27] Due to sluggish output growth and rapid wage growth after the 1973-1974 oil shock.

[28] Due to wage freeze policies implemented after 1982-1983 by the newly elected socialist government.

[29] Here we naturally look at the net outflow of divided and interest paid by the corporate sector. During the 1990s-2000s, gross outflows and inflows have increased enormously in absolute terms, reflecting a large rise in financial linkages within the corporate sector.

the profit share (resulting in large negative retained earnings in the late 1970s-early 1980s). But one can also notice that retained earnings were structurally higher in the reconstruction period than in recent decades: on average they made about 7% of net corporate product in the 1950s-1960s (3%-4% of national income), versus about 3% in the 1990s-2000s (1%-2% of national income).[30]

Available series for the 1896-1949 period broadly confirm the view of a long run stability of capital shares (with gross profit shares around 30%-35% and net profit shares around 20%-25%). They also show very large short run and medium run variations, and somewhat bigger average capital shares than contemporary levels.[31] The Villa series indicate that the net profit share was about 15%-20% around 1900, and rose to over 30% in 1910-1913.[32] It was again over 30% during the 1920s (a level unobserved in the post-1949 period), fell during the 1930s, and reached negative values in war years (when capital depreciation slightly exceeded gross profits). Prior to World War 1 there was no corporate income tax, retained earnings were small, so that distributed profits were as large as 15%-20% of net corporate product (far above all levels observed in the post-1949 period). The Villa series also indicate very large levels of retained earnings, especially during the 1920s. This seems consistent with the reconstruction story. Pre-1949 retained earnings estimates have been challenged by a number of scholars, however, and it is possible that the Villa's extremely high retained earnings levels for 1910-1913 and the interwar period are somewhat overestimated.[33]

Table A8: capital & labor shares in national income (1896-2008)

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[30] Note the relatively large "other corporate transfers" term (about 3%-4% of net corporate product, i.e. 1%-2% of national income, throughout the 1949-2008 period), which we define as the net value of various transfers paid by the corporate sector (D61+D62+D71+D72+D75 in ESA 1995 classification): D61-62 relate to employer provided social contributions and benefits transfers and sum to (close to) zero; D71-72 relate to insurance premiums and claims transfers and sum to (close to) zero; D75 is the only significant term; it relates to "miscellaneous current transfers" and typically includes unilateral transfers to the non-profit and personal sectors. This ought to be further investigated, especially given that such transfers are not properly recorded in pre-1949 series.

[31] See Figures A4 and A5. One should use the series reported here rather than the gross profit share series reported in Piketty (2001, pp.703-705; 2003, p.1022, fig. 4). Both sets of series are broadly similar, but our older functional shares series were less complete and suffered from various deficiencies. In particular, pre-1949 corporate capital shares were underestimated, due to the fact that the wage bill paid by households and unincorporated businesses was (wrongly) attributed to the corporate sector (see above).

[32] It is possible that this sharp rise is over-estimated somewhat. However all raw statistical series suggest that corporate output was indeed growing faster than corporate wages during the 1896-1913 period. We return below on 19th century functional share estimates (see Table A12).

[33] See Malissen (1953), who on the basis of interwar corporate income tax tabulations argues that the exploratory, semi-official national accounts constructed for year 1938 by Insee overestimate retained earnings. Since Villa anchors some of his series on these 1938 semi-official accounts, this criticism also applies to his series as well. See Piketty (2001, p.716).

Capital income does not come solely from the corporate sector. On Table A8 we break down the net product of the various sectors in order to compute capital and labor shares in total national income. We proceeded as follows.

Housing sector: by definition, the net product of the housing sector Y_{ht} solely generates capital income. I.e. $Y_{ht} = Y_{Kht}$ and $\alpha_{ht} = 100\%$.[34]

Self-employment sector: for simplicity, we choose to break down the net product of the self-employment sector Y_{set} into a capital income component Y_{Kset} and a labor income component Y_{Lset} by assuming the same capital share as in the corporate sector. I.e. we assume $\alpha_{set} = \alpha_{ct}$.[35]

Government sector: by definition, the net product of the government sector Y_{ht} solely generates labor income. I.e. $Y_{gt} = Y_{Lgt}$ and $\alpha_{gt} = 0\%$.[36] However, although the government does not generate capital income out of its productive economic activity,[37] it does generate capital income out of its public-finance, borrowing activity, namely government interest payments on public debt. The government also receives capital income on its financial assets (e.g. if the government owns equity shares in corporations). We define net government interest payments (which we note Y_{Kgt}) as the excess of capital income paid

[34] The fact that α_{ht}=100% is simply the consequence of the standard national-accounts definition of housing services: the value of housing services is defined is the pure rental value of housing, i.e. excluding all labor inputs that can increase the value of housing services (i.e. cleaning services, etc.).

[35] The other standard way of breaking down self-employed income into capital and labor income components is to attribute to self-employed workers the same average labor income compensation as the wage earners of the corporate sector. We found that this alternative computation delivers very similar results regarding the pattern of the aggregate capital share α_t. A third and somewhat more satisfactory way to break down self-employment income would be to attribute to self-employment capital stock the same rate of return as for the rest of the economy. This is more data demanding, however.

[36] The fact that α_{gt}=0% simply follows from the standard national-accounts definition of government net product: in national accounts, the gross value of non-market output is estimated on a cost basis, i.e. summing up labor cost, intermediate consumption and estimated capital depreciation; so that the net value-added is simply equal to labor cost. Note that the government sector also produces small (but positive) market output and receives residual payments from personal and corporate sectors for these goods and services, so that strictly speaking the net profit share of the government sector is not exactly equal to zero in national accounts. But it very small (always less than 0.5% of net government product in French accounts), so in order to simplify exposition and tables, we choose to conventionally set α_{gt}=0% and to attribute this small profit term to production taxes (see excel file for detailed series and formulas).

[37] Of course this is purely conventional: the government sector does use capital input (administrative buildings, schools, hospitals, etc.), and one could very well decide to attribute a positive return to these assets, which would raise national income Y_t. E.g. the estimated value of government tangible assets was around 75% of national income during the 2000s (see Table A13 below); if one attributes a 4% average return to these assets, this would raise national income by 3%. This not really relevant for our purposes, since this extra capital income is not distributed to any private individual (it is simply enjoyed by everyone), so this does not affect average returns to private wealth (in case the government sector uses capital inputs owned by other sectors, then the corresponding capital income flow is recorded).

by the government sector over capital income received by the government. Whether Y_{Kgt} should be taken into account in total capital income depends on the specific purpose one has in mind (see below). In order to compute average returns to private wealth (which is our primary purpose), the most consistent solution is to include net government interest payments in the definition of total capital income. In practice, this does not make a very large difference, as the detailed series reported on Table A8 illustrate (net government interest payments have usually been less about 1%-2% of national income).[38]

Foreign sector: we simply use the net foreign capital income FY_{Kt} and labor income FY_{Lt} series reported on Table A5 above.

We then define aggregate capital income Y_{Kt} (excluding government interest) and labor income Y_{Lt} by summing up the various components:

$$Y_{Kt} = Y_{Kct} + Y_{ht} + Y_{Kset} + FY_{Kt} \qquad \text{(A.9)}$$

$$Y_{Lt} = Y_{Lct} + Y_{Lset} + Y_{gt} + FY_{Lt} \qquad \text{(A.10)}$$

By construction the sum of these two terms is equal to factor-price national income:

$$Y_{Kt} + Y_{Lt} = Y_t - T_{pt} \qquad \text{(A.11)}$$

We define the aggregate capital share α_t (excluding government interest) and labor share $1-\alpha_t$ in factor price national income as follows:

$$\alpha_t = Y_{Kt} / (Y_t - T_{pt}) \qquad \text{(A.12)}$$

$$1-\alpha_t = Y_{Lt} / (Y_t - T_{pt}) \qquad \text{(A.13)}$$

[38] Except during the interwar period, following the large rise in public debt during World War 1. Also note that net government interest payments were negative during the late 1960s and early 1970s, i.e. interest and dividend on government financial assets slightly exceeded interest payments. It is maybe surprising that net capital income received by government was not more strongly positive during the period running from World War 2 to the 1980s, given the large government equity participations in corporations at that time. This could reflect the fact that the government was getting relatively low returns on its assets, and/or was keeping a large share of the profits as retained earnings to finance new investment in publicly owned companies (we know that aggregate retained earnings were very large during the 1950s-1960s, but we do not know the break down by ownership status; it seems likely that retained earnings were particularly large in publicly owned companies), and/or was implicitly using some of the returns to pay better wages in publicly owned companies. During the 2000s, the estimated value of government financial liabilities (public debt) was about 80% of national income, and that of government financial assets (e.g. shares in public utility companies) was about 50% of national income (see Table A13 below).

In order to compute average rates of return to private wealth, one needs to include government interest and to define total capital income $Y_{Kt}^* = Y_{Kt} + Y_{Kgt}$ and total capital share $\alpha_t^* = Y_{Kt}^*/ (Y_t – T_{pt}) = \alpha_t + \alpha_{gt}$ (with $\alpha_{gt} = Y_{Kgt}/ (Y_t – T_{pt})$).

On Table A8, we report primary (pre-tax) functional shares series using both definitions, i.e. including government interest (see col.(13)-(14)) and excluding government interest (see col.(15)-(16)). Note that when we include government interest the capital and labor shares do not exactly sum up to 100%. This is because government interest enters into the definition of total capital income Y_{Kt}^* but not in the definition of national income Y_t (it is treated as a pure transfer by national accounts, not as additional output). Both sets of series are very close and depict the same picture:[39] with the exception of the mid-century nadir, the capital share has been fairly stable over the 20th century, albeit at somewhat higher levels in the early 20th century (30%-35%) than in the late 20th century (25%-30%). This is due for the most part to the structural rise of the government sector (which does not distribute capital income out of its productive activity). Note also that the (sharp) U-shaped evolution of rental income generates a (moderate) U-shaped pattern for the overall capital share. I.e. the sharp rise of rental income explains why the capital share is now higher than what in the immediate postwar period, in spite of the fact that corporate capital shares are currently about the same level as in the 1950s-1960s.

Table A9: taxes & transfers (1896-2008)

On Table A9 we report national accounts series on taxes and transfers. Taxes raise complex general equilibrium tax incidence issues, which national accounts series alone are of course unable to solve. The computations reported in these tables rely on simple tax incidence assumptions (detailed below), which in our view are valid as a first approximation, but which would definitely deserved to be improved.

Following standard national accounts categories we distinguish four types of taxes:

$$T_t = T_{pt} + T_{ct} + T_{it} + SC_t \qquad \text{(A.13)}$$

With:

[39] See Figures A7 and A8.

T_t = total tax revenues

T_{pt} = production taxes revenues[40]

T_{ct} = corporate income and wealth taxes revenues[41]

T_{it} = personal income and wealth taxes revenues[42]

SC_t = social contributions revenues[43]

Total tax revenues rose from less than 10% of national income prior to World War 1 to about 15%-20% in the interwar, 30% by 1950, and 50% in the 1990s-2000s. In the early 20h century, tax revenues came mostly from production taxes. The interwar rise in tax revenues was largely due to the appearance of personal and corporate income taxes. The postwar rise was due to all type of taxes: production taxes, income taxes, and particularly social contributions (see Table A8, col. (1)-(5)).

We assume that corporate taxes T_{ct} fall entirely on capital, and that social contributions SC_t fall entirely on labor. Regarding personal taxes T_{it}, we proceed as follows. First we take away bequest and gift taxes T_{Bt} from T_{it}, and assume they fall on capital. Next, in order to decompose other personal taxes T_{it}-T_{Bt} (which in practice are mostly personal income taxes) into a capital tax component T_{Kit} and a labor tax component T_{Lit}, we assume that other personal taxes fall proportionally on 50% of capital income Y_{Kt}* and on 100% of labor income Y_{Lt}.[44] The 50% coefficient on capital income is supposed to take into account the fact that a large fraction of capital income is not subject to the personal income tax (imputed rent, retained earnings, tax exempt savings accounts, etc.) or benefits from lighter tax treatment or reduced rates.[45]

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[40] As explained above, we include in this category all "production taxes" (D2 in ESA 1995 classification), i.e. the sum of "product taxes" strictly speaking (D21) and "other production taxes" (D29), net of subsidies (D3).

[41] We include in this category all "current taxes on income and wealth" (D5 = D51+D59 in ESA 1995 classification) paid by the corporate sector (in practice D59=0 for corporations).

[42] We include in this category all "current taxes on income and wealth" (D5 = D51+D59 in ESA 1995 classification) paid by the personal (household) sector, as well as bequest and gift taxes, which are treated separately in national accounts (D91D in ESA 1995 classification).

[43] We include in this category all "social contributions" (actual and imputed) (D61 in ESA 1995 classification) received by the government sector.

[44] More precisely, we assume that $T_{Lit} = (T_{it}-T_{Bt}) \times (Y_{Lt}-SC_t+Y_{Rt})/(Y_{Lt}-SC_t+Y_{Rt}+0.5 \times Y_{Kt}^*)$ and $T_{Kit} = (T_{it}-T_{Bt}) \times (0.5 \times Y_{Kt}^*)/(Y_{Lt}-SC_t+Y_{Rt}+0.5 \times Y_{Kt}^*)$, where Y_{Rt} is replacement income defined below (social contributions are deductible for income tax purposes, but replacement income is taxable). Detailed computations are provided in the excel file.

[45] Assuming a constant (taxable Y_{Kt}*)/Y_{Kt}* factor equal to 50% throughout the 1896-2008 period is of course very rough and ought to be improved. The true factor was somewhat larger than 50% in the early 20th century (e.g. imputed rent was subject to the income tax at that time), and is somewhat below 50% in the late 20th century and early 21st century (special exemptions for capital income have become more and more numerous in recent decades). However we tried a number of alternative, less rough assumptions (such as using the observed capital income tax base), and we found that the impact on overall tax rates series was relatively limited, so we chose this simpler assumption. The complication comes from the fact that one would

Neglecting production taxes for the time being, we then define total capital taxes T_{Kt} and total labor taxes T_{Lt} as follows:

$$T_{Kt} = T_{ct} + T_{Kit} + T_{Bt} \qquad (A.14)$$

$$T_{Lt} = SC_t + T_{Lit} \qquad (A.15)$$

Note that bequest and gift taxes have generally raised about 0.5%-1% of national income, both around 1900-1910 and around 2000; at the mid 20th century inheritance nadir, it was as little as 0.1-.0.2%. This U-shaped pattern of inheritance tax revenues is for the most part the mechanical consequence of the U-shaped pattern of the inheritance flow itself. The average tax rate on bequests and gifts, defined as T_{Bt}/B_t (where B_t is the economic inheritance flow borrowed from Table A3), has been relatively stable around 5% throughout the 1896-2008 period (see Table A9, col. (15)). Prior to World War 1, bequest and gift taxes made most of capital taxes. The balance started shifting in the interwar period, and especially in the postwar period. Nowadays taxes on the capital income flows (either at the corporate or personal level) vastly dominate taxes on the transmission of capital (see Table A9, col.(6)-(7)).

By including inheritance taxes into total capital income taxes, we are in effect assuming in our simulations (Appendix D) that they are paid out of the yearly return to capital, so that they reduce after-tax returns to private wealth, just like other capital taxes. Maybe it would be preferable to treat them separately and to assume that inheritance taxes are paid out of inherited wealth, so that they reduce wealth transmission flows in the simulated model. One could then take into account the progressivity of inheritance taxes (most of the population pays inheritance taxes close to 0%, while a minority pays much more than 5%). Given our aggregate focus, however, it seems simpler as a first approximation to just include them into capital taxes.

By dividing T_{Kt} by Y_{Kt}^* and T_{Lt} by Y_{Lt} we obtain the average implicit tax rates on capital τ_{Kt0} and on labor τ_{Lt0} (excluding production taxes) reported on col.(9)-(10). Because social contributions are so large (almost half of total taxes, and about two thirds of total labor

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also need to take into account tax progressivity (capital incomes are typically higher up in the distribution than labor incomes); so a complete computation would require estimating the full joint distributions of capital income (including tax exempt capital income) and labor income. This falls far beyond the scope of the present research.

taxes), the labor tax rate vastly exceeds the capital tax rate: in the 1990s-2000s, the labor tax rate was about 45%, while the capital tax rate was less than 25%.

Note however that social contributions SC_t finance for the most part replacement income Y_{Rt}, i.e. transfers received by labor income earners when they do not work, and which are generally proportional to past labor income and social contributions (pensions, unemployment benefits). On col. (16)-(18) of Table A9 we report total government (monetary) transfers TR_t, which we break down into replacement income Y_{Rt} and "pure transfers" TR_{0t} ;[46] in the 2000s, total transfers made about 20% of national income, out of which about 18% were replacement income and 2% were pure transfers. In case one deducts "replacement taxes" from labor taxes, i.e. the fraction of social contributions financing replacement income (this amounts to treating these as forced savings rather than taxes), then the labor tax rate in the 2000s drops from about 45% to about 25%-30%, i.e. somewhat below the capital tax rate (see Table A9, col. (11)).[47]

As a first approximation, we choose to view production taxes T_{pt} as broad taxes falling proportionally on total factor income $Y_{Kt}+Y_{Lt}$ (or on total expenditures C_t+I_t, which in a closed economy setting is equivalent), with an implicit production tax rate $\tau_{pt} = T_{pt}/(Y_t-T_{pt})$. Under this assumption, the total tax rates on capital and labor (including production taxes) τ_{Kt} and τ_{Lt} are given by:[48]

$$\tau_{Kt} = 1- (1- \tau_{Kt0})/(1+ \tau_{pt}) = (\tau_{Kt0} +\tau_{pt})/(1+ \tau_{pt}) \qquad (A.16)$$

$$\tau_{Lt} = 1- (1- \tau_{Lt0})/(1+ \tau_{pt}) = (\tau_{Lt0} +\tau_{pt})/(1+ \tau_{pt}) \qquad (A.17)$$

[46] Replacement income Y_{Rt} is defined as the sum of "social security benefits in cash" (D621 in ESA 1995 classification) and "unfunded employee social benefits" (D623) paid by the governement; pure transfers TR_{0t} are defined as "social assistance benefits in cash" (D624) paid by the government; total government monetary transfers TR_t are defined as the sum of the two. D624 transfers include all means-tested cash transfers, while D621-D623 include earnings-related transfers (mostly pensions and unemployment benefits). We also report on col. (19) of Table A9 the value of in-kind government transfers, i.e. "social transfers in kind" (D63), defined as the sum of "social benefits in kind" (D631: health insurance reimbursement and benefits, housing benefits, etc.) and "transfers of individual non-market goods and services" (D632: value of free education services provided by the government, etc.).

[47] The slight superiority of capital tax rate over (net-of-replacement-taxes) labor tax rate comes from the fact that the corporate income tax is a flat tax with high tax rate (typically 30%-50% since the 1950s), while the personal income tax is progressive, with a lower average tax rate. However the many capital tax exemptions (imputed rent, etc.) tend to counterbalance this effect, and it is possible that the true capital tax rate is actually (slightly) below the labor tax rate.

[48] Alternatively, if one defines $\tau_{ft} = T_{pt}/Y_t = \tau_{pt}/(1+ \tau_{pt})$ the implicit factor income tax rate associated to production taxes, one gets the equivalent formulas $\tau_{Kt} = 1- (1- \tau_{Kt0})(1- \tau_{ft})$ and $\tau_{Lt} = 1- (1- \tau_{Lt0})(1- \tau_{ft})$.

The corresponding series for τ_{Kt} and τ_{Lt} are reported on col. (12)-(13) of Table A9. In the 2000s, the labor tax rate was about 55%, while the capital tax rate was about 35%. If one deducts from labor taxes the fraction of social contributions financing replacement income, then the labor tax rate drops to about 30% (see col. (14)).[49] It seems more justified to treat replacement income as part of (augmented) labor income (this is what we do in our theoretical and simulated models), so this second definition of the aggregate labor tax rate is more relevant, e.g. for the purpose of comparison with the capital tax rate.

Our methodological choice of treating production taxes as broad factor income taxes is not entirely innocuous, but seems like the most reasonable option, given data limitations. In practice, production taxes (in the D2 ESA 1995 national accounts classification sense) are a complex mixture of broad factor income taxes (or expenditure taxes) and pure consumption taxes.[50] Estimating their exact tax incidence would involve complicated open economy and asset pricing issues, and falls well beyond the scope of this research. In case a fraction of production taxes falls purely on consumption, then the formulas for τ_{Kt} and τ_{Lt} would still be valid, but they should be interpreted as averages over the different final uses of income: whether their income comes from capital or from labor, individuals would face higher tax rates when they use their income to purchase consumption goods than when use it purchase investment goods. In a homogenous good model, there would in effect be a price p_t=1 when the single good is purchased as a capital good, and a price p_t'=1+τ when it is purchased as a consumption good, where τ measures the fraction of production taxes falling on consumption, expressed in equivalent consumption tax rate.[51] E.g. in the 2000s, with full shifting of production taxes on consumption prices, the capital tax rate would be equal τ_{Kt0}=25% when capital income is saved, equal to τ_{Kt1}>35% when capital income is consumed, with a weighted average (using aggregate savings rate) equal to τ_{Kt} =35%. In practice less than half of total D2 revenues can be viewed as falling on consumption, so the effect would even be less strong. The overall impact on long run capital accumulation (in effect we are under-estimating the quantity of investment goods that savings can buy, whether savings come from capital or labor income) would be relatively small. In any case, this would simply lead us to revise downwards our estimate of

[49] We note τ_{Lt}* this corrected labor tax rate (i.e. after deduction of “replacement taxes”).
[50] See above (footnotes 21-22).
[51] Think of the price of cars under a VAT with full deductibility of capital goods (i.e. immediate expensing, such as the French VAT system): the price is lower when you buy cars for investment purposes than when you buy cars for consumption purposes.

the residual capital gain terms in our wealth accumulation equation (see below), with no impact on W_t, and on the rest of our analysis.[52]

Table A10: disposable income & savings (1896-2008)

We define personal disposable income Y_{dt} as follows:

$$Y_{dt} = Y_t – T_t + Y_{Rt} + Y_{Kgt} \quad \text{(A.18)}$$

I.e. disposable income equals national income minus taxes plus government monetary transfers (replacement income) plus net government interest payments.[53] We find that disposable income Y_{dt} was about 95% of national income Y_t around 1900-1910, dropped to about 80% by 1950, and stabilized around 70% in the 1990s-2000s (see Table A10, col. (1)). This simply comes from the fact that taxes currently represent about 50% of national income, while transfers are only 20% of national income (see Table A9). Note however that it is somewhat arbitrary to include only monetary government transfers in the definition of disposable income. In-kind government transfers, as recorded by national accounts, make almost 20% of national income in the 1990s-2000s (see Table A9, col. (16)), mostly in the form of free health and education services provided by the government. If in-kind transfers were added to the definition of disposable income, then disposable income in the 1990s-2000s would be about 90% of national income, i.e. roughly the same level as one century ago.[54] This is why in this research we prefer to use national income rather than disposable income as the proper income denominator when we compute aggregate wealth-income ratios or inheritance-income ratios.

Disposable income Y_{dt} can be broken into three terms:

$$Y_{dt} = Y_{Kdt} + Y_{Ldt} + Y_{Rdt} \quad \text{(A.19)}$$

[52] The only noticeable impact would be on our estimates of the share of inheritance resources in total disposable resources by cohort (see Appendix D4). If productions taxes fall entirely on factor incomes, as we assume, then inheritance resources pay no production taxes. However if part of production taxes are pure consumption taxes, then they also fall on successors when they use they inheritance resources to purchase consumption goods. So our estimates of α^{x*} and γ^x should be reduced somewhat in order to take this into account.

[53] Because replacement income Y_{Rt} represents the vast majority of government monetary transfers TR_t, and in order to simply tables and notations, we omit pure transfers TR_{ot} from our definition of disposable income (see Table A9, col. (19)). We also omit “other corporate transfers” (see below).

[54] If one were to add the value of all services produced by the government (police, national defence, justice, etc.), then by definition disposable income would be as large as national income.

With:

Y_{Kdt} = after-tax capital income[55]

Y_{Ldt} = after-tax labor income[56]

Y_{Rdt} = after-tax replacement income[57]

We find that the share of after-tax capital income in disposable income has been relatively stable in the long run, albeit at somewhat higher levels in the early 20th century (30%-35%) than in the late 20th century (20%-25%), which resembles closely the evolution of the pre-tax capital share in national income.[58] Because average tax rates on capital and labor have been fairly similar as a first approximation (once one takes away replacement-income payroll taxes), the tax system as a whole had a limited impact on the functional distribution. The main change from a long run perspective is the large rise of replacement income and the corresponding decline of labor income (see Table A10, col.(12)-(13)).

Note that we include net-of-depreciation corporate retained earnings in our definitions of after-tax capital income and disposable income. This seems like the most logical way to proceed: presumably retained earnings are in the interest of the owners of corporations (otherwise shareholders would opt for bigger dividends); as a first approximation they can be viewed as capital income that is immediately saved by shareholders and reinvested in the company. In general, this does not make a big difference in terms of levels.[59] E.g. in

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[55] After-tax capital income Y_{Kdt} is defined as capital income Y_{Kt}* (col. (1), Table A8), minus capital taxes T_{Kt} (col. (6), Table A9), minus "other corporate transfers" (col. (14), Table A7). This latter term raises difficult interpretation issues, and it is unclear whom it should be attributed to (see above). In effect, we choose to treat "other corporate transfers" as a tax: we include this term in the definition primary capital income (this is common practice in the analysis of profit shares), and we exclude it from the definition of disposable income (this is common practice in the analysis of household capital income). We note τ_{Kt}* this corrected capital tax rate, i.e. after inclusion of "other corporate transfers" (by construction, τ_{Kt}* = 1-α_{dt}/α_t*, where α_{dt}=Y_{Kdt}/Y_t is the after-tax capital income share in national income). In the 2000s, the corrected capital tax rate τ_{Kt}* appears to be over 40%, while the uncorrected capital tax rate τ_{Kt} is about 35% (see Table A11, col. (8) & (11)).

[56] After-tax labor income Y_{Ldt} is defined as labor income Y_{Lt} (col. (8), Table A8), minus social contributions SC_t (col. (5), Table A9), minus T_{Lit} x $(Y_{Lt}-SC_t)/(Y_{Lt}-SC_t+Y_{Rt})$, where T_{Lit} is the estimated labor share of personal taxes (see above). Personal taxes rely on net-of-social-contributions labor income and on replacement income, and we assume that the average tax rate is the same on the two.

[57] After-tax replacement income Y_{Rdt} is defined as replacement income Y_{Rt} (col. (17), Table A9), minus T_{Lit} x $Y_{Rt}/(Y_{Lt}-SC_t+Y_{Rt})$ (see above).

[58] See Figure A9 vs. Figure A7.

[59] Although this seems like the most logical way to proceed as a first approximation, our way of dealing with retained earnings is far from being fully satisfactory. In particular, one would need to take away the fraction of retained earnings which corresponds to government participations in the corporate sector (it is possible that this was a significant fraction in the 1950s-1960s), which unfortunately historical national accounts series do not allow to do. In principle one should also deduct the fraction which corresponds to foreign participations (and add the retained earnings of foreign companies which corresponds to the participations of French residents). Given that the net foreign asset position of France is close to zero, the net effect must be relatively small.

the 1990s-2000s, the total after-tax capital share in disposable income is about 24%-25%, with non-retained-earnings capital income around 22%-23% and retained earnings around 1%-2% (and sometime negative) (see Table A10, col. (14)-(15)). But this can affect the trends. E.g. retained earnings were higher in the 1950s-1960s (typically 3%-4% of disposable income) than they are today (typically 1%-2% of disposable income), so that if one takes away retained earnings from capital income, then the after-tax capital share would appear to be larger in the 1990s-2000s than in the 1950s-1960s.[60]

Finally, we break down disposable income between consumption and savings:

$$Y_{dt} = C_t + S_t \quad \text{(A.20)}$$

With:
S_t = private savings = personal (household) savings + corporate retained earnings
C_t = private consumption = $Y_{dt} - S_t$

For reasons explained above, we include net-of-depreciation corporate retained earnings in our definition of private savings. We find that private savings have generally fluctuated around 10% of national income over the 1896-2008 period, with the important exception of the reconstruction periods of the 1920s and the 1950s-1960s, when savings were significantly higher (in particular due to retained earnings).[61] These unusually high savings rate during reconstruction periods, especially during the 1920s, must of course be related to the large rates of capital destructions observed during war years (see Table A10, col.

[60] See Figure A10. Note that because we probably overestimate the effective tax burden falling on capital in the recent period (see above), we probably under-estimate somewhat the level of the after-tax capital share in the 2000s. In particular the apparent decline of the after-tax capital share in the late 1990s and early 2000s (and corresponding rise in the after-tax labor share) is likely to be exaggerated (it is parly due to the CSG tax reform – i.e. gradual transfer of some social contributions levied on labor income to a broad based proportional income tax – and the fraction of capital income subject to CSG is in fact smaller than our presumed constant-over-time 50% ratio). Note also the large rise of the capital income share in household income (from about 10% in the 1950s to about 20% in the 1990s) reported in our older series (Piketty (2001, pp.710-711; 2003, fig.4, p.1022) stems from the fact that we omitted in these older series to include retained earnings, and most importantly to include self-employment capital income. In our new corrected series we assumed for simplicity that the capital and labor shares were the same in the self-employment sector than in the corporate sector (which by construction implies than changes in the relative importance of the two sectors has no impact on aggregate capital share); this is imperfect, but better as a first approximation than the zero capital share assumption implicit in our older series.

[61] See Figure A11. As was noted above, it is possible that the retained earnings levels reported for the interwar period are over-estimated somewhat.

(9)); we return to this when we estimate the accumulation equation for private wealth (see section A5 below).[62]

Col. (1) to (17) of Tables A11-A12: summary macro variables (1820-2008)

On Table A11 we report the main macro variables obtained from the previous tables (capital and labor shares, tax rates, savings rates). On Table A12 we report the decennial averages corresponding to these annual 1896-2008 series, which we complete by providing decennial averages estimates covering the 1820-1900 period.

As was already noted, available national accounts series prior to 1896 are relatively rudimentary. The 19th century estimates reported on Table A12 for functional shares, tax rates and savings rates should be viewed as approximate and provisional. We proceed as follows.

Regarding capital and labor shares, there exists to our knowledge no estimate for 19th century France,[63] so we construct our own series using available wage indexes. I.e. the capital and labor shares α_t and $1-\alpha_t$ reported on col.(3)-(4) of Table A12 for years 1820-1829, 1830-1839, etc., until 1890-1899 are computed by dividing the estimated nominal wage bill for each decennial period by the corresponding national income, and by anchoring our labor share series $1-\alpha_t$ to the 1900-1909 and 1910-1913 values coming from our annual series.[64] Nominal wage bill estimates arere computed by multiplying the best available nominal wage index series by the relevant population index.[65]

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[62] Col. (9) of Table A10 is defined as col. (8) of Table A9 (private savings) plus col. (15) of Table A18 (estimated war destructions as a fraction of national income).

[63] In particular the 19th century national accounts constructed by Bourguignon and Levy-Leboyer (1985) and Toutain (1997) are pure production-based accounts: they offer decompositions by industrial production sectors, but not by income categories.

[64] The capital share α_t was then computed as one minus the labor share. The capital share $\alpha_t^*=\alpha_t+\alpha_{gt}$ (col. (5) of Table A12) was computed by assuming for simplicity that net government interest α_{gt} was equal to 2.0% of (factor-price) national income throughout the 1820-1900 period, i.e. the same approximate value as in 1900-1913. Government accounts show that α_{gt} was indeed relatively stable around 2% over the 1820-1913 period, except during the 1870-1900 period, when it reached 3%-3.5%, following the 1870-1871 war and the ensuing rise in public debt (see e.g. Toutain (1997, p.86); see also Fontvieille (1976)). This is negligible for our purposes.

[65] Detailed computations and formulas are reported on col.(3)-(4) of Table A12 and col.(13)-(14) of Table A18. We used the SGF-March manufacturing-sector nominal wage index (as reported by Toutain (1997, p.165)), and we multiplied it by total adult population in order to compute a nominal wage bill index. Of course wage earners made a smaller fraction of the labor force in the early 19th century than in the late 19th century. But because we attempt to compute the labor and capital shares in total national income, and since we do not have 19th century series on the relative shares of the corporate vs self-employment sectors in national income, this is the right thing to do, at least as a first approximation. In effect we are assuming that the SGF-March nominal wage index provides an acceptable approximation of how average individual labor

We find that the labor and capital shares in (factor-price) national income have been relatively stable in the long run over the 1820-1913 period, but with large medium run variations. According to our estimates, the capital share gradually rose from about 30%-35% in the 1820s-1830s to as much as 45% in the 1850s-1860s, then gradually declined to as little as 25%-30% around 1890-1900, and finally rose again, up to about 35% in 1910-1913. Given the data limitations we face, these series should be interpreted with caution. In particular, it is difficult (if not impossible) to estimate precisely the extent to which available 19th century wage indexes are representative of the whole workforce of the time. However the general pattern seems to be robust. In particular, all available wage series show that there was very little wage growth (if any) until the 1850s-1860s, in spite of the large growth in manufacturing output and total national income. We tried alternative series and methods, and we always find a large rise of the capital share between the 1820s-1830s and the 1850s-1860s.[66] Similarly, all series indicate very rapid wage growth during the second half of the 19th century (nominal wages almost doubled), with growth rates significantly larger than those of output and national income – thereby suggesting a marked decline in the capital share. Wage and output series also seem to indicate that the rebound of the capital share between 1900 and 1913 is statistically robust.[67] Whether the

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compensation per adult (wage labor and self employment labor, urban and rural) has evolved in France over the 1820-1900 period. Note also that rental income plays no role in our simple computations: in effect we are implicitly assuming that the share of rental income in national income has remained approximately constant over the 1820-1913 period; available estimates indeed suggest that housing rents have been relatively stable around 5%-8% of national income in the 19th century, possibly with a rise during the second half of the century; see the estimates of net housing rents given by Toutain (1997, p.113)).

[66] Our nominal wage index series seem to deliver lower-bound estimates of the rise of the capital share between 1820-1830 and 1850-1860. According to alternative wage series (such as those reported by Bourguignon and Levy-Leboyer (1985, pp.333-337)), there was virtually no wage growth at all until the 1850s; using such series, one would find capital shares over 50% at mid 19th century. Of course these wage series (including the SGF-March index we are using) are fragile and are generally restricted to industrial low-skill or medium-skill workers (i.e. a relatively small fraction of the workforce of the time). However one would need to assume enormous wage growth for other segments of the workforce in order to compensate manufacturing wage stagnation and undo the capital share pattern.

[67] The capital share pattern that we find for 1890-1899, 1900-1909 and 1910-1913 using our wage index method is consistent with the Villa 1896-1913 series. Villa's series might exaggerate somewhat the rise of the capital share for this time period. But qualitatively such a pattern seems more consistent than the 1900-1913 functional shares stability postulated by Colson (1918, livre 2, p.403): Colson constructs estimates for the structure of private incomes for years 1900 and 1913, using concepts and methods similar to those used by Dugé de Bernonville in his interwar series; however Colson provides limited information on his data sources, which appear to be much less sophisticated than the detailed output and wage indexes used by Villa; in addition there are reasons to believe that Colson (like a number of economists of the time) was strongly attached to the functional stability conclusion per se. Finally, note that the findings of Bouvier, Furet and Gillet (1965), who collected and analyzed book accounts of large companies in France during the 19th century, are consistent with the rise of the capital share in 1896-1913, as well as with our pattern for earlier periods: they find high profit growth during the 1850-1873 period, low profit growth during the 1873-1896 period, and again booming profits in 1896-1913; unfortunately their data is too incomplete to compute profit and labor shares (too small sample, and imperfect distinction between wage bill and intermediate consumption in book accounts), and they have even fewer companies with proper accounts prior to 1850.

levels of capital shares estimated for the 19th century (and particularly the very high levels obtained for the 1850s-1860s) are also statistically robust, and can be compared to 20th century levels in a meaningful way, is a more complicated issue.[68]

Regarding government taxes (col. (7)-(9) of Table A12), we simply assume an aggregate tax rate equal to 8.0% of national income throughout the 1820-1899 period, i.e. approximately the same level as in our 1900-1913 annual series.[69] For simplicity we also assume that this tax rate of 8.0% fell proportionally on capital and labor income throughout the 1820-1899 period.[70] We make a similar stability assumption regarding government transfers.[71]

Regarding savings (col.(12)-(14) of Table A12), we computed average decennial private savings rates from the investment series constructed by Bourguignon and Levy-Leboyer (1985). These series do not allow to differentiate between personal savings and corporate retained earnings, so the private and personal savings rates reported on Table A12 for the 1820-1899 period are the same. We find that private savings have been relatively stable around 8%-10% of national income during the 19th century.[72]

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[68] One would need to collect new raw data on wages in larger segments of the urban and rural economy of the time in order to settle the issue. Note however that the possibility that capital shares attained levels as large as 45% (or even larger) in 19th century economies is certainly plausible. In today's less developed countries one often finds capital shares closer to 40%-50% (or even larger) than to the standard 20%-30% figures found in today's developed countries. Possible explanations for this range from technological stories (e.g. human capital might play a structurally less important role in ancient production functions than in modern ones) to institutional stories (e.g. unions and strikes were virtually banned in a country like France prior to the 1850s-1860s; it is possible that labor bargaining power was particularly low in France during the first half of the 19th century, and more generally in less developed economies). To our knowledge this is very much an open issue.

[69] The government accounts of the time indeed suggest that aggregate tax revenues during the 19th century were approximately stable around 8%-9% of national income (see e.g. Toutain (1997, p.86); see also Fontvieille (1976)). Small time variations around this approximately constant level are negligible for our purposes.

[70] For 1900-1909 and 1910-1913 we found that the average capital tax rate was slightly larger than the average labor tax rate, due to relatively large bequest and gift tax revenues. The bequest and gift tax however raised somewhat lower revenues before the 1901 estate tax reform. Note also the tax on interest and dividend income that was in force in 1900-1913 did not exist during most of the 19th century (the "impôt sur le revenu des valeurs mobilières" – IRVM – was introduced in 1872; this rudimentary income tax system was extended to other income sources in 1914-1917). In any case, the tax rates of the time were all pretty small by modern standards, and that these small variations can be neglected as a first approximation.

[71] I.e. we assumed that government transfers (replacement income) were permanently equal to 1.0% of national income throughout the 1820-1899 period, i.e. approximately the same level as in 1900-1913 (see references above). It follows from our assumptions on government interest (2.0%), taxes (8.0%) and transfers (1.0%) that personal disposable income was equal to 95.0% of national income throughout the 1820-1899 period ($Y_{dt} = Y_t - T_t + Y_{Rt} + Y_{Kgt}$).

[72] In order to compute total private savings, we summed up the net domestic investment and net foreign investment series constructed by Bourguignon and Levy-Leboyer (1985, pp.323-327 & 339-342). Note that according to these estimates, from the 1850s up until 1913, net foreign investment made a substantial part of total private savings (at least 20%-30%, and up to 40% during the 1860s and 1890s-1900s). This is qualitatively and quantitatively consistent with the rising share of foreign assets in total private wealth during

We also report on Tables A11 and A12 estimates of average rates of return on private wealth. In order to compute the pre-tax return on private wealth r_t, one simply needs to divide the primary capital share α_t* in (factor-price) national income by the wealth-income ratio $\beta_t = W_t/Y_t$:

$$r_t = \alpha_t^*/\beta_t \qquad (A.21)$$

The after-tax rate of return to private wealth r_t is similarly defined as the ratio between the after-tax capital share α_{dt}* in national income and the wealth-income ratio β_t :

$$r_{dt} = \alpha_{dt}^*/\beta_t = (1-\tau_{Kt}^*)\ r_t \qquad (A.22)$$

With a wealth-income ratio β_t equal to 600% and a capital share α_t* equal to 30%, we would get a pre-tax rate of return r_t equal to 5%. If we compute a simple arithmetic average of our r_t estimates over the 1820-2009 period, we find an average pre-tax return equal to 6.8% (see Table A12, col.(6)). This reflects the fact that on average over the two centuries wealth-income ratios have actually been less than 500% (due to the low ratios which prevailed during most of the 20th century), and capital shares have actually been (slightly) above 30%. The average pre-tax rate of return was equal to 5.9% during the 1820-1913 period and 7.7% during the 1913-2009 period, again reflecting the lower average wealth-income ratios prevailing in the 20th century. Within the 1913-2009 period, the large variations in the wealth-income ratios have generated large variations in pre-tax rates of return: r_t was as large as 10% both during the 1920s and the 1950s (see Figure A12). Taxes played a relatively small role during the 19th century and a much larger role in the 20th century: on average, the after-tax rate of return r_{dt} appears to be equal to 5.4% both during the 1820-1913 period and the 1913-2009 period. Over the past thirty years (1979-2009), the pre-tax return r_t was 6.9%, while the after-tax return r_{dt} was 4.3% (see Table A12, col. (11)).[73]

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this period (see section A4 below). These BLL investment series should be preferred to the investment series provided by Toutain (1997, pp.77-78), which do not include foreign investment; in addition, Toutain's domestic investment series look definitely too low (with net domestic investment rates below 5% of national income during most of the 19th century), and inconsistent with the observed pattern of private wealth-national income ratios.

[73] We might however overestimate the tax burden falling on capital during the recent period.

To the extent that asset prices rise as much as consumer prices in the long run (see section A5 below), the rates of return r_t and r_{dt} are better thought as real rates of return. In the short run and medium run, however, large variations in asset prices relatively to consumer prices often generate large capital gains and losses, which need to be added to the flow returns r_t and r_{dt} in order to compute the full real returns to private wealth. War-induced capital destructions also need to be taken into account. On Table A12, we use the (imperfect but consistent) real rates of capital gains q_t and war destructions d_t estimated in section A5 below in order to compute augmented after-tax rates of return $r_{dt}^* = r_{dt}+q_t+d_t = (1-\tau_{Kt}^*)r_t+q_t+d_t$.[74] The impact on century-long averages is small; but the impact on decennial averages can be extremely substantial, both during the chaotic world war periods and during peacetime asset price boom periods such as the 2000s. Over the 1913-1949, the after tax flow return r_{dt} was 6.4%, but the augmented return $r_{dt}^* = r_{dt}+q_t+d_t$ was only 1.9%, due to large capital losses and destructions. Conversely, in the 2000-2009 decade, the after tax flow return r_{dt} was only 3.5%, but the augmented return r_{dt}^* was as high as 7.7%, due to large capital gains. Note however that if we take averages over several decades the size of capital gains and losses is usually much smaller than the flow return itself (except naturally during destruction periods). E.g. over the past 30 years (1979-2009), the after tax flow return r_{dt} was 4.3%, and the augmented return r_{dt}^* was 5.3% (see Table A12, col. (15)-(17)).

Capital gains and losses seem to play a smaller role in explaining the 1820-1913 evolution of aggregate rates of returns, which is primarily driven by the relatively large movements of the capital share (and to a lesser extent of the wealth-income ratio). According to our computations, the aggregate rate of return r_t rose from 5%-6% in the 1820s-1830s to over 7% during the 1850s-1860s (when the profit share reached its record level), then declined to little more than 4% during the 1870-1900 period, and then rose again to 5%-6% at the eve of World War 1 (see Table A12, col. (6)). The exact levels we obtain for these 19th century rates of return are obviously fragile, given the data limitations of the time (see above); however the general pattern seems to be relatively robust, and consistent with a number of other data sources.[75]

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[74] Capital gains sometime pay (moderate) taxes. However capital gains tax revenues are already taken into account in corporate and personal income taxes (in effect we attribute them to the capital income flow), so they do not need to be added here.

[75] In particular, the book accounts of large 19th century French companies collected by Bouvier et al (1865) display high profit rates in the 1850s-1860s, declining profit rates in the 1870-1900 period, and rising profit rates from the late 1890s until 1913 (see above). According to net rental income estimates of Toutain (1997, p.113), the rate of return on housing assets was relatively stable around 4% during the 1800-1913 period. I.e. most of the 19th century movements in the overall rate of return on private wealth seems to come from

These historical rates of return, the role they have played in the wealth accumulation and transmission process, and the role they might be playing in the future, are further analyzed in the working paper (see especially section 6) and in Appendix D.

A.4. Supplementary series on the structure of private wealth W_t (Tables A13-16)

Detailed annual series on the structure of private wealth W_t in France over the 1970-2009 period (using official national wealth accounts) are reported on Tables A13 to A15. Available pre-1970, non official estimates of national and private wealth are reported on Table A16. In section A5 below we use these estimates together with savings series in order to estimate the private wealth accumulation equation and to construct continuous private wealth series. Here we describe how Tables A13 to A16 were constructed.

Table A13: private wealth vs government wealth (1970-2009)

All series reported on Tables A13-A15 come directly from official Insee-Banque de France balance sheets.[76] We report values expressed as fractions of national income Y_t (or other aggregates).[77]

On Table A13 we report the following basic decomposition of private wealth W_t :

$$W_t = K_{pt} + A_{pt} – L_{pt} \qquad (A.23)$$

With: K_{pt} = tangible (non-financial) assets of the personal sector[78]
A_{pt} = financial assets of the personal sector

non-housing productive assets (particularly in the booming manufacturing sector). Note also that Banque de France (and Bank of England) interest rates were on average larger between the 1820s and 1860s (typically 4%, and sometime as much as 5%) than during the 1870-1900 period (when they declined to about 3%); see e.g. the series reported by Bourguignon and Lévy-Leboyer (pp.338-342). Such interest rate series raise serious interpretation issues, however: they could also reflect changes in central bank and government finances credibility, at least in part; in any case, given than public debt represents such a small part of aggregate private wealth (usually less than 10%), it seems unlikely that movements in this particular return (wherever they come from) have driven movements in the overall rate of return on private wealth.

[76] See section A1 above for exact references to the Insee-Banque de France tables. We did not make any correction to the Insee-Banque de France raw balance sheets.

[77] Raw values expressed in current billions euros are provided in the excel file.

[78] We use the words “tangible assets” for the sake of concreteness, but we actually include in this category all non-financial assets, as defined by international balance sheets official guidelines (“AN” in ESA 1995 classification code; this also includes a number of intangible assets such as computer software and patents, as well as inventories and valuables; see ESA 1995 manual).

L_{pt} = financial liabilities of the personal sector

We find that the total market value of tangible assets owned by French households was relatively stable around 200%-220% of national income between the early 1970s and the late 1990s, and then nearly doubled during the 2000s (see Table A13, col. (2)). Note that the decomposition of personal tangible assets into housing tangible assets (residential real estate) and non-housing tangible assets (typically business assets owned by the self-employed for conducting their unincorporated production activity: non-residential real estate, commercial dwellings, structures, equipment, land, etc.) has also changed a lot over the period.[79] The rise of personal financial assets was more gradual, from about 100% of national income in the 1970s and early 1980s to about 150% in the 1990s and over 200% in the 2000s. Household debt also rose gradually from 20%-30% of national income in the 1970s-early 1980s to 40%-50% in the 1990s and 60%-70% in the 2000s. Net private wealth W_t rose from 280%-300% of national income in the 1970s-early 1980s to 330%-350% in the 1990s and over 500% in the 2000s (see Table A13, col. (1)-(4)).

We also report on Table A13 the same decomposition for government wealth W_{gt} :

$$W_{gt} = K_{gt} + A_{gt} - L_{gt} \qquad \text{(A.24)}$$

With: K_{gt} = tangible (non-financial) assets of the government sector[80]

A_{gt} = financial assets of the government sector

L_{gt} = financial liabilities of the government sector

We find that net government wealth W_{gt} has always been (slightly) positive and trendless during the 1970-2009 period, usually around 20%-40% of national income, in spite of the large rise of government debt, from 30%-50% of national income in the 1970s-1980s to 60%-80% in the 1990s and 90%-100% in the late 2000s. This is because government tangible and financial assets have always been larger than government debt, and rose from about 80% of national income in the 1970s to 130%-140% in the 2000s (see Table A13, col. (5)-(8)). The value of government tangible assets (administrative buildings, public schools and hospitals, etc.), as estimated by national accounts statisticians on the basis of observed market values for land and similar buildings, has always been larger than the

[79] See Table A15 below for the decomposition between housing and non-housing tangible assets.

[80] In the same way as for income accounts (see above), we include in the government sectors all government levels, as well as the (tiny) non profit sector. Detailed series are available in the excel file.

value of government financial assets. The long term rise of government tangible assets must naturally be related to the long term rise of the government sector share in national income and employment.[81] The value of government financial assets did not decline significantly in the recent past, in spite of the large privatization waves of the late 1980s and 1990s.[82]

We also report on Table A13 the same decomposition for national wealth W_{nt}, which we define as the sum of private wealth W_t and government wealth W_{gt}:

$$W_{nt} = W_t + W_{gt} \qquad \text{(A.25)}$$

We find that private wealth has always represented the vast majority of national wealth throughout the 1970-2009 period: about 85%-90% during the 1970s-1980s, 90%-95% during the 1990s-2000s (see Table A13, col. (13)-(14)).

Table A14: corporate wealth and net foreign asset position (1970-2009)

On Table A14 we report supplementary data on the structure of corporate assets and on net foreign asset position. In principle this should be useless for our purposes: foreign assets were already counted in private wealth W_t and government wealth W_{gt} (we simply isolate them on Table A14 for illustrative purposes), and in theory the net worth of corporations should simply be equal to their equity value, which was also already counted in the balance sheets of the personal and government sector. However in practice there are many reasons why Tobin's Q ratio might differ from 100%, and it is useful to briefly

[81] From a conceptual perspective, this naturally raises the issue as to whether we should exclude the government sector net product from our national income denominator Y_t (given that the corresponding tangibles assets used to produce government net product are excluded from the private wealth numerator W_t). I.e. with our definitions a country with a rising proportion of tangible assets owned by the government (and a rising proportion of the population employed by the government) will go through a mechanical decline in W_t/Y_t and B_t/Y_t ratios (this would simply reflect the fact that public schools, hospitals, museums etc. cannot be privately owned and transmitted through inheritance). The reason why we finally decided to stick to our definitions is because in practice the large and growing tangible assets owned by the government have been approximately compensated by the rise of public debt, so that the share of government net wealth in national wealth is almost as small today as what it was one century ago. In effect, it is almost as if private individuals owned public tangible assets via public debt.

[82] This seems to be due to the fact that during the 1990s-2000s a number of public utilities services were turned into private corporations with large and rising equity capitalization (and in which the government kept important financial participations). For instance, today's government financial portfolio includes large equity positions in France Telecom and EDF. To some extent these have compensated for the privatization of financial assets in the banking and manufacturing sectors (especially given that the latter were lowly valued in the 1970s-1980s, prior to their privatization).

discuss how this might bias our private wealth estimates (and therefore our economic inheritance flow series).

We report on Table A14 the value of corporate net worth, defined according to the standard definition, and compare it to corporate equity value, in order to compute Tobin's Q ratio:

$$NW_{ct} = K_{ct} + A_{ct} - L_{ct}^{d} \qquad (A.26)$$
$$Q = L_{ct}^{e} / NW_{ct} \quad (A.27)$$

With: NW_{ct} = net worth of the corporate sector (non-financial + financial)[83]
K_{ct} = tangible (non-financial) assets of the corporate sector
A_{ct} = financial assets of the corporate sector
L_{ct}^{d} = financial (non-equity) liabilities of the corporate sector
L_{ct}^{e} = equity value of the corporate sector[84]
$Q = L_{ct}^{e} / NW_{ct}$ = ratio between corporate equity value and corporate net worth

We find that Tobin's Q ratio has been less than 100% throughout the 1970-2009 period: it was about 60% in the early 1970s, went as low as 30%-40% in the late 1970s-early 1980s, and stabilized around 70%-80% in the 1980s-1990s (see Table A14, col. (7)). As a consequence, if we compute net corporate wealth, defined as net worth minus equity value, which by definition should be equal to zero in case Tobin's Q was exactly equal to 100%, we find that net corporate wealth was positive throughout the 1970-2009 period. If we express net corporate wealth as a fraction of national wealth (defined above as the sum of private wealth and government wealth), we find that the value of net corporate wealth was the equivalent of about 20%-25% of national wealth throughout the 1970-2009 period (see Table A14, col.(8)).

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[83] In the same way as for income accounts (see above), and in order to simplify notations and tables, we include in the corporate sector both non-financial corporations and financial corporations. Separate series for non-financial and financial corporations are provided in the excel file. Note also that throughout this appendix we use unconsolidated balance sheets (i.e. financial assets and liabilities of the corporate sector include claims of French corporate entities on other French corporate entities). Consolidated balance sheets covering the 1978-2009 period were recently released by Banque de France, but are not reported here.
[84] L_{ct}^{e} is defined as the total value of "shares and other equities" financial liabilities of the corporate sector (AF5 in ESA 1995 classification codes), while L_{ct}^{d} is defined as the total value of debt-like financial liabilities, i.e. all non-equity financial liabilities (AF1+AF2+AF3+AF4+AF6+AF7 in ESA classification codes for financial assets and liabilities; see ESA 1995 manual). Note that in international balance sheets guidelines (ESA 1995), as well as in French private accounting practice, equity value is conventionally included in the financial liabilities of the corporate balance sheet.

In this research we certainly do not intend to solve the complex issue of Tobin's Q ratios and balance sheets measurement errors and statistical discrepancies. For our purposes the key question is the following: should we raise our national wealth and private wealth estimates by about 20%-25%? Or is our private wealth concept W_t (as defined above) the best available approximation that one should use in order to compute the economic inheritance flow? Of course there are lots of reasons why Tobin's Q ratios might differ from 100%: equity pricing is a notoriously complicated and uncertain business, and corporate net worth (book value) is often a poor guide to evaluate future profit prospects. Given that we care about the market value of wealth (inheritance is valued at the asset market prices of the day), we should not care too much as to whether Tobin's Q is momentarily above or below 100%. However it is a bit puzzling that Q ratios appear to be systematically below 100%, including during time periods which are generally viewed as stock market booms and equity overpricing. For instance, the Q ratio appears to be equal to 86% on January 1st 2000: this is more than in every other year, but this is still below 100%.[85]

The reason why we finally decided not to make any correction to our private wealth W_t series is the following. The most plausible explanations as to why Tobin's Q is systematically below 100% are corporate tangible assets overpricing on the one hand, and control rights valuation on the other hand; in both cases, this is not relevant for our purposes.

The corporate tangible assets overpricing story has been recently advocated by Wright (2004) using U.S. data.[86] Many tangible assets owned by corporations (e.g. the Paris headquarters of a large financial firm or the specific machinery and infrastructure used by

[85] This is particularly puzzling if one considers the fact that many non-financial assets (typically intangible assets such trade marks, firm reputation, etc.) are not properly taken into account in the balance sheets of corporations. In principle the stock market should take these assets into account, which should push measured Tobin's Q ratios structurally above 100%. Note also that Q ratios seem to be approximately the same for publicly traded and non publicly traded firms (following ESA 1995 guidelines, the value of private equity in Insee-Banque de France balance sheets is estimated using observed valuations for quoted shares and Q ratios, controlling for industrial sector and company size, and including a discount for lower liquidity, based upon recent private equity transactions; this is of course imperfect, but we have no reason to believe that we can improve these estimates).

[86] Wright finds Tobin's Q ratios around 70%-80% for the US corporate sector in the 1990s, i.e. approximately the same levels as those we find for France. Wright uses balance sheets released by corporations (i.e. book accounts relying upon private accounting concepts and methods) rather than national accounts balance sheets, but to a large extent his arguments also apply to the latter. Note that one also finds U.S. Q ratios around 70%-80% in the 2000s if one uses the balance sheets released by the Federal Reserve (see e.g. *Flows of Funds Accounts of the United States*, Sept. 18 2008, p.95, table B102, line 38). For such computations it is important to use Federal Reserve balance sheets (which include all tangible assets, including land values, using concepts and methods that are broadly similar to the Insee-Banque de France balance sheets) rather than NIPA fixed assets tables (which exclude land values).

manufacturing or utilities company) are difficult to value: readily available market prices are often missing, so that national accounts statisticians (as well as private accountants) generally use a mixture of perpetual inventory methods and market valuation methods in order to put a market price on these assets. Wright argues that on average this might lead to systematic overvaluation of corporate tangible assets (and therefore of corporate net worth), e.g. because rates of capital depreciation on these assets are underestimated. If this is the explanation as to why Q ratios are below 100%, then we should definitely not correct upwards our national wealth and private wealth estimates.[87]

The control rights valuation story is the following. Estimates of aggregate equity value are based upon observed stock market prices, which typically reflect prices for small marginal transactions. In practice, one typically needs to pay a premium as large as 20%-25% in order to purchase sufficient stock to take the control of a corporation, e.g. in order to liquidate its book value. This could mechanically explain why Tobin's Q ratios might be structurally lower than 100%. More generally, the fact that equity values are lower than book values might reflect the fact that shareholders have imperfect control over corporations (and in particular over future profit streams): depending on the country and the specific time period and institutional set-up, other stake holders (such as wage earners or the broader public opinion) might have a say on how corporations should behave, shareholders might have various beliefs about future tax policies or expropriation threats, etc.[88] But whatever the exact story might be, we feel that it is justified for our purposes to use our market value based definition of private wealth W_t (inheritance is valued at prevailing market prices). For other purposes, e.g. if one wants to compute the fundamental economic value of personal wealth, one might prefer to re-attribute the corporate value that is not included in marginal stock market valuation to the ultimate

[87] Except if the national accounts statisticians overvaluation of corporate tangible assets also applies to personal tangible assets, in which case our private wealth estimates should actually be corrected downwards. It is likely however that the valuation problems for personal tangible assets (which currently consist mostly of real estate property) are less severe than for corporate assets. Also one additional reason explaining the overvaluation of corporate assets might be that private companies have an obvious incentive to make their book value look larger than their equity value (national accounts statisticians are in principle immune to corporate creative accounting, since they develop their own methods and concepts to estimate corporate balance sheets; but in practice they have little choice except relying – at least in part – on values reported in corporate book accounts). Clearly this problem does not apply to the personal sector (if anything, private individuals tend to under-report their assets in wealth surveys; but national accounts statisticians rely very little on wealth surveys anyway, at least in France).

[88] This kind of political threat argument can hardly explain why Tobin's Q ratios are lower than 100% in the 1990s-2000s. But it certainly contributes to explain the very low Q ratios observed in France in the late 1970s-early 1980s (when a socialist-communist alliance came to power with a large nationalization programme), and more generally the historically low levels of asset prices observed in the West during the Cold War period (and particularly in the immediate postwar period).

owners of corporations, and raise personal wealth accordingly (see e.g. Atkinson (1972)).[89]

We also report on Table A14 the value of net foreign assets W_{Ft}, defined as the difference between total foreign financial assets owned by French residents FA_t and and total French financial assets owned by foreign residents FL_t. The net foreign asset position of France appears to have been (slightly) positive during most of the 1970-2009 period, except in 1990-1994 and 2009, when it was (slightly) negative. Most importantly, it has always been extremely small. Expressed as fraction of national wealth, the value of net foreign assets has been in the -1% to +5% range throughout the 1970-2009 period (see Table A14, col. (14)).[90] Note however that gross asset positions appear to have risen enormously in recent decades (due to financial globalization): in the 1970s, gross capital foreign asset positions were around 30% of national income; in the 2000s, they were around 300% of national income (see Table A14, col. (9)-(13)). This is qualitatively and quantitatively consistent with the income account data reported on Table A5 above.

Tables A15a & A15b: composition of private wealth (1970-2009)

On Tables A15a-A15b, we report detailed series describing the changing composition of private wealth, using asset categories that can be compared to the categories available in bequest and gift tax returns. Values are expressed as a fraction of national income on Table A15a, and as a fraction of private wealth on Table A15b.

We find that the value of housing assets (residential real-estate tangible assets, net of mortgage debt) has increased significantly over the period, from about 30% of total private wealth in the 1970s to about 50% in the 2000s. The value of non-housing personal tangible assets, which mostly consist of business assets owned by the self-employed for conducting their unincorporated production activity (non-residential real estate, commercial

[89] According to Atkinson (1972, pp.6-7), such a correction can lead to upgrade aggregate U.K. personal wealth by as much as 25%. In the case of France, our best guess is that a substantial part of the upgrade (if any) should be attributed to the government sector rather than to the personal sector (it is likely that government equity participations in a number of public or quasi public unquoted corporate entities are undervalued). In any case, note that measurement errors of the order of 20%-25% of private wealth (at the very most) would not seriously affect our key results regarding long run patterns.

[90] Note that by definition the net foreign asset position is equal to total financial assets minus total financial liabilities of French resident sectors (personal, government and corporate sectors). I.e. $W_{Ft} = FA_t\text{-}FL_t = A_{pt} + A_{gt} + A_{ct} – L_{pt} – L_{gt} – L_{ct}$. E.g. on January 1st 2009 the negative foreign asset position equal to -5% of national income is equal to +135% (positive financial asset position of personal sector) – 50% (negative financial asset position of government) – 90% (negative financial asset position of corporate sector, including equity value in liabilities).

dwellings, structures, equipment, land, etc.), has declined enormously, from over 35% of private wealth in the 1970s to about 10% in the 2000s. This largely reflects the sharp decline of self employment in France during the past 40 years.[91] The share of financial assets in private wealth was about 35% in the 1970s and gradually rose over 50% by 2000, and went down to about 40% during the 2000s, due to the housing market boom (see Table A15b, col. (2), (5) and (6)).

The composition of financial assets has also changed in important ways since 1970. The share of equity assets has always been around one third of total financial assets (with public equity and mutual funds gradually taking over private equity), and the non-equity share has always been around two thirds.[92] This reflects the fact private individuals in France have limited direct stock market ownership. Within non-equity assets, one observes a very large rise of life-insurance assets, which made only 2% of private wealth in the 1970s, up to about 15% in the 2000s, i.e. about a third of total financial assets (see Table A15b, col. (7)-(12)).

The large development of life insurance in France has certainly been encouraged by its very favourable tax treatment. In particular, life insurance has always been (almost) entirely exempt from bequest and gift tax: the corresponding wealth can be transmitted tax free to children, surviving spouses and other beneficiaries. Note also that in France life insurance is often used as a long term, old-age saving vehicle, in the absence of explicit

[91] The share of self-employment in total employment was as large as 25%-30% in France during the 1960s, and it is now less than 10% (see Piketty (2001, p.51, graph 1-4)). Note however the share of non-housing personal tangible assets in private wealth (as we measure it) is almost certainly an overestimate of the true share of the productive assets of the self-employed in private wealth. First, non-housing personal tangible include many assets (e.g. valuables, non-agricultural land, etc.) that have little to do with self-employed productive assets. We computed the value of housing tangible assets K_{pt}^{h} as the value of "residential dwellings" (AN1111 in ESA 1995 classification codes), plus an estimate of the corresponding land value (we allocated the value of "land underlying buildings and structures" AN2111 proportionally to AN1111 and to AN1112 "other buildings and structures"); non-housing tangible assets K_{pt}^{n} was then simply computed as a residual K_{pt} - K_{pt}^{h} (see excel file for raw data and formulas). So by construction K_{pt}^{n} includes all non-housing, non-self-employed assets. Also, we attributed total personal financial liabilities to housing assets, thereby assuming that household debt consists entirely of mortgage debt. This is an acceptable approximation for the 1990s-2000s, but in the 1970s it is likely that a larger fraction of personal debt should be attributed to the self-employed. As a consequence the series reported on Tables A15a-A15b probably underestimate the net value of housing assets and over-estimate the value of non-housing assets in the early 1970s. Finally, as was noted above, pre-1978 balance sheets are more rudimentary and less precise than post-1978 series.
[92] Equity assets are defined as "shares and other equities" (AF5 in ESA 1995 classification codes). Public equity and mutual funds are defined as the sum of "quoted shares" (AF511) and "mutual funds shares" (AF52); private equity is defined as the sum of "unquoted shares" (AF512) and "other equity" (AF513). Non-equity assets are defined as the sum of all other financial assets: "currency and deposits" (AF2), "securities other than shares" (AF3), "loans" (AF4), "insurance technical reserves" (AF6), and "other accounts" (AF7).

pension funds.[93] In principle, we should make a correction for the annuitized fraction of life-insurance assets, i.e. for the fact that a fraction of what is counted as life-insurance assets cannot be bequeathed at death. However this annuitized fraction is difficult to estimate, and in any case appears to be relatively small (at most 20%).[94] Life-insurance assets currently represent about 15% of aggregate private wealth W_t, so this implies that the non-bequeathable fraction of aggregate private wealth W_t is at most 3%. In addition, note that we did not attempt to make corrections for the fact that a number of bequeathable assets are not included in our private wealth W_t estimates. In particular, consumer durables (such as cars or furnitures), which usually represent less than 5% of total wealth,[95] are excluded from the Insee-Banque de France balance sheets,[96] and therefore are also excluded from our private wealth W_t, in spite of the fact that durables are in principle subject to the estate tax. Because these two corrections terms (annuitized fraction of life insurance assets, consumer durables) are small, hard to estimate with precision, and tend to compensate one another, we feel that it is more reasonable not to make any explicit correction at this stage, and to leave these issues for future research.[97]

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[93] According to ESA 1995 classification, "insurance technical reserves" (AF6) can be broken down into the value of "life insurance reserves" (AF611) and the value of "pension funds reserves" (AF612). However in the French balance sheets compiled by Insee-Banque de France, "pension funds reserves" are equal to zero by construction, i.e. 'insurance technical reserves" are entirely allocated to "life insurance reserves". We took the full value of "insurance technical reserves" (AF6) as our estimate of life insurance assets (col. (11) of Tables A15a-A15b), with no correction.

[94] Unfortunately, the data published by insurance companies (FFSA) appears to be insufficient to compute a precise estimate of the annuitized fraction of life-insurance assets. One could think of using published payment flows to beneficiaries at the death of policy-holders. These annual flows currently appear to be relatively small, typically less than 1% of total life insurance reserves (around 5-10 billions euros, out of over 1200 billions euros in life insurance reserves; see *Rapport annuel FFSA 2008*, pp.31-33), i.e. slightly less than the aggregate mortality rate m_t, and substantially less than the aggregate inheritance-wealth ratio $B_t/W_t = \mu_t^* m_t$ (see Table A3 above). However it is unclear how exactly insurance companies compute these payment flows to non-policy-holders beneficiaries: they apparently include only the payments corresponding to the explicit death insurance clause stipulated in life insurance contracts (i.e. the lump sum payment to beneficiaries conditional upon the death of the policy-holder). Most life insurance contracts in France are merely temporary term savings contracts (typically 8-year-long), with a small explicit death insurance dimension, so that most payment flows mechanically return to policy-holders themselves, and possibly to their heirs in case they die (but these payments to heirs then do not seem to be counted as "death insurance" payments). Also note that we do not know from available data which fraction of the payment flows going to policy holders is used to repurchase new life insurance contracts and which fraction is used to purchase other assets, which may end up being transmitted to heirs (life insurance assets in France carry tax advantages not only at the time of wealth transmission, but also during accumulation: for the most part flow returns are being re-capitalized net of income tax).

[95] According to wealth surveys and to estate tax returns.

[96] This is because consumer durables do not generate flow returns in income accounts, and therefore are not treated as investment goods.

[97] Note that in their computation of bequeathable aggregate wealth using Federal Reserve balance sheets, Kopczuk and Saez (2004, NBER WP version, pp.44-47) keep the full value of life insurance reserves (as we do here), i.e. they assume that 100% of the value of life insurance reserves is bequeathable; but they keep only the cash surrender value (CSV) of pension funds reserves (i.e. the value of pensions that remains upon death), ranging from 5% of pension funds reserves for traditional defined benefits pension schemes to 100% for recent defined contributions pension schemes. Social security pensions cannot be transmitted to heirs and are naturally excluded from bequeathable wealth, both by Kopczuk-Saez and in the present research.

We also report on Table A15b estimated fractions of assets that are subject the bequest and gift tax. These estimates are used in Appendix B1 in order to upgrade the fiscal inheritance flow series; they were computed on the basis of estate tax law and of asset composition observed in estate tax returns.[98]

Table A16: Raw national wealth estimates in France (1820-2008)

On Table A16 we report the various non-official, pre-1970 national and private wealth estimates that we used in this research (for comparison purposes we also report official estimates for 1978, 1990 and 2008). Pre-1970 national and private wealth estimates are more rudimentary and offer fewer (and less homogeneous) break downs than post-1970 Insee-Banque de France balance sheets, so we only report the decomposition between private and government wealth, as well as estimates of the share of foreign assets in private wealth.

As we explain the working paper (see section 3.2), national and private wealth estimates for the 1820-1913 are plentiful and relatively reliable. The national and private wealth concepts used by the economists of the time are broadly similar to the concepts of W_{nt} and W_t that we defined using modern, post-1970 official balance sheets. In particular, 19th century and early 20th century economists defined aggregate private wealth ("fortune privée") as the market value of all tangible and financial assets owned by private individuals, minus their financial liabilities. They relied mostly upon the decennial censuses of tangible assets organized by the tax administration (the tax system of the time relied extensively on the property values of real estate, land and business assets, so such censuses played a critical role). They took into account the growing stock and bond market capitalisation and the booming foreign assets, and they usually explained in a precise and careful way how they made all the necessary corrections in order to avoid all forms double counting. The most sophisticated estimates, e.g. those of Colson (1903), compare explicitly the equity value of corporations obtained from stock market capitalization (deducting cross holdings), to the book value of corporations obtained by summing up the value of tangible assets (minus debt), and find similar results using both methods (i.e. they find that Tobin's Q ratios were close to 100% on average). The most important point to be careful about is the following: one should use only the national wealth estimates that were

[98] See Appendix B1.

explicitly based upon wealth-census-type methods, and ignore estimates based upon estate-multiplier-type computations.[99] All estimates reported on Table A16 are based upon wealth-censuses methods.

For 1913, we take the reference estimate due to Colson, with aggregate private wealth of 297 billions old francs, including an estimated 41 billions in foreign assets.[100] For 1896, there are variations across authors within the 190-230 billions range, and we take an average estimate of 205 billions.[101] For earlier decades (1820-1829, …, 1880-1889), the confidence interval between the various authors is usually less than 10%, and we report on Table A16 the average estimates of private wealth available in the literature, from about 62 billions old francs in the 1820s to about 195 billions old francs in the 1880s.[102] The published estimates usually include separate computations for government wealth (government tangible and financial assets, minus government debt), showing that government wealth was a positive but small fraction of national wealth throughout the 1820-1913 period: between 2% and 5%, i.e. private wealth always represents 95%-98% of national wealth (see Table A16, col. (8)-(9))).[103] All estimates also show a large and gradual rise of foreign assets, from about 2%-3% of aggregate private wealth in the 1820s-1840s to about 10% in the 1860s-1870s and almost 15% in 1900-1913 (see Table A16, col.(3)).[104]

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[99] It would indeed make no sense at all to use estate-multiplier-based national wealth estimates in order to compute the inheritance flow. Nineteenth century economists were so confident in the W/B=H estate multiplier formula (see working paper, section 2.4) that they often mixed up both methods. For instance, following Foville (1893), Colson (1903, vol.2, pp.282-283) presents a census-based estimate of national wealth for 1898, then notes that it is roughly equal to 30-35 times the inheritance flow, and a few pages later presents national wealth estimates for the 1820s, 1840s, 1860s, 1880s and 1890s based on the estate multiplier method, i.e. by multiplying by 30-35 the inheritance flow. We did not use such estimates.

[100] See Colson (1918, livre 2, p.372). See also Divisia, Dupin and Roy (1956, vol.3, p.69), who start their 1954 computations from this 1913 Colson estimate.

[101] E.g. Colson provides an estimate of 230 billions for 1898 (see Colson (1903, vol.2, pp.282-283), while Leroy-Beaulieu provides an estimate of 195 billions for 1900 (see Danysz (1934, p.141)).

[102] For a compilation of various national wealth estimates from 1800 to 1913, see e.g. Fovile (1893, pp.604-605), Danysz (1934,p.141); “Quelques données statistiques sur l’imposition en France des fortunes privées”, unsigned article, *Bulletin Mensuel de Statistique*, Insee, 1958, p.34; and Lévy-Leboyer (1977, p.396).

[103] The estimates for government assets and debt reported on Table A16 for the 1820-1880 should be viewed as approximate and illustrative (the corresponding raw estimates are less sophisticated than the Colson-type estimates computed around 1890-1913).

[104] The other asset categories used in these estimates are not sufficiently homogenous through time to produce detailed composition series. Around 1900-1913, the total value of real estate and land was typically about 45%-50% of aggregate private wealth in most estimates (roughly 20%-25% for real estate, and 25% for land); around 1800-1820, the total value of real estate and land was as large as 65%-75% of aggregate private wealth in most estimates (roughly 20%-25% for real estate, and as much as 45%-50% for land). This appears to be consistent with the decline of the agricultural sector and the rise of the manufacturing and services sector. It is difficult to go much beyond this at this stage, because the frontiers between cultivated land, rural and urban real-estate properties and non-land, non-real-estate tangible business assets are not fully homogenous over time in the raw data coming from the tax administration decennial censuses of

The 1914-1969 period is the most problematic one from the viewpoint of national and private wealth estimates in France. This was a chaotic time for wealth (war destructions, large inflation, wide variations in real estate and stock prices, not to mention the fact that large segments of banking and manufacturing sector were nationalized in 1945). This certainly discouraged the economists of the time from pursuing the private wealth computations that were so popular until 1913. We used only two estimates of private and national wealth over the 1914-1969 period. Both are based upon methods and concepts that are broadly similar to the 1820-1913 and 1970-2009 estimates: one for year 1925 due to Colson (1927), and one for year 1954 due to Divisia, Dupin and Roy (1956). Both are reported on Table A16.

The advantage of the 1925 estimate that it was constructed by Colson using the same methods and concepts as his estimates for years 1898 and 1913. One central difficulty with this period is that asset prices declined significantly relatively to consumer prices between 1913 and 1925, with large movements in the relative prices of various assets.[105] Colson carefully explains how he computed the market value of 1925 private wealth W_t using the asset prices prevailing in 1925 for the various assets, which is what we want.[106] Colson uses the same method to estimate the net market value of government wealth W_{gt}, which for the only time in our long run series appears to be negative in 1925: the French government accumulated so much public debt during the World War 1 and the early 1920s that by 1925 the value of government debt significantly exceeded the value of government tangible and financial assets (see Table A16, col. (4)-(6)). According to these computations, private wealth W_t according to these computations was equal to 293% of national income, but national wealth $W_{nt}= W_t+W_{gt}$ was equal to 241% of national income (see Table A16, col. (10)-(14)). This 1925 Colson estimate could possibly be improved by returning to the raw statistical material of the time.[107] But at this stage one can consider

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property values (and consequently in the national and private wealth estimates constructed by the economists of the time).

[105] See the raw price index series reported on Tables A20-A22 below.

[106] See Colson (1927, pp.484-486). Colson then converted his 1925 private wealth estimates expressed in 1925 asset prices (1060 billions francs) into an estimate expressed in 1913 prices using consumer price inflation between 1913 and 1925 (he assumes consumer prices were multiplied by 4.0 between 1913 and 1925, which is very close to the 4.1 ratio we obtain with our CPI series, see Table A20, col.(1)), and found that 1925 private wealth was equal to 265 billions 1913 francs, i.e. about 10% less than his 1913 estimate. The subsequent literature usually refers to this 265 billions number (see e.g. Danysz 1934 p.141 and *BMS* 1958 p. 34), but it is important to realize that it was computed by Colson using 1925 relative asset prices, not 1913 relative asset prices.

[107] We made two important downward corrections to the Colson raw estimate. Fist, for the sake of consistency with modern estimates, we took durable goods and furnitures (20 billions out of the 265 billions

that this is relatively reliable and well documented estimate – and in any case by far the best available estimate for the interwar period.

Similarly, the advantage of the 1954 estimate is that Divisia-Dupin-Roy use the same methods and concepts as Colson, and make a systematic comparison with the 1913 Colston estimates for the different types of assets. Also, Divisia-Dupin-Roy are very careful at distinguishing between market values and book values (including for unincorporated businesses). The estimate that we report on Table A16 for private wealth W_t corresponds to their market value of private wealth (i.e. evaluated at the asset prices prevailing in 1954).[108] They also provide interesting estimates of government wealth W_{gt}, which unlike in 1925 was significantly positive in 1954: this comes from the fact that public debt vanished in the immediate postwar period (due to inflation), while at the same time government tangible and financial assets rose substantially (due to 1945 nationalisation policy). In effect, the government was the owner of substantial segments of the French corporate sector in the 1950s. According to the Divisia-Dupin-Roy estimates, the share of government wealth W_{gt} in national wealth $W_{nt}=W_t+W_{gt}$ was as large as 32% in 1954, while the share of private wealth W_t was only 68% (see Table A16, col. (8)-(9)).[109] It took several decades for government debt to build up again (and also for corporate privatizations to occur) and finally for the government share in national wealth to return to about 5% in the 1990s-2000s, i.e. about the same level as during the 1820-1913 period (see Figure A14).

We also borrowed to Divisia et al (1956) their estimates of physical capital destructions during both world wars. After a careful review of the various existing computations on wartime physical destructions (real estate, structures, equipment, machinery, etc.), Divisia-Dupin-Roy come with the conclusion that total capital destructions represented the equivalent of about 11% of 1913 aggregate private wealth W_t during World War 1, and the

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total). Next, although Colson does attempt to use current equity value for publicly traded corporations, it is apparent that he did not make the corresponding correction for private equity and unincorporated businesses: in effect he uses the book value of tangible assets for non-public traded firms, which is problematic at a time when Tobin's Q ratios were probably substantially below 100%. In order to take this into account, we applied a 30% downward correction to the corresponding values (see excel file for formulas).

[108] See Divisia, Dupin and Roy (1956, vol.3, and particularly pp.65-67). More precisely, we used their market value ("valeur vénale") of private wealth of 32 000 billions old francs, minus durable goods and furnitures (6 400 billions), plus net foreign assets (500 billions), plus government debt (4 500 billions), so that W_t = 30 600 billions old francs (i.e. 47 billions euros). See formulas in the excel file.

[109] More precisely, we used the Divisia-Dupin-Roy estimate of 24 600 billions old francs of government tangible and financial assets; unfortunately this is a book value estimate ("valeur d'inventaire"; see Divisia et al (1956, p.47)); we converted into a market value by assuming the same market-to-book-value ratio as for private wealth, i.e. we multiplied 24600 by 32000/42300 (this is the "valeur vénale"-"valeur d'inventaire" ratio found by Divisia-Dupin-Roy for private wealth). See formulas in the excel file.

equivalent of about 22% of 1913 aggregate private wealth W_t during World War 2.[110] We also added to these physical destruction numbers available estimates for foreign assets losses during World War 1 (typically, Russian bonds repudiation), which to some extent can be assimilated capital destruction. Total foreign assets losses during World War 1 appear to be as large as physical capital destructions strictly speaking: the equivalent of about 12% of 1913 aggregate private wealth.[111] Overall, total private wealth destructions (including foreign assets losses) amount to about 23% of 1913 aggregate private wealth W_t during World War 1, and about 22% during World War 2. We use these estimates when we compute the private wealth accumulation equation below.[112]

How reliable are our aggregate private wealth estimates for the chaotic 1914-1969 period? With our data we find that private wealth W_t was 660% of national income Y_t in 1913, 293% in 1925, 203% in 1954, and 289% in 1970 (see Table A16, col. (10)). We certainly do not pretend that the 1925 and 1954 numbers are perfectly comparable to the pre-1913 and the post-1970 numbers: the quantitative precision of such ratios should not be over-

[110] Expressed in 1913 old francs, total destructions are estimated to 34 billions in 1914-1918 and 61 billions in 1939-1945 (see Divisia et al (1956, pp.62-63)). Physical fights were shorter during World War 2 (there was no fighting in 1941-1943) than during World War 1, but the bombing technology used in 1940 and 1944-1945 was much more devastating than that used during World War 1. Note that these Divisia-Dupin-Roy estimates of total physical destructions during wars (about 10% of aggregate wealth during World War 1 and about 20% during World War 2) are much more plausible than the Cornut-Sauvy estimates, according to which as much as one third of the capital stock was destroyed during World War 1, and as much as two thirds during World War 2. We (carelessly) reported these Cornut-Sauvy estimates in our previous work (see Piketty (2001, p.137; 2003, p.1020)). These estimates are dubious, because they are entirely based on estate-multiplied methods, rather than on census-based national wealth estimates, and should therefore be ignored. Cornut (1963, p.399) computed national wealth estimates for 1908, 1934, 1949 and 1954 by multiplying observed fiscal inheritance flows by a sequence of somewhat arbitrary estate multiplier coefficients (Cornut realized that the estate multiplier coefficient should be upgraded over time, but was uncertain as to how this should be done; in the end he picked upgraded coefficients pretty much on a ad hoc basis, or at least with no clear written justification); Sauvy (1984, p.323) then divided these Cornut national wealth estimates by national income estimates, and found wealth-income ratios of 570% for 1908, 350% for 1934, 120% in 1949 and 140% in 1953; from which he concluded that the war-induced wealth destruction rate was about one third during World War 1 and two thirds during World War 2. Our new, consistent series show that Sauvy probably underestimates wealth-income ratios in 1908 and 1949-1953 (and overestimates the 1935 ratio). Most importantly, our new series show that wartime physical destructions explain a much smaller fraction of the overall decline in wealth-income ratios than what was implicitly assumed by Sauvy, and that asset price changes played a bigger role. See below.

[111] Total foreign assets losses during World War 1 are estimated to as much as 90% of the 1913 foreign asset portfolio, i.e. about 37 billions francs (see Divisia et al (1956, pp.62-63)). Note that this includes not only foreign asset repudiation (exemplified by the pure case of Russian bonds), but also the loss in foreign asset values due to inflation and stock market collapse. The exact decomposition between wealth destruction via repudiation and wealth destruction via inflation is dfficult to compute, and inrrelevant for our puposes, so we simply add up all foreign asset losses to wartime physical destructions. National income and national wealth data consistently show that foreign assets never recovered from the Wolrd War 1 shock and remained relatively low in the interwar and at the eve of World War 2 (see Tables A5 and A16), so we neglect foreign asset losses during Wolrd War 2.

[112] In order to annualize the destruction estimates we assumed that these private wealth destructions could be splitted equally (in real terms, as measured by 1913 consumer prices) over the four years 1915-1918 and over the six years 1940-1945. See the excel file for the resulting raw series (Table A0).

estimated, especially in times of economic crises. However, there are several reasons to believe that these numbers provide a relatively accurate quantitative picture of changes in the wealth-income ratio (at least as a first approximation). First and foremost, as we show below, this 1913-1925-1954-1970 profile of the wealth-income ratio is broadly consistent with the aggregate accumulation equation for private wealth, i.e. with the savings rates coming from national income accounts and the (imperfect) asset price indexes at our disposal (see section A5 below). Next, our 1954 private wealth total is consistent with a number of independent computations that were made in France in the 1960s-1970s, at the time when Insee was starting to construct official balance sheets.[113] Finally, it is reinsuring to see that economic inheritance flows that we obtain from our national wealth estimates are consistent with the fiscal inheritance flows, including during the 1914-1969 period. Note however that the gap between our fiscal and economic inheritance flow series is significantly larger in the 1920s-1930s and 1950s-1960s than in the pre-World War 1 and post-1970 period.[114] If anything, this suggests that our wealth-income W_t/Y_t for 1925 (293%) and 1954 (203%) are over-estimated, i.e. that aggregate private wealth was even lower than what the Colson-Divisia-Dupin-Roy computations indicate. In order to obtain the same economic flow-fiscal flow ratios as for the other periods, one would need to assume that the aggregate wealth-ration W_t/Y_t was as low as 210%-230% in 1925 (instead of 293%), and as low as 150%-170% in 1954 (instead of 203%).[115] Given the very large asset price movements of the time, and the data imperfections we face, this is certainly a possibility that cannot be excluded. However it is likely that the higher economic-fiscal flow ratios also reflect higher estate tax evasion during this period, and/or higher unmeasured

[113] In particular, it is reinsuring to note that our final estimate for 1954 private wealth (30 600 billions old francs) turns out to be almost identical to the private wealth estimate given by Masson and Strauss-Kahn (1978, p.38), who find 30 700 billions old francs for 1954. Note however that the fact that both estimates turn out to be so close is largely a coincidence, since Masson and Strauss-Kahn use a completely different method: they start from a 1975 estimate of national wealth and work it backwards through savings until 1949; given that they do not take into account capital gains, they should find a smaller number than ours for 1954; the explanation seems to be that on the other hand they underestimate savings with their pre-B2000 national accounts. See also Masson (1986), and Babeau (1983), who uses a similar method. To our knowledge these Masson-Strauss-Kahn-Babeau papers are the only attempt to construct private wealth estimates in France for 1950s-1960s, i.e. prior to the introduction of official Insee balance sheets in 1970. The only other attempt (based on direct evaluation method close to national wealth accounts) seems to be due to Campion (1971), who gives estimates of total private wealth for 1962 (967 billions francs), 1965 (1242 billions francs) and 1967 (1465 billions francs), which are very close to our estimates (slightly bigger).

[114] The B_t/B_t^f ratio is generally about 110% both in 1820-1910 and 1980-2010, but is as large as 130%-150% between the 1920s and the 1970s. See Table A4, col. (9). Note also that the B_t/B_t^f ratio reaches 125% in the 1850s: it could be because our private wealth estimate for the 1850s is 10%-15% too high (as compared to other 19th century estimates), and/or because we under-estimate legal estate tax exemptions in this period, and/or because we over-estimate the μ_t coefficient (see Appendix B).

[115] The Cornut-Sauvy wealth-income ratios are as low as 120%-140% in 1949-1953, but this of course is tautological and uninformative, since they were computed by applying estate multiplier coefficients to the fiscal inheritance flow (see above).

legal estate tax exemptions.[116] So these alternative wealth-income ratios for 1925 and 1954 should probably be viewed as absolute lower bounds, and our Colson-Divisia-Dupin-Roy-based ratios should be viewed as more realistic and consistent. Available data does not allow us to push this analysis much further. Given that this (limited) residual uncertainty has little consequence for our overall long run empirical and theoretical analysis, we leave this issue for future work.

A.5. Computation of the private wealth accumulation equation (Tables A17-A19)

On Tables A17-A19 we report the series resulting from the computation of the private wealth accumulation equation. We need to estimate such an accumulation equation in order to overcome the incompleteness of historical national wealth accounts and to obtain annual series for private wealth W_t (especially regarding the data-poor 1914-1969 period). More generally, estimating such an accumulation equation in the long run offers an opportunity to assess the internal consistency between national income and wealth accounts, and also to test standard capital accumulation models. Here we describe how Tables A17-A19 were constructed. We start by describing the basic accumulation equations, and then explain how they were applied to the 1896-2008 period (annual series) and to the 1820-1913 period (decennial averages).

A.5.1. Capital accumulation equation with no price inflation

In a world with no price inflation, the relationship between private wealth at the beginning of year t (W_t), private savings during year t (S_t) and private wealth at the beginning of year t+1 (i.e. end of year t) (W_{t+1}) would be straightforward:

$$W_{t+1} = W_t + S_t \qquad (A.28)$$

[116] In principle estate tax law always required taxpayers to report market value of assets (at the time of death or gift). In practice, however, it is possible that the tax administration allowed taxpayers to report lower values during times of large inflation, which were numerous during the 1920-1970 period. It is very difficult to estimate the magnitude of this effect, but it can be large. Also, many temporary, asset-specific estate tax exemption regimes were created in the aftermath of both world wars (e.g. for specific public bonds or savings accounts, or for new real estate constructions), and some of them applied for several decades. We attempt to take these into account, but it is possible that we underestimate the fraction of tax exempt assets during this period. The fact that the B_t/B_t^f ratio appears to be almost as large in the 1970s (when we use official Insee-Banque de France balance sheets) than in the 1920s-1930s and 1950s-1960s (when we rely on Colson-Divisia-Dupin-Roy estimates) suggests that the under-evaluation of the fiscal flow (due to tax evasion and exemption, broadly understood) plays a larger role than the over-evaluation of national wealth in Colson-Divisia-Dupin-Roy estimates (unless the official balance sheets of the 1970s are also over-evaluated).

Dividing both terms by national income Y_{t+1}, and re-arranging the terms, one gets the following equation:

$$\beta_{t+1} = W_{t+1}/Y_{t+1} = [\beta_t + s_t]/[1+g_{t+1}]$$

I.e.: $$\boldsymbol{\beta_{t+1} = \beta_t\ [1+s_t/\beta_t]/[1+g_{t+1}]} \qquad \textbf{(A.29)}$$

With: $\beta_t = W_t/Y_t$ = (private wealth)/(national income) ratio
$s_t = S_t/Y_t$ = savings rate (private savings as a fraction of national income)
$1+g_{t+1} = Y_{t+1}/Y_t$ = growth rate of national income between t and t+1

Intuitively, equation (A.29) says that the wealth-income ratio $\beta_{t+1}>\beta_t$ iff $s_t/\beta_t > g_{t+1}$ (i.e. if $\beta_t < s_t/g_{t+1}$). Note that $s_t/\beta_t = S_t/W_t$ is simply equal to private savings as a fraction of private wealth, which can be labelled "savings-induced wealth growth rate". E.g. if s_t=10% and β_t =500%, then s_t/β_t = 2%: private savings during year t represent 2% of wealth at the beginning of year t, and therefore allow wealth to grow at 2% per year between t and t+1. Intuitively, he wealth-income ratio rises if and only the savings-induce wealth growth rate exceeds the growth rate of national income. In case s_t and g_t are stationary (i.e. s_t=s g_t=g), then β_t converges toward a steady-state value β^* = s/g. E.g. if s=10% and g=2%, then β^* = 500%. This is simply the standard Harrod-Domar formula (see working paper, section 5).

In order to clarify this interplay between income growth and wealth growth, it is useful to note g_{t+1}^s the savings-induced wealth growth rate between t and t+1, and to rewrite equation (A.29) in the following manner:

$$\boldsymbol{\beta_{t+1} = \beta_t\ [1+g_{wst+1}]/[1+g_{t+1}]} \qquad \textbf{(A.30)}$$

With: $g_{wst+1} = s_t/\beta_t = S_t/W_t$ = savings-induced growth rate of private wealth

A.5.2. Capital accumulation equations with capital gains

Taking price inflation into account complicates the capital accumulation equation. We note P_t the consumer price index (average consumer prices during year t), and Q_t the asset price index (asset prices at the beginning of year t). In practice, we observe nominal national income Y_t (measured at current market prices for consumer goods and investment goods), nominal private wealth W_t (measured at current market prices for assets) and nominal wealth/income ratios $\beta_t=W_t/Y_t$. The year-to-year variations of β_t generally reflect both relative volume effects (as determined by savings S_t) and relative price effects (as determine by the evolution by the relative asset vs goods price index Q_t/P_t). We do have (reasonably) good series on consumer price indexes P_t, which allow us to compute consumer price inflation p_t and real growth rate of national income g_t:

$1+p_t = P_t/P_{t-1}$ = consumer price inflation

$1+g_t = (Y_t/P_t)/(Y_{t-1}/P_{t-1}) = (Y_t/Y_{t-1})/(1+p_t)$ = real growth rate of national income

However, we usually do not have good measures of the asset price index Q_t: we do have all sorts of price series for various assets (real estate prices, stock prices, etc.), but it is very difficult to weight them properly, especially given the very large variations in asset price inflation over different types of assets (more on this below). If we know the evolution of W_t, then we can define an implicit asset price index Q_t directly from the wealth accumulation equation:

$$W_{t+1} = (Q_{t+1}/Q_t)\ (W_t + S_t)$$

$$\text{I.e.}\quad W_{t+1} = (1+q_{t+1})\ (1+p_{t+1})\ (W_t + S_t)$$

Dividing both terms of the equation by Y_{t+1}, and re-arranging the terms, one gets the following equation:

$\beta_{t+1} = [1+q_{t+1}]\ [\beta_t+s_t] / [1+g_{t+1}]$

I.e. :

$$\boldsymbol{\beta_{t+1} = [1+q_{t+1}]\ \ \beta_t\ [1+s_t/\beta_t]/[1+g_{t+1}]} \qquad \textbf{(A.31)}$$

With:

$1+q_{t+1} = (Q_{t+1}/P_{t+1})/(Q_t/P_t)$ = asset price inflation relatively to consumer price inflation[117]

[117] Note that P_{t+1}/P_t measures inflation between average consumer prices during year t and average consumer prices during year t+1, while Q_{t+1}/Q_t measures inflation between asset prices on January 1[st] of year t and January 1[st] of year t+1. Given our long run focus, this six-month time inconsistency does not really matter (one solution would be to re-compute national wealth accounts in average year prices, but that did not seem worth while).

In case asset prices increase (or decrease) just as much as consumer prices, then q_t=0%, and equation (A.31) boils down to equation (A.29): capital accumulation involves pure volume effects, as determined by savings. However, in case wealth holders experience real capital gains (q_t>0), then equation (A.31) says that the wealth-income ratio can increase even though there is little savings (i.e. even though $s_t/\beta_t < g_{t+1}$, providing that real capital gains are strong enough), and conversely in case of real capital losses (q_t>0).[118] Alternatively, one could view real unrealized capital gains or losses as capital income that is being saved at a 100% rate. I.e. if we note $Y_{Kqt} = q_{t+1}\ (W_t+S_t)$ the real unrealized capital gains (or losses) made during year t, and if we define a corrected saving rate $s_t^* = (s_tY_t+Y_{Kqt})/Y_t$, i.e. $s_t^* = s_t + q_{t+1}\ (\beta_t+s_t)$, then equation (A.31) can simply be rewritten as follows: $\beta_{t+1} = \beta_t\ [1+s_t^*/\beta_t]/[1+g_{t+1}]$.

In order to clarify this interplay between income growth and wealth growth, it is again useful to rewrite equation (A.31) in the following manner:

$$\boldsymbol{\beta_{t+1} = \beta_t\ [1+g_{wt+1}]/[1+g_{t+1}]} \qquad \textbf{(A.32)}$$

Where $g_{wt+1} = (W_{t+1}/P_{t+1})/(W_t/P_t) = (W_{t+1}/W_t)/(1+p_{t+1})$ is the real (relative to CPI) total growth rate of private wealth between t and t+1, which by construction can be decomposed into two terms, a savings effect and a capital gain effect:

$$\mathbf{1+g_{wt+1} = (1+q_{t+1})\ \ (1+g_{ws+1})} \qquad \textbf{(A.33)}$$

With:

[118] Note that in writing equation (A.31) above, we assumed implicitly that savings S_t are used to purchase assets at the beginning of year t (i.e. at Q_t prices). Taken literally, this assumption does not make much sense, given that production and savings are supposed to take place throughout year t. Again, the consistent way to deal with this would be to re-compute mid-year national wealth estimates, but this did not seem worth while given our long run focus (in order to analyze short term fluctuations, one would need to be more careful about this). One advantage of our modelling is that savings and capital gains enter multiplicatively (rather than additively) into the wealth growth equation (see equation (A.33) below), which facilitates growth decomposition. Had we assumed that savings S_t were used to purchase assets at the end of year t (i.e. at Q_{t+1} prices), then the accumulation equation would have been: $W_{t+1} = (Q_{t+1}/Q_t)W_t + S_t$. That is, equation (A.31) would become: $\beta_{t+1} = \beta_t\ [1+q_{t+1}+s_t/\beta_t]/[1+g_{t+1}]$. Equation (A.33) would then be: $1+g_{wt+1} = (1+q_{t+1}+g_{wst+1})$, rather than$1+g_{wt+1} = (1+q_{t+1})\ (1+g_{ws+1})$. I.e. the equation for total wealth growth would become additive rather than multiplicative. In practice, given that these annual growth rates are usually very small (g_{wst+1} is typically 1%-2% per year), opting for the multiplicative or additive formulation makes virtually no difference (i.e. capital gains on current-year savings are negligible as compared to both aggregate capital gains and aggregate savings).

g_{wt+1} = total real growth rate of private wealth between t and t+1

q_{t+1} = capital-gain-induced growth rate of private wealth

$g_{wst+1} = s_t/\beta_t = S_t/W_t$ = savings-induced growth rate of private wealth

A.5.3. Applying the equations to France 1896-2009

We can now apply these equations to French series. On Table A17 we report the findings using two alternative methods. In method n°1, we use total private savings (personal savings plus corporate retained earnings) in order to compute the accumulation equation. In method n°2, we use personal savings alone. For reasons explained above (see section A3), we prefer method n°1, which is conceptually more consistent. The corresponding savings rate used on Table A17 (col. (5) & (11)) are borrowed from Table A10, col.(7)-(8). National income Y_t, expressed in 2009 euros using the CPI deflator, reported on Table A17 (col.(1)) is taken from Table A1, col.(1).

Regarding the 1970-2009 period, we observe the true annual series for the wealth-income ratio β_t, thanks to the Insee-Banque de France private wealth series, so we do not need to estimate annual β_t series. We can instead use the above equations in order to compute the implicit real rate of capital gains q_t on an annual basis:[119]

$$\mathbf{1+q_t = \beta_t[1+g_t] / \beta_{t-1}[1+s_{t-1}/\beta_{t-1}] = (1+g_{wt})/(1+g_{wst}) \qquad (A.34)}$$

For years 1971-2009, the q_t series reported on col. (7) and (13) of Table A17 were obtained by applying mechanically equation (A.34).[120] Unsurprisingly, the real rate of capital gains displays very large year-to-year variations, and capital gains effects often largely dominate savings effects for any given year. For instance the real rate of capital gains was strongly positive during the asset price boom of the mid-2000s (e.g. q_t=+10.0% in 2005), and strongly negative the asset price collapse of the late 2000s (e.g. q_t= -5.3% in 2009).[121] However the important point is that these large year-to-year asset price variations tend to compensate each other if one looks at longer time periods. If we

[119] Note that the detailed balance sheets released by Insee-Banque de France (and available on-line) actually include for each asset category (using ESA 1995 classification codes) the yearly decomposition of asset variation into a saving flow effect and a valuation effect (i.e. an asset specific capital gain). By construction our aggregate real rate of capital gains q_t is simply the average Insee-Banque de France valuation effect (weighted over all asset categories), minus CPI inflation.

[120] All formulas used to estimate the wealth accumulation equation are available in the excel file.

[121] On Table A17 we assumed that the same real rate of capital loss prevailed for t=2010 (i.e. between January 1st 2009 and January 1st 2010) as for t=2009 (i.e. between January 1st 2008 and January 1st 2009).

compute the average over the 1970-2009, we find that the average rate of capital gains q_t=+0.6%, while the average rate of savings-induced real wealth growth was g_{wst}=+3.2%, thereby generating a total real wealth growth g_{wt}=+3.8% (see Table A17, line 1970-2009). That is, over the entire 1970-2009 period, savings explain 85% of total wealth growth, while capital gains explain only 15%. When we do the same computations using personal savings (method n°2), then the average rate of capital gains rises to q_t=+0.9%, while the average rate of savings-induced real wealth growth was g_{wst}=+2.9%. That is, the savings share in total wealth accumulation declines to 76%, while the capital gains share rises to 24% (see Table A17, line 1970-2009). This can be interpreted as saying that about one third of total capital gains over the 1970-2009 can be accounted for by corporate retained earnings (through additional investment and increased shareholder value).[122]

For the pre-1970 period, we observe the wealth-income ratio β_t only for a few isolated years (1896, 1913, 1925, 1954, 1970), and we want to use the equations above to construct annual series passing through these observations. We proceeded as follows. We first consider the 1954-1970 sub-period. We start from our estimated wealth-income ratio β_t=203% for 1954, and we compute the constant rate of capital gains q_t over the 1955-1970 period which generates the observed wealth-income ratio β_t=289% for 1970, given observed growth rates and savings rates over the 1955-1970 period and dynamic equations (A.32) and (A.33). We find that in order to reproduce the observed 1954-1970 pattern of wealth-income ratios we need to assume constant capital gains q_t=+2.4%. Given that the savings-induced wealth growth rate over this period was g_{wst}=+5.5%, this means that savings explain 69% of aggregate wealth accumulation between 1954 and 1970, while capital gains explain 31%. In case we use personal savings (method n°2), we find a savings share of 55%, and a capital gains share of 45% (see Table A17, line 1954-1970). Retained earnings again seem to account for about a third of capital gains.

[122] The existence of corporate retained earnings is the simplest explanation as to why asset prices might grow structurally faster than consumer prices, thereby generating permanent real capital gains. E.g. in the Gordon-Shapiro equity pricing formula, $Q_t=D_t/(r-g)$, with Q_t = equity price index and D_t = dividend flow), dividend payments are supposed to grow at the same rate as national income ($D_t=D_0e^{gt}$ & $Y_t=Y_0e^{gt}$), and the equity price index is also supposed to grow at the same as national income ($Q_t=Q_0e^{gt}$), thereby generating a permanently positive real rate of capital gains q equal to the real growth rate g. But this can be a steady-state growth path only if the corporate capital stock K_t also grows at rate g, i.e. if the representative corporation permanently saves and invests a fraction g/r of its profits $\pi_t=rK_t=\alpha Y_t$ as retained earnings (i.e. $E_t=(g/r)\pi_t=gK_t$), and distributes a fraction 1-g/r as dividends (i.e. $D_t=(1-g/r)\pi_t=(r-g)K_t$). In effect the total equity return r is the sum of a dividend yield equal to r-g and of a real capital gain term equals to g generated by retained earnings. Our series show that retained earnings do indeed explain a significant fraction of capital gains, but are too small to generate permanent real capital gains q as large as g. See Gordon (1959) for the original derivation of the formula (Gordon explicitly assumes that companies keep a fraction g/r of their profits as retained earnings). See Baker, DeLong and Krugman (2005) for a discussion of how the formula can be used to think about the long term macro relationship between r and g.

We then consider the 1925-1954 sub-period and proceed in the same manner. We start from our estimated wealth-income ratio β_t=293% for 1925, and we compute the constant rate of capital gains q_t over the 1925-1954 period which generates the observed wealth-income ratio β_t=293% for 1970, given observed growth rates and savings rates over the 1929-1954 period, wealth destructions rates observed during World War 2, and dynamic equations (A.32) and (A.33).[123] We find that in order to reproduce the observed 1925-1954 pattern of wealth-income ratios we need to assume constant negative capital gains q_t = -1.2% during those years. We do the same for the 1913-1925 period, and we find that we need to assume constant negative capital gains q_t = -5.6% during those years in order to reproduce the decline from β_t=660% in 1913 to β_t=293% in 1925. Finally, we do the same for the 1896-1913 period, and we find that we only need to assume negligible capital gains (q_t=0.0%) during those years in order to reproduce the observed pattern of β_t between 1896 and 1913: observed savings rates are sufficient to predict almost perfectly aggregate wealth accumulation during this period.

This estimation method delivers annual series for the wealth-income ratio β_t and aggregate private wealth W_t over the 1896-1970 period. We keep the series obtained under method n°1 (private savings), i.e. col. (2) of Table A1 is equal to col. (4) of Table A17 times col. (1) of Table A1. Note that the series obtained under method n°2 (personal savings) are almost identical.[124]

A.5.4. Applying the equations to France 1820-1913

In order to further test the consistency of our method and of our simple wealth accumulation model, we also applied the accumulation equations to the 1820-1913 period. On Table A18 we report the results obtained with decennial averages.[125] Real rates of capital gains q_t reported on col. (8) of Table A18 were obtained by applying equation

[123] With wealth destruction rates d_t, equation (A.33) simply becomes: $1+g_{wt+1} = (1+q_{t+1})(1+g_{ws+1})(1+d_t)$. See excel file for simulation formulas.

[124] See Table A17, col. (4) vs col. (10). By construction both methods deliver similar results for years 1896, 1913, 1925, 1954 and 1970-2009, and differ only in the relatively short run. To the extent that short-run variations in corporate retained earnings are informative about short run variations in corporate market values, method n°1 delivers more precise series than method n°2. Also, the savings vs capital gains decomposition of wealth accumulation obtained under method n°1 is arguably more consistent from a conceptual viewpoint. But as far as estimating decennial-averages wealth-income and inheritance-income ratios is concerned, the choice between the two series is virtually irrelevant.

[125] In the simulation appendix (see Appendix D, Tables D1-D2), we use the results obtained on Table A18 in order to annualize our 1820-1913 series on national income and private wealth.

(A.34) above. That is, $1+q_t$ is again defined as a residual fraction of total private wealth growth rate that cannot be accounted for by savings, i.e. as the ratio between total private wealth growth rate $1+g_{wt}$ and savings-induced wealth growth rate $1+g_{wst}$. We find that the observed 1820-1913 pattern of wealth-income ratios β_t is very well accounted for by observed savings flow, and that capital gains seem to play a negligible role during the entire 1820-1913 period. The real rate of capital gains q_t appears to be sometime positive and sometime negative, but in any case relatively small, i.e. between -0.4% and +0.4% per year during each decennial period, with the single exception of the 1870s (-1.3%).[126] Between 1820 and 1913, the average real growth rate of national income was g_t=1.0%, while average the real growth rate of private wealth was g_{wt}=1.3%, which can be decomposed into a savings-induced private wealth growth rate g_{wst}=1.4% and a real capital gains effect q_t=-0.1% (see Table A18). Given that we face data limitations regarding the measurement of wealth-income ratios and savings rates, it is fairly obvious that real rates of capital gains as small as -0.1% cannot really be distinguished from zero.[127] It could well be that our 19th century savings rates are slightly over-estimated, or that the rise in the wealth-income ratio is slightly under-estimated. In any case, the important finding is that both during the 19th century and during the 20th century, the bulk of private wealth accumulation seems to be well accounted for by savings flows.[128]

A.5.5. Two centuries of wealth accumulation

We report on Table A19 summary statistics on the sources of wealth accumulation (saving vs capital gains) in France over the entire 1820-2009 period. The main lesson is that there does not seem to be large movements in the relative price of assets in the very long run. However the decompositions by subperiod reported on Table A19 also show that over a few decades capital gains and losses matter a lot. If we examine the 1913-1949 period as a whole, the main conclusion is that most of the decline in the wealth-income ratio is due to the decline in the relative price of assets, rather than by war destructions. If we add up

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[126] The relatively large capital losses of the 1870s seem to be due the capital shocks incurred by French private wealth holders following the 1870-1 war with Germany. We did not attempt to investigate how much can be accounted for by the large capital payment subsequently made to Germany, vs the annexation of Alsace-Moselle (about 5% of the French territory, population-wise), vs physical capital destructions due to the war (which appear to be limited). In any case, it is apparent that such 19th century-style conflicts had a limited impact on aggregate wealth accumulation, as compared to the devastating effects of 20th century wars and ensuing government interventions.

[127] Taken literally, this would mean that while consumer prices have increased at 0.5% per year during the 1820-1913, asset prices have increased at 0.5%-0.1%=0.4% per year.

[128] We also report on Table A18 (col. (13)-(14)) the raw wage series used to estimate our 19th century capital and labor shares series (see Table A12 above).

war destructions estimates for World Wars 1 and 2, total destructions seem to represent the equivalent of about 30% of the 1913 capital stock. Given that the wealth-income ratio declined by about 60% between 1913 and 1949, one might be tempted to conclude that destructions explain about half of the fall. However this is misleading, because savings were relatively high during this period, particularly in the 1920s and late 1940s, presumably as a response to the destructions (as least in part). On the basis of destructions and savings flow, private wealth should have risen at about 0.9% per year between 1913 and 1949, i.e. only slightly less than national income (1.3%). However in fact it declined by 1.7% per year, due to a negative asset price effect (-2.6%). So relatively to national income, private wealth declined at a rate of 3.0% per year, out of which only 0.4% can be attributed to volume effects (destructions and savings), i.e. about 10%, and 2.6% can be attributed the decline in the relative price of assets, i.e. about 90%.

Regarding the recent period, the interesting lesson from Table A19 is the following. The recovery of asset prices has played an important role in the rebound of the wealth-income ratio, but the bulk of private wealth accumulation and private wealth recovery came from saving. Between 1949 and 1979, national income grew at 5.2% per year, while private wealth grew at 6.2% per year. Out of these 6.2% per year, 5.4% can be accounted for by savings, and 0.8% are left for capital gains. Between 1979 and 2009, national income grew at 1.7% per year, while private wealth grew at 3.8% per year. Out of these 3.8% per year, 2.8% can be accounted for by savings, and 1.0% are left for capital gains.

Of course, if one looks at the detailed decennial and annual data, one can see much bigger contributions of capital gains (or capital losses). E.g. between 1999 and 2009, national income grew at 1.4% per year, while private wealth grew at 6.7% per year, out of which 2.3% can be accounted for by savings and 4.3% by capital gains. The large rise of asset prices during the 2000s is largely responsible for the booming wealth-income ratio, which was gradually rising from about 200% in the 1950s to about 350% in the 1990s, before suddenly reaching 500%-550% in the 2000s. According to the latest data (January 1st 2009), the wealth-income ratio declined from 563% in 2008 to 552% in 2009. How far this is going to continue and whether asset prices are going to keep falling is certainly a complicated issue. The important point that we would like to stress, however, is that when we take a medium run perspective, one should not exaggerate the importance of the asset price boom of the 2000s. During each single decade of the 1949-2009 period (including the 2000s), the growth rate of national income was substantially below the savings-induced growth rate of private wealth. The only exception was the 1960s: the savings rate was high (13.8%), but the growth rate of national income was so high (6.2%) that this was not sufficient to make private wealth grow faster. More generally, the savings rate was somewhat bigger during the 1949-1979 period (13.4%) than during the 1979-2009 (9.5%), but growth was so much smaller during the second period that the gap between income growth and savings-induced wealth growth was substantially bigger in the 1979-2009 period than in the 1949-1979 period. This is what explains – in an accounting sense – why the wealth-income ratio grew faster during the 1979-2009 period than during the 1949-1979 period. The capital gains effect appears to have been similar during the two subperiods (1.0% vs 0.8%). If we take the 1949-2009 period as a whole, the recovery of asset prices relatively to consumer prices appears to have been relatively steady – or at least less chaotic than one might think at first stance. The 1949-2009 increase in asset prices (at 0.8%-1% per year) seems to have almost fully compensated the 1913-1949 fall in asset prices (at -2.4% per year), so that the overall capital gain effect between 1913 and 2009 appears to be fairly modest (-0.3%).

A.5.6. Discussion of the method and comparison with asset price indexes

How reliable is our estimation method? We feel that it is reasonably reliable, given our purposes in this research. First, as long as our raw wealth-income ratio estimates for 120-1896, 1913, 1925, 1954 and 1970-2009 are reliable, the choice of what is essentially an interpolation method for missing years is not going to make an enormous difference – at least as far as decennial averages are concerned.

Next, and most importantly, we find it reinsuring – and interesting in its own right – that the wealth accumulation equation works so well in the medium and long run. In particular, the average real rates of capital gains that we need to assume in order to reproduce the pattern of wealth-income ratios over each sub-period (1820-1896, 1896-1913, 1913-1925, 1925-1954, 1954-1970) are consistent with available asset price series. Take the 1954-1970 sub-period. Consumer prices grew on average by 4.9% a year during this period. But available real estate and stock market indexes show that nominal housing prices (17.4% a year) and equity prices (7.0%) grew substantially faster (see Table A22, line 1954-1970). So it is not surprising that we need to assume positive real rates of capital gains to account for observed wealth accumulation over this period. Conversely, during the 1913-1925 and 1925-1954 sub-periods, consumer price inflation was very large (resp. 12.4% and 13.4%

per year during each sub-period), and available asset price indexes show that nominal housing price inflation (resp. 5.4% and 9.0%) and equity price inflation (resp. 6.0% and 10.1%) stood at substantially lower levels (see Table A22, lines 1913-1925 and 1925-1954). So it is not surprising that we need to assume negative real rates of capital gains to account for observed wealth accumulation during the 1913-1954 period, and particularly so in the 1913-1925 sub-period.[129] Prior to 1913, and in fact during the entire 1820-1913 period, both consumer and asset price inflation was generally low (usually less than 1%-2% a year from a decennial average perspective), so it is not surprising that we only need to assume negligible rates of capital gains to reproduce the observed pattern of wealth-income ratios between 1896 and 1913, and more generally during the entire 1820-1913 period (see below).

Of course it is highly unsatisfactory and arbitrary to assume fixed real rates of capital gains q_t during each sub period 1896-1913, 1913-1925, 1925-1954 and 1954-1970 (this is probably less important for the 1820-1896 period). Annual wealth accounts available for the 1970-2009 period show that real rates of capital gains can vary enormously on a year-to-year basis, and available asset price indexes show that the same conclusion certainly applies as well to the 1914-1969 chaotic period. The reason why we finally decided to use our simple method to construct our annual private wealth series, and not to use annual asset price indexes, is because is the latter appear to be of insufficient quality.

There are two conceptual and practical problems with existing historical asset price indexes (see Tables A20-A22). First, they typically cover a limited set of broad asset categories, and it is unclear how one should weight them in order to reproduce the average asset portfolio owned by private individuals at a given point in time. In France, economic historians and statisticians have constructed an index for Paris housing prices starting in 1840, an housing price index for the all of France starting in 1936, and an aggregate index for equity prices starting in 1886.[130] Using these raw indexes, we attempted to construct a composite asset price index. By assuming a simple, constant

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[129] Note that the much higher capital losses estimated over 1913-1925 than over 1925-1954 (q_t=-5.6% vs q_t=-1.2%) reflects the fact that the prices of private assets (real estate and equity) lagged behind consumer prices during the first sub-period (possibly because consumer price inflation in 1913-1925 came after a century-long period of almost complete price stability), and the fact that public debt reached very high levels in France in the 1920s (see Table A16 above): private individuals lent a lot of money to the French government during World War 1 and the early 1920s, and suffered enormous implicit capital losses on this investment (the real rate of capital gains q_t on nominal assets such as public debt is by definition equal $-p_t$). We did not attempt to disentangle the shares of each effect.

[130] We also have various indexes for total returns on stocks and bonds (see Tables A20-A22). See section A6 below for the various sources where these raw indexes can be found.

portfolio allocation,[131] one can easily generate a composite index which broadly resembles our real rates of capital gains q_t over each sub period.[132] However in order to match perfectly our real rates of capital gains q_t, one would need to make somewhat arbitrary assumptions about changing portfolio shares.

Most importantly, we noticed that using such a composite price index in order to construct annual series for wealth-income ratios β_t and aggregate private wealth W_t would generate series with implausibly large year-to-year variations (both downwards and upwards). This seems to be due to the fact that existing asset price indexes give excessive weights on a few specific assets (real estate indexes typically rely on a limited set of housing sales in Paris and a few other cities; equity indexes exclusively rely on quoted shares), while private individuals taken as a whole own a very diversified portfolio of assets, whose short run price variations tend to offset one another, at least partly. Using such a method would also lead us to overestimate some of the medium-run, asset-prices-induced changed in aggregate private wealth. In particular, the 1913-1954 fall in real estate and equity prices (relative to CPI) is so large in raw asset price indexes that such series are bound to lead to implausibly small wealth-income ratios β_t in the 1950s, even if we put very small portfolio weight on real estate and equity.[133] Given that we are mostly interested in decennial averages in the context of this research on long run trends, we decided to leave this complicated issue for further work and to keep our simplifying assumptions about constant real rates of capital gains during each sub-period.

Together with the fact that they tend to overestimate short-run and medium-run variations, the other important problem with existing asset price indexes is that they seem to overestimate the long term rise of asset prices relatively to consumer prices. According to our q_t series, which were computed as the residual term to the wealth accumulation

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[131] Namely 30% real estate, 30% equity, 20% CPI-type assets (i.e. assets with prices rising like consumer prices), and 20% nominal assets (i.e. assets with fixed nominal prices like public debt or checking accounts). See Table A21.

[132] See Table A22, col. (11) vs col. (14), lines 1896-1913, 1913-1925, 1925-1954 and 1954-1970.

[133] Expressed as a fraction of CPI, raw real estate and equity indexes on the early 1950s are worth about 10% their 1913 value. See Table A20, col. (1)-(4). If we were to use such indexes we would find wealth-income ratios substantially below 200% in the early 1950s. There are several reasons why these indexes might overstate the 1913-1949 fall in asset prices: Paris housing prices probably fell more than average French housing prices, and there exists no national index before 1936; even after this date it is unclear whether the index relies on a truly representative sample of housing units sales, or whether large cities are oversampled; also, the tough rent control policies applied in the aftermath of world wars led to a dual price system for occupied and non-occupied housing (typically rents can be raised only after a change in tenants, which also explain why rent recovery can span over several decades after the end of rent control), and it is unclear how this was dealt with by sales-based indexes (biases can go both ways); finally, it is likely that the 1913-1949 fall in public equity prices largely overstates the aggregate fall in (public and private) firm value.

equation (i.e. without using asset price indexes), asset prices have risen approximately at the same pace as consumer prices over the course of the 20th century. The real rate of capital gains q_t was equal to 0.0% in 1896-1913, -2.8% in 1913-1949, +0.8% in 1949-1979 and +1.0% in 1979-2009, so that the 1913-1949 fall and the 1949-2009 rise almost exactly compensate one another: the cumulated annualized rate of capital gains over the entire 1896-2009 period appears to be as small as -0.3% (see Table A19, col. (5)). To put it differently: savings appear to be the primary determinant of aggregate private wealth accumulation in the long run; if anything, capital gains have played a (small) negative role over the 1896-2009 taken as a whole.

Note that if we do the same computations using method n°2 (with the personal savings definition, i.e. excluding corporate retained earnings), then the small negative real rate of capital gains q_t = -0.3% becomes a small positive real rate of capital gains q_t = +0.4% (see Table A19, col. (9)). Taken literally, these estimates mean that assets prices are about 40% larger in 2009 than what they were at the eve of World War 1 (relatively to consumer prices), but that if we take into account the value of accumulated retained earnings within corporations, then they are actually 30% smaller. Of course, given the data limitations, and particularly given the uncertainty about savings rates and depreciation rates, it does not make much sense to pretend that one can really distinguish between a -0.3% and +0.4% annualized average real rate of capital gains over a century-long period. What these findings indicate is simply that the average real rate of capital gains over the 20th century was apparently relatively close to 0%. We certainly do not infer from this finding that real capital gains will be 0% during the 21st century. The experience of the 20th century certainly show that major shocks can create large gaps between asset and consumer prices that last over several decades. Also, one can easily construct theoretical wealth accumulation models with two goods and long run divergence between the price of the asset good (say, real estate) and the price of consumer good. We have nothing to say as to whether such models might be relevant for the future. At a more modest level, the conclusion we draw from our computations is that the 1913-1949 drop in asset prices seems to have been more or less compensated by the 1949-2009 rise in asset prices, and that over the whole 1896-2009 aggregate wealth accumulation seems to be well accounted for by measured savings flows.

In any case, we feel that this conclusion (and the national-accounts-based computations leading to this conclusion) is more meaningful than the conclusions and computations one can draw from existing historical asset price indexes. E.g. according to available real estate price indexes, housing prices in Paris grew 1.2% faster than consumer prices on average over the 1896-2009 period: real housing price inflation (relatively to CPI) was +0.6% in 1896-1913, -6.9% in 1913-1949 and +7.1% in 1949-2009 (+11.1% in 1949-1979, +3.1% in 1979-2009), but on the whole the balance stood positive at +1.% (see Table A22, col. (8)). This can look like a small number, but this is much larger than 0.3%-0.4%: if we cumulate 1.2% over 113 years, we obtain about 400%, i.e. the Paris real estate price index is currently four times larger than the consumer price index (relatively to 1896-1913 levels).[134] If this kind of long run asset price movement was representative of average asset prices, this would imply that wealth-income ratios $\beta_t = W_t/Y_t$ should have risen enormously over the 20th century, even in the complete absence of savings. This does not make much sense.

One key reason why such computations are not really meaningful is because historical real estate indexes generally include no adjustment whatsoever for quality improvement: in effect we are comparing the price of a 1900 Paris apartment with no toilet and limted water and heating supply with the price of 2000 Paris apartment with multiple bathrooms and cable tv. Consumer price indexes do include substantial corrections for quality improvements (otherwise consumer price inflation would look much larger, and real growth in living standards would look much smaller). Assuming that quality improvements are not properly included in price indexes for capital goods such as housing (and they are arguably even more difficult to include for capital goods than for consumer goods), then it is not too surprising to find that the price of assets mechanically rises in the long run relatively to consumer prices. Long run biases regarding equity prices involve other effects generally going in the same direction.[135] Generally speaking, there are good reasons to believe that existing historical asset price indexes (both real estate indexes and stock indexes) do not properly take into account quality and composition effects in the long run,

[134] See Table A20, col. (1)-(3). The national real estate index currently looks even bigger (about six-seven times the CPI), but this is simply because it starts in 1936 (at a time when real estate prices were already very low as compared to 1913).

[135] One standard reason why existing stock price indexes might rise structurally faster than consumer prices in the long run is of course the existence of corporate retained earnings. Other explanations include a structural rise in the share of the national economy quoted in the stock market (either because of the share of publicly quoted firms in national output rises, or because publicly quoted firms start to rely more on equity finance than on debt finance; such a structural evolution seems to have occurred since the 1970s-1980s) and the rise of cross holdings within the corporate sector (which can create an artificial rise in stock market capitalization, and possibly a rise in stock price indexes, depending on how weights are computed in commonly available stock indexes). Also, because in practice the set of publicly traded firms changes many times over the course of a century, long run stock indexes necessarily rely upon fairly specific assumptions about portfolio reallocation and reweighting; such assumptions are relatively innocuous in the short run, but can have huge effects in the long run.

and are therefore ill suited for volume vs price decomposition analysis. We feel that it is conceptually and practically more consistent to compute implicit real rates of capital gains q_t from the wealth accumulation equation, i.e. from the observed patterns of wealth-income ratios and savings flows, as measured by national accounts.[136]

<u>A.6. Supplementary series on price indexes</u> (Tables A20-A22)

On Tables A20-A22 we report supplementary series on long run price indexes in France. We use these series at various points in this appendix, particularly in the previous section. Here we briefly describe how Tables A20-A22 were constructed.

The consumer price index (CPI) reported on col. (1) of Table A20 is the official Insee-SGF consumer price index.[137] The real estate price indexes for Paris and the whole of France reported on col. (2)-(3) of Table A20 are borrowed to Friggit (2007), whose important work represents the most systematic historical data collection effort on real estate markets in France so far.[138] The Friggit data base also includes historical indexes for total stock returns (dividend reinvested) and total bond returns (interest reinvested), which we report on col. (5)-(6) of Table A20.[139] We also report on col. (4) a simple equity price index (no dividend reinvested) based upon series from Friggit (2007) and Villa (1994).[140] All other series reported on Tables A20-A22 were computed from these raw series (and/or from previous tables).

[136] Note that our savings-based method implicitly takes quality improvements into account. E.g. if the raw (non-quality-corrected) price of Paris apartments doubles between 1900 and 2000, and if observed savings flows appear to be sufficient to account for the observed doubling of the wealth-income ratio, then it is reasonable to infer that the doubling of the real estate price is entirely due to savings-financed quality improvements in Paris apartments (i.e. savings flows were implicitly used to finance investment in Paris housing so as to improve their quality).

[137] We borrowed 1891-1998 annual CPI inflation rates from Piketty (2001, pp.690-691, Table F1, col. (5)). We updated the series by using the latest 1999-2008 CPI inflation rates released on www.insee.fr (15/09/2009). For 2009 we used the latest projections available (0.4%). For 1800-1890 we used the consumer price inflation series included in the Friggit data base (see below).

[138] The Friggit historical data base (ltseries.v4.0.xls, available on-line, downloaded on 15/09/2009) ends in 2005, so for 2006-2009 we uptaded the Notaries-Insee (BMS) official real estate indexes. This is the same data source as the one used by Friggit for the recent period.

[139] For 2006-2009 we updated the historical Friggit series by using the SBF 250 total stock return index released by Euronext.com (15/09/2009); this is the same data source as the one used by Friggit for the recent period. Euronext does not seem to release total bond return indexes, so we updated the Friggit series by assuming 5.0% returns for 2006-2009 (this is consistent with previous years).

[140] Unfortunately the Friggit data base does not include the decomposition of the total stock return index into a pure price index and a dividend index. Euronext series do provide such a breakdown (on average over the 1991-2009 period the total stock return of 6.8% can be broken down into a 3.8% price effect and a 3.0% dividend effect), but do not go beyond 1991. So we used the 1890-1985 equity price index published by Villa (see Villa (1994, p.146, series "Q: Indice du cours des valeurs françaises à revenus variables"), which we complete for 1856-1890 using the equity price index published in AR 1966, Insee, p.541. For 1986-1990 we used Friggit's total return index, minus 3.0% (i.e. the average dividend yield over 1991-2009).

Appendix B: Estate Tax Data

The other key data source used in this research is estate tax data. The main data sources and methodological issues regarding French estate tax data and the way we use it, in particular in order to compute the fiscal inheritance flow B_t^f and the μ_t ratio, are discussed in the working paper (see sections 3.1 and 3.3). In this appendix we provide the complete series used in this research, as well as additional details about sources, methodology and concepts.

In section B1 we describe how we computed our fiscal inheritance flow series B_t^f. In section B2 we describe how we used estate-tax-based age-wealth profiles $w_t(a)$ in order to compute our μ_t ratio series.

B.1. Computation of the fiscal inheritance flow B_t^f series

Our fiscal inheritance flow series are reported on Tables B1 (annual series) and B2 (decennial averages). We start from the raw fiscal series B_t^{f0} (col.(1)), to our final corrected series B_t^f (col. (10)). Here we describe the data sources and methods used to make the relevant corrections and construct these tables.

Tables B1-B2, col. (1) : B_t^{f0} = raw fiscal bequest flow

We start from the raw bequest flow reported on estate tax returns (with no adjustment whatsoever), which we note B_t^{f0} (col. (1)). In the same way as in the national accounts appendix, all money values reported on Tables B1 and B2 are expressed in current billions currency, by which we mean current billions euros for the 1949-2009 period and current billions old francs for the 1820-1948 period.[141]

The raw bequest flow series B_t^{f0} reported on col.(1) come directly from the estate tax data published by the French Finance Ministry during the 1826-1964 period, and from the so-called "DMTG" micro-files of estate tax returns compiled by the French Ministry of Finance

[141] The old franc was replaced by the new franc on January 1st 1960 (1 new franc = 100 old francs), and the new franc was replaced by the euro on January 1st 2002 (1 euro = 6.55957 new francs). In order to convert 1949-2001 current currency values into what we call current euros, we simply divided 1960-2001 new francs values by 6.55957, and 1949-1959 old francs values by 655.957. Current prices national accounts series released by Insee adopt the same monetary convention.

during the 1977-2006 period.[142] Throughout the period, we use a net wealth concept: the raw bequest flow B_t^{f0} is defined as the aggregate market value of all tangible and financial assets (minus financial liabilities) transmitted at death during a given year, as reported by heirs to tax authorities.[143] I.e. in 1826 a total net wealth value of 1.270 billions francs was left by decedents; in 1913 this total value was equal to 5.612 billions francs; in 2006 it was equal to 58.850 billions euros.

Between 1826 and 1964, detailed estate tax data was published quasi-annually by the French Finance Ministry. In particular, the basic annual series on aggregate bequest flows cover the entire 1826-1964 period on a continuous, annual basis (with the single exception of years 1914-1920, 1923-1924, 1961 and 1963). Complete retrospective 1826-1964 series on aggregate flows were published in the historical statistics yearbook compiled in 1966 by Insee, which constitutes our basic source for col. (1) of table B1.[144] One can also find the same aggregate series in the annual tabulations reporting the number and value of estates broken down by estate bracket. The French Finance Ministry started compiling such tabulations in 1902, when the estate tax became progressive, and published them until 1964.[145]

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[142] "DMTG" stands for "Droits de mutation à titre gratuit" (the official name of the estate tax in France).

[143] The aggregate (net wealth)/(gross assets) ratio has varied very little in the long run, from about 93%-94% prior to 1914 to about 94%-95% during the interwar period, 96%-97% during the 1950s-1960s and again 94%-95% in the 1980s-2000s. I.e. aggregate liabilities have always been around 5% of aggregate gross assets transmitted at death in France. In the DMTG files, there are very few cases where liabilities exceed assets. In these cases we set net wealth equal to zero (heirs generally choose not to take up such negative bequests). In published Finance Ministry data, bequests with negative net wealth were excluded. Because published net wealth series start in 1903 (prior to the 1901 estate tax reform liabilities were not deductible from gross assets, so we only observe gross assets series in 1826-1902), we assumed a constant (net wealth)/(gross assets) ratio equal to 95% over the 1826-1913 period, and reduced accordingly the published gross assets series for this period.

[144] See "Annuaire statistique de la France 1966, Résumé Rétrospectif", Insee 1966 (thereafter AR 1966), p.530. The numbers reported on col.(1) of Table B1 for 1826-1964 are taken directly from AR 1966 p.530. More precisely: for 1826-1913 we took 95% of col. "Successions – Valeur totale de l'actif brut" (gross assets) (see excel file for formulas and original raw gross assets series); for 1924-1964, we took 100% of col. "Successions – Valeur totale de l'actif net" (net wealth). For 1921-1922 the bequest flow was not published, so we take the gift flow series, divided by the estimated (gift flow)/(bequest flow) ratio (see below). For 1949-1959 we divided the raw numbers published in AR 1966 by 655.957, and for 1960-1964 we divided the raw numbers published in AR 1966 by 6.55957 (e.g. in 1964: 8.427 billions francs divided by 6.55957 equals 1.285 billions euros).

[145] These quasi-annual 1902-1964 tabulations were used in Piketty (2001, 2003) in order to estimate top estate fractiles using Pareto interpolation techniques. The exact references of the French Finance Ministry statistical bulletins were these tabulations were originally published ("Bulletin de Statistique et de Législation Comparée" (thereafter BSLC) for years 1902-1938, "Bulletin Statistique du Ministère des Finances" (thereafter BSMF) for years 1939-1946, and "Statistiques et Etudes Financières" (thereafter S&EF) for years 1947-1964) are given in Piketty (2001, Appendix J, p.749). The wealth concept used in these tabulations was net wealth until 1956, and gross assets afterwards (but we know aggregate net wealth from the aggregate AR 1966 series). Note also that the aggregate bequest flow reported in 1902-1956 tabulations is sometime slightly smaller than 100% of the aggregate bequest flow series published in AR 1966 (e.g. the ratio is about 96%-99% in 1902-1905), which indicates that a small fraction of estate tax returns was not included in size tabulations. For the entire 1826-1964 period we used the AR 1966 series, as explained

In 1964, the French administration stopped compiling and publishing annual estate tax statistics altogether. The only data available on an annual basis since 1964 are the total number of estate tax returns and the value of aggregate estate tax receipts – from which it is impossible to infer the value of the aggregate bequest flow in a reliable way, given tax progressivity.[146] Fortunately, the French Finance Ministry has been collecting every 6-7 years since 1977 nationally representative samples of estate tax returns, primarily for internal tax simulation purposes. These DMTG micro files exist for years 1977, 1984, 1987, 1994, 2000 and 2006.[147] Each file contains between 3,000 and 5,000 individual estate tax returns (as compared to a total of about 300,000 estate tax returns filed each year, i.e. the average sampling rate is typically slightly above 1/100), but is heavily stratified, with a sampling rate as high as 1/4 within the top percentile of decedents. Each file includes all variables reported in the estate tax return, and in particular detailed information on the value of the estates (broken down using a large number of asset categories: residential vs non-residential real estate, public vs private equity, bonds, cash, etc.), the share of total estate going to each successor, as well as basic socio-demographic information on the decedent and on each heir.

These DMTG files provide very rich information on intergenerational wealth transmission in France, and the present research relies heavily on this data source. Although these files have been compiled by the tax administration primarily for internal purposes, they have regularly been used by researchers outside the tax administration since 1984, both at Insee and outside Insee.[148] The 1977 DMTG file has apparently not been archived in an accessible computer format, so we used the aggregate bequest flow estimated and published by Insee researchers who had access to this file during the 1980s.[149] The 2006

above. The French Finance Ministry also compiled tabulations broken down by age of decedents, which we use in section B.2 below.

[146] These rudimentary estate tax statistics are currently published in the « Annuaire Statistique de la DGI » (the yearly statistical publication of the French tax administration, available on-line).

[147] In addition to the first DMTG file compiled in 1977, the tax administration and Insee also attempted to link up the income tax returns files compiled in 1975 and 1979 with the bequests and gifts that occurred since 1962 (i.e. since the time annual estate tax tabulations were abandoned by the tax administration). See Canceill (1979) and Lollivier (1986). However these files only cover real estate transmission and include too few annual observations to be of interest for our purposes.

[148] See e.g. Arrondel and Laferrère (1992, 1994, 2001) and Arrondel and Masson (2006) for examples of research work using the DMTG files for 1984, 1987, 1994 and 2000.

[149] See Laferrère (1990, p.5) and Laferrere and Monteil (1992, p.11): (81/95) x (57.8-9.0)/6.55957 = 6.3 billions €. The 81/95 adjustment factor comes from the fact that Fouquet and Meron chose to upgrade the raw fiscal values reported in tax returns by a 95/81 corresponding to their estimated average inflation rate between time of death and year 1977 (see Fouquet and Meron (1982, pp.86-87)), while we choose to use raw fiscal values (see below). The -9.0 term is an estimate of tax exempt assets made by these authors, which we later include (here we look only at the taxable bequest flow).

DMTG file has not been made available to researchers outside the Finance Ministry yet, so we used the aggregate bequest flow recently estimated and published in an official tax administration report.[150] The raw fiscal flows reported on col.(1) of Table B1 for years 1984-1987-1994-2000 come from our own computations using the corresponding DMTG micro files, and are consistent with available published estimates.[151]

Finally, note that we did not make any adjustment in order to correct for the time gap between time of death and time of tax filing. That is, throughout the period of study, estate tax data always refers to the calendar year when the estate tax return was filed, rather than the calendar year of death. Both calendar years do not perfectly correspond, because successors are given by law a six-month delay following the date of death in order to fill an estate tax return. E.g. the aggregate bequest flow of 58.9 billions euros reported on Table B1 for year 2006 represents the total value of bequests reported in estate tax returns filed in 2006, and includes a number of estates of individuals who died in early 2006 and a number of individuals who died in late 2005 (and in some rare cases in early 2005 or even in 2004, when successors are running late). Estate values are always estimated at the time of death (rather than at the time the return is filled and registered), this can potentially create a non-trivial downward bias in our estimated fiscal flows during periods of rapid asset price inflation. Our fiscal inheritance flows are primarily meant to be compared with economic inheritance flows based upon national wealth estimates (which are estimated on January 1st of each year, see Appendix A), so we decided that the simplest strategy was to make no adjustment whatsoever: if successors take about six months to fill their return, then on average asset values correspond approximately to January 1st prices. Detailed data from the DMTG files for the recent period (unlike published statistics, micro-files do include full details about date of death and date of registration) suggests however that the

[150] See “La repartition des prélèvements obligatoires entre generations et la question de l’équité intergénérationnelle”, Rapport du Conseil des Prélèvements Obligatoires, 2008 (thereafter Rapport CPO 2008), p.227. See also « Le patrimoine des ménages », Rapport du Conseil des Prélèvements Obligatoires, 2009 (thereafter Rapport CPO 2009), p.151.

[151] Basic summary statistics extracted from DMTG files have regularly been published in official Finance Ministry reports. See e.g. “L’imposition du capital”, Rapport du Conseil des Impôts, 1986, pp.69-83 ; « L’imposition du patrimoine », Rapport du Conseil des Impôts, 1998, pp.210-211 ; « Les mutations à titre gratuit », Notes Bleues de Bercy n°148, 2002 ; Rapport CPO 2008 pp. 225-230 ; Rapport CPO 2009, p.151. Other extractions from DMTG files are also occasionally published in parliamentary reports. See e.g. “Rapport d’information sur la fiscalité des mutations à titre gratuity”, Rapport du Sénat n°65, 2002 (thereafter Rapport Senat 2002), pp.15-27. The only substantial inconsistency between the numbers reported in these publications and our own estimates is the following: the 2000 aggregate bequest flow published in Rapport CPO 2009 p.151 (34.5 billions €) is about 10% lower than our own estimate computed from the 2000 DMTG file (38.9 billions €), and also about 10% lower than the estimate published in Rapport Senat 2002 p.19. There are other statistical inconsistencies in Rapport CPO 2009, including inconsistencies with the numbers published in Rapport CPO 2008, which provides more complete and reliable tables and should be viewed as the reference source for recent French estate tax statistics.

six-month delay rule is not being enforced very strictly, and therefore that our simplifying assumption probably results into a slight downward bias for our fiscal inheritance flow estimates.[152]

Tables B1-B2, col. (2) to (4): correction for non-filers

The first adjustment that needs to be made to the raw fiscal series has to do with non-filers, i.e. with the fact that in a number of cases successors do not file an estate tax return. That is, we upgraded the raw fiscal series B_t^{f0} (col. (1)) in order to obtain corrected estimates B_t^{f1} (col. (3)) of the aggregate fiscal flow including non-filers. Our estimated upgrade factor B_t^{f1}/B_t^{f0} is reported on col. (2), and the corresponding share of non-filers in the corrected aggregate flow $(B_t^{f1}-B_t^{f0})/B_t^{f1}$ is reported on col. (4). Col. (3) of Table B1 was obtained by multiplying col. (1) by col. (2). Although our estimated upgrade factor B_t^{f1}/B_t^{f0} is fairly small (usually 105%-110% at most, except in the late 1950s-early 1960s, when it reaches 120%-130%), with no long run trend, we try to be precise about where our estimates come from. Before we describe the formulas we use for the non-filers corrections, it is useful to briefly summarize how and why the fraction of non-filers decedents has evolved over time.

Until 1956, all successors were required by law to fill an estate tax return, no matter how small the estate was. In particular, there was no tax exemption threshold of any kind. Tax rates – and graduated tax schedules, following the introduction of estate tax progressivity in 1901 – did vary widely, both over time and across categories of successors, as they have always done in France (children and spouses have always faced much lower tax rates than other heirs). But the key point is that until 1956 every positive bequest was subject to a positive tax, i.e. there was no base exemption, no zero rate bracket, no matters who the heirs were. So in principle there should be no need to make any correction for non-filers prior to 1956. In practice, the number of estate tax returns filled each year fluctuated around 50%-70% of the annual number of decedents aged 20-year-old during the 1826-1955 period (typically, about 300,000-400,000 annual tax returns, vs

[152] For instance, 57.9% of of estate tax returns filled in 1977 actually correspond to individuals who died in 1977, 26.0% to individuals who died in 1976, 8.1% to individuals who died in 1975 and another 8.0% to individuals who died in 1974 or before (see Fouquet and Méron (1982, p.86)). According to Fouquet-Meron, reported estate values should be increased by as much as 17% (95/81) in order to correct this bias and express all values in 1977 prices. In 2006, tax filling delay still seems to be higher than 6 months for a significant proportion of estate tax returns (see Rapport CPO 2008, p.227). However we know very little on average delay prior to 1977, and in order to preserve the continuity of our series it seemed more appropriate to make no adjustment at all.

about 500,000-600,000 adult decedents per year).[153] Given that the bottom 50% of the population generally holds very little wealth (always less than 10% of aggregate wealth, and usually about 5%), this suggests that the law was indeed applied very strictly: only successors with very small estates could escape their tax filing duties. I.e. the effective filling threshold was probably positive but extremely small.[154] For the sake of consistency, however, we do compute a non-filers upgrade factor for the 1826-1955 period, using the same method as for the 1956-2006 period (see below).

In 1956, for the very first time, a tax exemption threshold was introduced into the French estate tax system. The number of estate tax returns suddenly dropped from 250,000 in 1955 to 65,000 in 1956, i.e. from 50% of the number of adult decedents to less than 15%.[155] However, the nominal exemption threshold introduced in 1956 was updated very rarely since then – and in any case much less rapidly than inflation. As a consequence the annual number of estate tax returns gradually returned to its original level: 25% of the number of adult decedents by 1964, about 50% in the 1970s-1980s, and approximately 60%-70% during the 1990s-2000s (i.e. again around 300,000-350,000 annual returns, vs 500,000-550,000 adult decedents). Note that many of these estate tax returns are currently facing no tax liability.[156] E.g. during the 1990s-2000s, the number of taxable estate tax returns (i.e. with returns with positive tax liability) was only about 100,000-150,000 each year, i.e. approximately 20% of the number of adult decedents.[157] This is

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[153] See Table B1, col. (8)-(9). The annual number returns reported on col. (8) are taken from published tabulations for 1902-1964, and from DMTG files for 1977-2006. Prior to 1902, the tax administration did not bother collecting data on total numbers of returns. However according to the so-called TRA survey (which follows the estate tax returns of descendents of all couples married in France between 1800 and 1830 and whose family name started with the letters “TRA” up to 1940), the annual number of estate tax returns has been relatively stable around 50%-60% of the annual number of adult decedents throughout the 1820-1910 period (at least as a first approximation). See Bourdieu, Postel-Vinay and Suwa-Eisenmann (2002, 2003). Note that in Paris (where wealth concentration has even been more extreme than in the rest of France at that time), the tax-filing fraction of decedents was as low as 30% during the 1820-1910 period, before slowly converging towards the national average in the interwar and postwar periods. See Piketty, Postel-Vinay and Rosenthal (2006).

[154] It is difficult to know precisely how tax inspectors dealt in practice with successors of decedents with very little wealth. E.g. in case a decedent only leaves low value furniture worth a few months income, are tax inspectors going to chase the children until they fill a return? According to the law, they should: they start from the list of deceased individuals in their city and are supposed to make sure that all transmitted wealth gets recorded. In practice there has probably always been some tolerance with very poor individuals. E.g. the costs of funerals (which for poor individuals often exceed the net estate value) have apparently always been treated as being deductible from the estate (though this is formally not written in the law). The exact effective filling threshold probably varied over time and space. What really matters for our purposes is that given the functioning of the tax administration it has always been impossible to transmit real estate property or non-cash financial assets without filling a return. This is confirmed by the very large tax filers fractions observed throughout this period.

[155] See Table B1, col. (8)-(9).

[156] See Table B1, col. (8)-(9).

[157] Annual series on the number of taxable estate tax returns are currently published in the « Annuaire Statistique de la DGI » (see above). Note that non-spouse, non-children heirs are over-represented in

because the filling threshold (i.e. the wealth level above which all estates need to be reported to tax authorities, whether or not heirs end up paying a positive tax) is currently much lower than the tax exemption threshold (i.e. the wealth level above which one starts paying estate taxes), which for spouses and children heirs was raised much faster than the filling threshold since 1956, particularly in the most recent period. For children heirs, following a series of increases in the 2000s (most recently in 2007), the tax filing threshold is currently 50,000 euros (in terms of total gross assets left by the decedent), while the tax exemption threshold is currently to 150,000 euros (in terms of per children bequest).[158] Note that the latest rise in the tax filling threshold (2007) was not in force at the time of our latest data point (2006). The number of returns (338,000 returns in 2006, i.e. 66% of the number of decedents) probably declined somewhat in 2007 and subsequent years (no data is available yet). Of course, in case the tax filling threshold of 50,000 euros (about 25% of average per adult wealth, currently around 200,000 euros)[159] is further raised importantly in the future, then the tax filers fraction of decedents might decline more significantly. In case this happens, the non-filers correction would then become a more

taxable estate tax returns (and in aggregate estate tax receipts): although other heirs (i.e. non-spouse, non-children heirs) receive only 15%-20% of the aggregate inheritance flow (see Appendix C, section C.2), they benefit from no or little base exemption (see below), i.e. almost all of them pay positive taxes. Among children heirs, the fraction paying taxes has fluctuated a lot over time, because of large changes in the real value of the children tax exemption (see below). In the 1990s-2000s, it was typically around 5%-10%; following the 2007 tax reform (increase in tax exemption thresholds, new rules regarding inter vivos gifts), it could fall below 1%-2%, depending on how intensively future decedents use the new legal provisions regarding inter vivos gifts (see below).

[158] From a strict legal viewpoint, the threshold introduced in 1956 was actually a tax exemption threshold, not a filling threshold: all estates with gross assets below one million old francs (i.e. 10,000 new francs, i.e. 1,524 euros, at a time when per adult average wealth in current currency was about 2,000 euros; see Appendix A, Table A1, col. (6)) were entirely exempted from estate taxation (no matter who the heirs were); in principle, the universal tax filling obligation was unaffected by this reform; but in practice, tax inspectors received instructions not to chase heirs with gross assets below this threshold, and the (presumably very few) tax returns filled after 1956 with gross assets below 10,000 new francs were entirely excluded from tax publications and statistics. This nominal 10,000 new francs threshold was never updated since 1956 (it simply became 1,500 euros with the 2002 currency change), and it remained until 2004 the only general tax exemption threshold for non-spouse, non-children heirs. In 1960, a tax exemption threshold of 100,000 new francs (15,240 euros) was introduced for spouses and children heirs; it was raised to 175,000 francs in 1974, 200,000 francs (175,000 for children) in 1980; 275,000 francs (250,000) in 1981; 300,000 francs (275,000) in 1984; 330,000 francs (300,000) in 1992; 400,000 francs (300,000) in 1999; 500,000 francs (400,000) in 2000; 76,000 euros (50,000 for children) in 2005; finally, in 2007, spouses were wholly exempted from estate taxation, and the tax exemption threshold for children heirs was raised to 150,000 euros (with automatic CPI adjustment for subsequent years). An official tax filling threshold of 10,000 euros (in total gross assets) was also introduced in 2004 for spouses and children heirs; this threshold was raised to 50,000 euros in 2006. It does not apply however in case the same heirs benefited from inter vivos gifts from the decedent (in which case all estates must be reported to tax authorities; see below). An official tax filling threshold of 3,000 euros for non-spouse, non-children heirs was introduced in 2004 (not upgraded since then); a tax exemption threshold of 5,000 euros was created in 2006 for brothers/sisters; it was raised to 15,000 euros in 2007, together with the introduction of a 7,500 euros threshold for nephews/nieces.

[159] See Appendix A, Table A1, col. (6).

serious issue, and French estate tax data would loose some of its exceptional quality in comparison to other countries.[160]

To summarize: except during a brief period in the late 1950s-early 1960s, the fraction of tax filers has generally been about 50%-60% of the annual number of adult decedents throughout the 1820-2006 period.

In order to compute the non-filers correction factor we proceed as follows. We note N_{dt}^{f} the number of estate tax returns, N_{dt}^{20+} the total number of adult decedents, $n_{dt}^{f} = N_{dt}^{f}/N_{dt}^{20+}$ the fraction of tax-filers decedents, and $w_{dt}^{f} = B_{t}^{f0}/N_{dt}^{f}$ the average wealth reported by tax filers. All we need to estimate is the average wealth of non-filers w_{dt}^{nf}. We note $z_{dt}^{nf} = w_{dt}^{nf}/w_{dt}^{f}$ the ratio between non-filers and filers average wealth. Once we know z_{dt}^{nf}, we can simply compute the non-filers correction factor by applying the following equation:

$$B_{t}^{f1} = N_{dt}^{f}\, w_{dt}^{f} + (N_{dt}^{20+}-N_{dt}^{f})\, w_{dt}^{nf} = B_{t}^{f0}\, [\, 1 + (1-n_{dt}^{f})\, z_{dt}^{nf}\,]$$

$$\text{I.e.:} \qquad B_{t}^{f1}/B_{t}^{f0} = 1 + (1-n_{dt}^{f})\, z_{dt}^{nf} \qquad \text{(B.1)}$$

As a first approximation, one could think of the non-filers as decedents with wealth below some effective filling threshold w_{dt}^{*}, with $1\text{-}F_{t}(w_{dt}^{*}) = n_{dt}^{f}$, where $F_{t}(w)$ is the cumulative distribution function for wealth-at-death (i.e. $F_{t}(w)$ is the fraction of decedents with wealth-at-death less than w, and $1\text{-}F_{t}(w)$ is the fraction with wealth above w). Ideally, it would certainly be interesting to model explicitly the functional form of the wealth distribution $F_{t}(w)$ and its endogenous dynamics, and then from there to derive explicit estimates for the wealth ratio z_{dt}^{nf} between the bottom and upper parts of the distribution. However such an explicit modelling of distributions would fall far beyond the scope of the present research, where we concentrate primarily upon aggregate ratios and their evolution. Also we know that the effective filing threshold w_{dt}^{*} has always been relatively small, but we do not know its exact value: in the 1826-1955 period, it was officially supposed to be equal to zero, but in practice it was probably slightly positive; in the 1956-2006 period, it was officially slightly positive, but varied with the family structure (in particular the existence of children heirs),

[160] For the purpose of comparison, note that the estate tax filling threshold in the U.S. was 2,000,000$ (gross assets) in 2008, and that the number of returns was less than 2% of the total number of adult decedents (less than 40,000 returns, out of a total of 2.5 millions decedents). See IRS estate tax statistics available on-line. The US estate tax has always been an elite tax since its creation in 1916 (with a tax filers fraction typically less than 2%-3%; see Kopczuk-Saez (2004)). It seems unlikely that the French tax filers fraction of decedents drops to such low levels in the foreseeable future.

so the observed wealth distribution is actually truncated downwards at slightly different levels for different sub-populations.

So instead we make the following simple approximate assumptions about the wealth ratio z_{dt}^{nf} (which in any case is bound to be very small). For the recent decades, we have several data sources to estimate the average wealth of non-filers. First, the wealth surveys carried out by Insee in the 1990s-2000s (similar to the U.S. Survey of consumer finances) consistently show that the bottom half of the population owns at most 5%-10% of aggregate wealth (this is true at all ages).[161] In the U.S., the bottom 50% wealth share, as estimated in the SCF surveys of the 1990s-2000s, is even less than 5%.[162] By definition, note that a 5% aggregate wealth share for the bottom 50% means that the bottom half average wealth w_{dt}^{b} is equal to 10% of aggregate average wealth w_{dt}, and to about 5.3% of the upper half average wealth w_{dt}^{u}.[163] I.e. this corresponds to a bottom-top wealth ratio $z_{dt}^{b} = w_{dt}^{b}/w_{dt}^{u} = 5.3\%$. Similarly, a 10% aggregate wealth share for the bottom 50% means that w_{dt}^{b} is equal to 20% of aggregate average wealth w_{dt}, and that the bottom-top wealth ratio $z_{dt}^{b} = w_{dt}^{b}/w_{dt}^{u}$ is about 11.1%.[164] So on the basis of wealth surveys, and considering that the non-filers approximately correspond to the bottom half of the wealth distribution, one might be tempting to assume values of about 5%-10% for the z_{dt}^{nf} ratio. However a special survey conducted by the tax administration in 1988 in order to estimate the wealth of non-filers suggests that the true ratio is somewhat higher, with z_{dt}^{nf} around 15%.[165] This

[161] Using the raw wealth levels reported by households (with no correction whatsoever) in the wealth surveys conducted by Insee in 1986, 1992, 1998 and 2004 (about 10,000 households per survey), we find a bottom 50% wealth share of about 7.5% of aggregate wealth in all three surveys. E.g. in 2004 the average net wealth reported by all households was approximately 200,000€, while average wealth reported by the bottom 50% of the distribution was about 30,000€, i.e. 15% of 200,000€. There are good reasons to believe that high-wealth individuals under report their wealth in surveys (omission of various assets such as life insurance, top coding issues, etc.), and the true wealth share of the bottom 50% is probably closer to 5% than to 7%-8%. If we compute the bottom 50% share for the various age groups, we find a slightly rising profile (from about 5% for lower age groups to slightly above 10% for older age groups), but the pattern is not entirely clear cut, and in any case pretty small.

[162] The bottom 50% wealth share appears to be about 2% of aggregate wealth in all SCF surveys conducted between 1989 and 2007. See Kennickell (2009, p.35, table 4). Note that the exact figure one obtains for bottom half wealth shares depends on a number of measurement issues, e.g. how one counts negative net wealth individuals. In this research we conventionally set them to zero, since negative net wealth cannot be transmitted (see above). Kennickell also adopts this convention.

[163] (5/0.5)/(95/0.5) = 5.3%.

[164] (10/0.5)/(90/0.5) = 11.1%.

[165] See Laferrère and Monteil (1994) and Accardo and Monteil (1995). This 1988 “Wealth at death” was carried out jointly by Insee and the tax administration, and its specific purpose was to learn more about the wealth of non-filers decedents. It was based on a representative sample of all adult deceasing in 1988, for which the tax administration gathered not only the estate tax returns of the tax filers subsamble (about 50% of dedecents at that time), but also all other tax forms available for non tax filers (past income tax and local tax returns, bank forms on assets and asset returns, past bequest and gift tax returns, registration duties for sales of real estate assets, etc.), so as to compute relatively precise estimates of the average non-filers wealth w_{nf}. They found that the aggregate wealth share of non-filers was about 13%, which corresponds to a z_{dt}^{nf} ratio of about 15%: (13/0.5)/(87/0.5) = 14.9%.

seems to due to the fact that in the recent decades the effective filling threshold has been substantially higher for a sub-fraction of decedents (particularly those with children), thereby raising somewhat the non-filers average wealth.

So for the 1977-2006 sub-period we assume z_{dt}^{nf} = 15%. For the 1826-1955 period, given that filling obligations were the same for all decedents and were applied very strictly, and given that the bottom 50% wealth share was probably at most 5% during this period (at that time top wealth shares were even larger than they are today),[166] we assume z_{dt}^{nf} = 5%. For the 1956-1964 period, on the basis of the Finance Ministry tabulations by estate size, we also find that the best approximation is z_{dt}^{nf} = 5%.[167]

The non-filers upgrade factor B_t^{f1}/B_t^{f0} was therefore computed by applying equation (B.1) and by assuming z_{dt}^{nf} =5% for 1826-1964 and z_{dt}^{nf} =15% for 1977-2006. We find an upgrade factor B_t^{f1}/B_t^{f0} around 103%-105% throughout the 1826-1955 period;[168] the upgrade factor then jumps to over 130% in 1956-1957, but quickly diminishes towards 115%-120% in the late 1950s-early 1960s, and then stabilizes around 110%-115% in the period going from the 1970s to 2000s (see Table B1, col. (2)). We tried several alternative assumptions, and we found that the impact on upgrade factors was relatively small (less than 5%).

Tables B1-B2, col. (5) to (7): correction for tax-exempt assets

The second adjustment that needs to be made to the raw fiscal series has to do with tax-exempt assets, i.e. with the fact that a number of assets are legally exempt from estate taxation and are generally not reported on estate tax returns. That is, we upgraded the

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166 On the historical evolution of top and middle wealth shares, see working paper, section 7.2, and Piketty, Postel-Vinay and Rosenthal (2006, appendix tables A4 and A7). These top wealth share estimates rely on estate tax data (and crudely estimated aggregate wealth series), so by construction they do not give very precise estimates of the bottom shares. But with top 10% wealth shares as large as 80%-90% in 1820-1913 and as large as 70%-80% in the interwar period and the 1950s, bottom 50% wealth shares are bound to be very small, probably less than 5% (today top decile wealth shares are about -60%, and bottom 50% shares are about 5%-10%)

167 Comparing the estate-size tabulations for 1950-1955 and 1956-1960 one can approximately compute the average wealth of the non-filers of the second sub-period (who were filers during the first sub-period), and one finds z_{dt}^{nf} ratios around 5% (for the raw tabulations, see Piketty (2001, appendix J)). Alternatively, note that the filers fraction of decedents was about 20% in the late 1950s, and that the top 20% wealth share was approximately 80%-85% at that time (see Piketty et al (2006, appendix table A7)); if one assumes that the non-filers were the bottom 80% of the distribution, then one again finds a bottom-top wealth ratio z_{dt}^{nf} around 5%: (15/0.8)/(85/0.2) = 4.4%, and (20/0.8)/(80/0.2) = 6.3%.

168 Our annual series on tax filers fractions n_{dt}^{f} start in 1902, but we know that the filers fraction was approximately stable during the 19th century (see above), so we assume that n_{dt}^{f} was the same in 1826-1901 as in 1902 (see formulas in excel file).

non-filers-corrected fiscal series B_t^{f1} (col. (3)) in order to obtain corrected estimates B_t^{f2} (col. (6)) of the aggregate fiscal flow including non-filers and tax-exempt assets. Our estimated upgrade factor B_t^{f2}/B_t^{f1} is reported on col. (5), and the corresponding share of tax exempt assets in the corrected aggregate flow $(B_t^{f2}-B_t^{f1})/B_t^{f2}$ is reported on col. (7). Col. (6) was obtained by multiplying col. (3) by col. (5).

In order to estimate the fraction of tax exempt assets in the corrected aggregate flow, we proceed as follows. For the 1970-2009 period, we have detailed annual series on aggregate private wealth broken by asset categories coming from Insee-Banque de France balance sheets (see Appendix A, Table A15b). On the basis of estate tax law, and by comparing the asset composition of aggregate private wealth and the asset composition of the fiscal estate flow (as measured by 1977-2006 DMTG files),[169] we make the following assumptions about the taxable and tax-exempt fractions of each asset category.[170] We assume that 80% of the value of housing assets (residential real estate), as estimated by Insee-Banque de France balance sheets, was subject to the estate tax, and that 20% was tax exempt.[171] For non-housing tangible assets (which include unincorporated business assets), we assume a taxable fraction of 70% and a tax exempt fraction of 30%.[172] For financial assets other than private equity and life insurance (i.e. for

[169] Using 1984-2000 DMTG micro files we obtained the following break down for the aggregate bequest flow. The share of residential real estate went from 44% of total gross assets in 1984, 47% in 1987, 42% in 1994 and 39% in 2000. The share of non-housing tangible assets went from 13% of total gross assets in 1984 to 9% in 1987, 6% in 1994 and 4% in 2000. The share of financial assets (including private equity) went from 44% of total gross assets in 1984 to 44% in 1987, 51% in 1994 and 57% in 2000. The share of financial liabilities went from 5% of total gross assets in 1984 to 7% in 1987, 5% in 1994, and 5% in 2000. Note that these series cannot easily be broken down in a more detailed manner, because asset categories used in DMTG files are not fully homogenous over time.

[170] These estimated fractions are reported on Table A15b.

[171] This 20% tax exemption coefficient might be somewhat underestimated, especially at the beginning of the 1970-2009 period. First, housing assets currently benefit from a 20% rebate on market values whenever the asset serves as the primary residence of the decedent and the surviving spouse, or of the decedent and one of the children. In DMTG micro files we do not know how often this rebate is used (reported values are after-rebate values, if applicable), but this is probably a very large fraction. Next, in order to foster reconstruction a general estate tax exemption was introduced in 1947 for the first intergenerational transmission of all real estate properties built between 1947 and 1973. According to some estimates, the loss in estate tax revenues due to this specific exemption was as large as 25% in the 1970s (see Rapport du Conseil des Impôts, 1986, p.44). See also Laferrère (1990, p.5), who on the basis of the DMTG 1977 micro-file estimates that this specific exemption accounts for an aggregate loss in bequest tax base as large as 20%.

[172] Family firms have always benefited from various exemptions and special tax rebates, whether they take the form of unincorporated businesses (e.g. commercial dwellings or agricultural assets directly owned by self-employed individuals) or the form of corporate unquoted firms (private equity financial assets). The rules required to qualify for the "biens professionnels" tax rebates have been repeatedly relaxed in the 1990s-2000s (e.g. currently successors only need to commit to operate the family business for two years after the decedent passed way in order to obtain a 100% tax rebate, with no ceiling). We did not attempt to enter into the complicated history of these special exemptions (for more details, see e.g. Rapport CPO 2008 and 2009). Given the very low levels of business assets reported in estate tax returns (see above), our estimated 30% tax exempt fraction for non-housing tangible business assets and 50% tax exempt fraction for private equity appear to be reasonable (and probably slightly under-estimated at the end of the 1970-2009 period).

public equity, mutual funds, bonds, checking and savings accounts, etc.), we assume a taxable fraction of 90% and a tax exempt fraction of 10%.[173] For private equity financial assets, we assume a taxable fraction of 50% and a tax-exempt fraction of 50%.[174] Finally, for life insurance financial assets (the major tax exempt asset), we assume a taxable fraction of 5% and a tax-exempt fraction of 95%.[175] We then weighted these tax exempt fractions by the relative importance of each asset category in aggregate private wealth in order to estimate the overall fraction of tax-exempt assets in total wealth, which according to these computations gradually rose from about 24%-25% in the 1970s to about 33%-34% in the 2000s (see Table B1, col. (7)).[176] This is mostly due to the rise of life insurance.

These estimates are approximate – and if anything are probably conservative, especially for the more recent period. In particular, we implicitly assume that average asset composition is the same for decedents and for aggregate private wealth.[177] Insee wealth surveys suggest that the elderly actually own a larger fraction of their wealth in tax exempt assets such as life insurance, so that we probably underestimate our upgrade factor.[178] Also, note that the top estate tax rate for children heirs was raised from 20% to 40% in

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Note also that a number of non-housing, non-business tangible assets have long benefited from special exemption regimes in France, e.g. a number of specific rural assets like forests (see Rapport du Conseil des Impôts, 1986, p.44).

[173] In principle, all non-private-equity, non-life-insurance financial assets are subject to estate taxation, on the basis of their full market value. In particular, the general exemption for public bonds was suppressed in 1850, and never re-introduced. However, a number of special exemption schemes were introduced by various governments for specific assets, especially for specific public bonds issued at a given point time (many governments used this as a debt policy tool during and in the aftermath of both world wars, and the habit continued afterwards: e.g. the “emprunt Pnay” issued in the 1950s was wholly exempted from estate taxation, and so was the “emprunt Balladur” in the 1990s). In order to take this into account, we assume that 90% of the overall market value of non-equity, non-life-insurance financial assets (as measured by Insee-Banque de France balance sheets) is subject to tax, and that 10% is tax exempt. This is of course approximate and ought to be refined.

[174] See above.

[175] Between 1930 and 1990, life insurance assets were entirely tax free (i.e. 100% exemption rate). Since 1991, the fraction of life insurance premiums paid after age 70 and above 30,500€ is subject to estate tax (not the corresponding interest). In order to take this into account we assume that a 5% fraction of life insurance assets is taxable (according to DMTG files for 1994-2000, which include virtually no life insurance assets, this is probably even lower than 5%). Also note that a special 20% tax on the fraction of life insurance payments to successors above 152,500€ was instituted in 1998. However this special tax is administered completely separately from the general estate tax, and the corresponding asset values are not reported on estate tax returns.

[176] See formulas in excel file.

[177] We attempted to compute the tax exempt fractions for various assets so as to match the observed composition of taxable estates, so in principle we correct for such biases. However the asset categories used in Insee-Banque de France balance sheets and in DMTG estate tax returns files are not exactly the same, so such computations are bound to be approximate. Also, there are virtually no life insurance assets in estate tax returns, so the age bias correction does not work for this asset.

[178] It is also possible that the annuitized (non-bequeathable) fraction of life-insurance assets rises with age (an issue on which we know very little), in which case the bias would go in the other direction. Given however that the overall annuitized fraction of life insurance assets is relatively small in France (see Appendix A.5), it seems unlikely that this second effect dominates.

1984:[179] this possibly raised incentives for straight tax evasion, which by choice we do not attempt to include in our legal tax exemption upgrade factor.[180]

For the pre-1970 period we proceed as follows. We use the detailed decomposition by asset categories (including estimated tax exempt assets) regularly published by the Finance Ministry during the 1898-1964 period. These estimates show that tax-exempt assets were relatively small in 1898-1899 (about 5% of total assets, taxable and tax-exempt), then fastly rose to about 15%-20% following the 1901 estate tax reform (the introduction of tax progressivity was accompanied by the development of legal exemptions, and according to some observers of the time by the rise of tax evasion, which we do not take into account), then stabilized at about 20% during the interwar period, and finally rose somewhat during the 1950s and early 1960s.[181] Since these numbers are consistent with our independent 1970-2009 estimates, we simply link them up by assuming that the aggregate fraction of tax exempt assets rose gradually rose from 20% in 1950 to 25% in 1970.[182] For the 1826-1897 period we have very limited data to compute the fraction of tax exempt assets. However we know from estate tax law that the major exemption during the 19th century was public debt: government bonds were entirely exempted from estate tax until 1850, while bonds issued after 1850 were all subject to tax.

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[179] See working paper, section 3.3.

[180] In our methodology, the gap between our economic and fiscal inheritance flows can be interpreted as an indirect measure of tax evasion. From that perspective, tax evasion would appear to be trendless in the long run: in particular the gap between the two series does not seem to increase after 1984 (see working paper, Figures 1 and 2). However the gap between the two series also reflects all other measurement errors, so it is hard to reach precise conclusions about tax evasion from this kind of comparison (apart from the fact that it does not seem to affect long run patterns).

[181] See AR 1966, p.530. Here we look at the ratio between the sum of all taxable and tax-exempt gross assets (i.e. “valeurs soumises ou non aux droits”, defined as the sum of “valeurs mobilières – fonds d’Etat, actions, obligations”, “autres biens meubles”, “biens immeubles urbains et ruraux”) and the value of taxable gross assets (“valeur total de l’actif brut”). The tax administration started compiling estimates on tax exempt assets only in 1898; until 1897 the tax administration asset composition series solely refer to taxable assets; so by construction this ratio is equal to 100% over the 1826-1897 period. The ratio is always above 100% over the 1898-1964 period and offers the best available estimate of tax exempt assets for this period. Note however that this ratio displays intriguing variations around World War 2, e.g. it is as high as 140% in 1943 and 1949, while it is about 120%-125% for all surrounding years; it is possible that for these years the tax administration wrongly included into tax exempt assets the fraction of community assets belonging to surviving spouses; this would need to be further investigated; we neglected these high ratios and assumed that tax exempt assets were a constant 20% fraction of total assets over the 1910-1950 period; but it is possible that by doing so we under-estimate somewhat the importance of tax exempt assets in the 1940s. Note also that according to these asset composition series the overall fraction of real estate (urban and rural properties, including land values) in the aggregate bequest flow gradually declined from as much as 60%-70% in the 1820s to about 50% around 1900-1910, which is consistent with the evolution of asset composition observed in national wealth estimates (see Appendix A, section A.5).

[182] That is, we assumed that the ratio $(B_t^{f2}-B_t^{f1})/B_t^{f2}$ was rose from 5% in 1900 to 20% in 1910 (using Finance Ministry ratios), then stabilized at 20% in 1910-1950, then rose linearly from 20% in 1950 to our estimate of about 25% in 1970 (see formula for col. (7) of Table B1 in excel file). The 1950-1970 rise if consistent with the development of new exemption regimes for specific housing and public bonds assets during this period (see above).

Based on approximate estimates on the total value and maturity structure of government bonds,[183] we assumed that the aggregate fraction of tax exempt assets rose gradually from 15% in 1826 to 20% in 1840, stabilized at 20% between 1840 and 1855, and then declined gradually from 20% in 1855 to 5% in 1880, before stabilizing at 5% until 1900.[184]

Tables B1-B2, col. (8) to (12): correction for inter vivos gifts

The third and last adjustment that needs to be made to the raw fiscal series has to do with inter vivos gifts, i.e. with the fact that a number of assets are transmitted before death and are therefore not included in the bequest flow strictly speaking. As was explained in the working paper (section 3.1), the simplest way to take gifts into account is to add the gift flow of a given year to the bequest flow of the same year.[185] This is what we do on Table B1. That is, we report on col. (8) the raw fiscal gift flow V_t^{f0}, the total net wealth value transmitted via inter vivos gifts during year t, as reported to tax authorities. We then compute the raw gift-bequest ratio $v_t = V_t^{f0}/B_t^{fo}$ by dividing col.(8) by col.(1). We find the gift-bequest ratio was relatively stable around 30%-40% from the 1820s to the 1850s, then declined somewhat and stabilized around 20%-30% from the 1870s to the 1970s, and then gradually rose to about 40% in the 1980s, 60%-70% in the 1990s and over 80% in the 2000s (see Table B1, col.(9)). We compute the corresponding upgrade factor $1+v_t$ (col. (11)), which we multiply by non-filers-and-tax-exempt-assets-corrected fiscal series B_t^{f2} (col. (6)) in order to obtain our final estimates B_t^{f2} of the fiscal inheritance flow (col.(10)). In effect, we are assuming that the same upward correction for non-filers and tax-exempt assets apply to bequests and gifts, which as a first approximation seems like the most natural assumption (though it probably understates the true economic importance of gifts).[186]

[183] See Appendix A, section A.5.
[184] For simplicity we again assumed linear trends. See Table B1, col. (7).
[185] In the simulations, we re-attribute gifts to the proper generation of decedents. See Appendix D.
[186] Regarding tax exempt assets, the same rules apply to bequests and gifts, so it makes sense to assume the same correction factor (though the 1977 estimates published by Laférrère (1990, p.5) suggest a significantly larger fraction of tax exempt assets for gifts than for bequests; unfortunately we do not have similar estimates for other years, so we decided that it was more reasonable to keep the same correction for bequests and gifts for the entire period under study). Regarding non filers, note that there has never been any official filling threshold for inter vivos gifts: in principle, from 1791 up until the present day, all gifts are supposed to be reported to tax authorities in France, no matter how small they are (otherwise individuals could just fractionalize gifts indefinitely and transmit large wealth levels entirely tax free). In practice however, according to French case law, it is of course allowed to make birthday presents and other “small gifts” (as long as they are of “reasonable” value, which according to case law should be interpreted as varying with the living standards of the donor, among other things) without reporting them to tax authorities. It is likely that many not-so-small gifts never get reported, so that the true non-filers upgrade factor is probably larger for gifts than for bequests.

Our raw fiscal gift flow series (col.(8)) comes from the same data sources as the raw fiscal bequest flow, i.e. published Finance Ministry aggregate annual series for the 1826-1964 period,[187] and DMTG micro files estimates for the 1977-1984-1987-1994-2000-2006 period.[188] Given the importance of the gift-bequest ratio parameter v_t, it would obviously be preferable to have annual series on the gift and bequest flows for the entire period.[189] However we feel reinsured by the fact that the data points at our disposal do show a relatively regular and gradual evolution of the gift-bequest ratio in the long run, including during the recent decades.[190]

Note that inter vivos gifts have always benefited from a number of tax advantages in the French system of bequest and gift taxation. Prior to 1901, there was no explicit tax advantage for gifts: bequests and gifts were subject to similarly low proportional tax rates (varying only with the identity of the heir or donee). The main tax advantage was due to capital gains (and capitalized interest): by giving an asset earlier in life one pays lower taxes, simply because its value is generally lower than at the time of death. With the

[187] For 1826-1964, we simply reported on col. (8) of Table A1 the raw gift flow series published in AR 1966, p.530, col. "Donations". Note that the Finance Ministry sadly did not compile gift flow series for years 1923-1943 (i.e. the annual 1826-1964 series published in AR 1966 p.530 display years 1923-1943 as missing years, and the Finance Ministry publications of the interwar period do not provide any gift data either). Since the gift-bequest ratio appears to be relatively stable around 20%-30% both in the 1870s-1910s and in the 1940s-1970s, we simply assumed a 25% ratio for the 1920s-1930s (see col. (9), Table B1). Gift tax receipts vs bequest tax receipts series (which are available on an annual basis since the 1820s up until today, and in particular are available for the interwar period) seem to be consistent with this approximate assumption (because of tax progressivity, it is unfortunately difficult to obtain precise tax base estimates from tax receipts series, particularly for the interwar period). The gift flow data published for 1921-1922, as compared to the bequest flow data published after 1925, suggests that the gift-bequest ratio was closer to 25% than to 20% during the interwar period (i.e. was intermediate between pre World War 1 and post World War 2 levels).

[188] For 1984-2000 we report gift flow estimates coming from our own computations using DMTG micro-files. The series we obtain, and the corresponding evolution of the v_t ratio, are similar to those published in various official reports. See e.g. Rapport du Conseil des Impôts, 1998, pp.210-211, for gift and bequest flows similar to ours (corresponding to v_t=29% in 1984 and v_t=64% in 1994). For 2006, we take the estimates published in Rapport CPO 2008: V_t^{f0} = 48.0 billions was computed as the sum of the regular gift flow (39.4 billions) (see Rapport CPO 2008 p.273) and the average yearly 2004-2006 flow under the special cash-gifts regime (8.6 billions) (see CPO 2008 p.241; apparently this extra flow was not included in the regular flow statistics; if we were to exclude it, we would find v_t=67% rather than v_t=82%, which would still be high by historical standards; given the regular flow v_t=81% obtained in the DMTG 2000 micro file, at a time when there was no such special regime, it seems more justified to look at the full flow in 2006). For 1977, the gift and bequest flow estimates computed by Laferrère (1990, p.5) correspond to a gift-bequest ratio v_t=49% (=(37.7-14)/(57.8-9)). However this does not seem fully consistent with the gift tax receipts vs bequest tax receipts series, which suggest that the gift-bequest ratio in the 1970s was similar to the levels observed in the 1964 and in 1984 (i.e. v_t=25%-30%). So we did not use this 1977 gift estimate and instead assumed that the ratio v_t evolved linearly between 1964 and 1984 (see Table B1, col. (10)). This would need to be further investigated.

[189] In 2006 the Finance Ministry started to computerize all gift tax returns on an annual basis, so in principle data quality should improve soon. So far this however does not apply to bequest tax returns.

[190] We need annual series on the gift-bequest ratio v_t for the economic inheritance flow computation (see Appendix A, section A2), which we obtained by simple linear interpolation (see Table B1, col. (9)). Given the data at our disposable, this looks like the most reasonable approximation.

introduction of tax progressivity in 1901, another implicit tax incentive was created: by splitting an estate into several pieces one could end up in lower tax brackets and hence pay lower total taxes.[191] With the rise of top tax rates in the interwar period, this became increasingly problematic, and in 1942 a major reform was enacted in order to unify the bequest and gift taxes. Since 1942 until the present day, the general rule is that the same graduated tax schedules apply to both bequests and gifts, and most importantly that all inter vivos gifts are "recalled" when the donor dies and are added to the bequest left at death, so that each heir ends up paying taxes on the basis of the total estate he or she received from the decedent. In principle, the system is designed so as to achieve full tax neutrality between gifts and bequests: if you want to transfer a given asset to your kid, then the total tax burden is the same whether you transmit half now and half at your death or you transmit it entirely at your death.[192]

In practice, however, gifts remained less taxed than bequests after 1942, and these tax advantages were significantly reinforced in the late 1990s and in the 2000s. First, the 1942 reform did not eliminate the capital gains (and capitalized interest) tax advantage: recalled gifts have always been valued at the time they were made, not at the time of death. In times of high inflation and even more rapidly rising asset values, this can make a big difference.[193] Next, the 1942 reform created a special 25% tax rebate for so called "sharing gifts" ("donations-partages"), i.e. inter vivos gifts with equal sharing between all children. This special 25% tax rebate regime was abolished in 1981, but then re-introduced in 1986 for "sharing gifts" made by donors aged less than 65-year-old (no such age condition existed in the 1942-1981 regime). This was then extended in 1996-1998 to all gifts made

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[191] In addition the tax rates themselves differed: between 1901 and 1942, bequests were subject to graduated tax schedules, while gifts were subject to quasi-proportional tax rates (always varying with the identity of the donee).

[192] The tax paid at the time of the gift is deducted from the tax liability computed at the time of death on the sum of gift and bequest. I.e. if t(.) is the relevant tax schedule, v is the gift and b is the bequest, then one pays tax t(v) at the time of the gift, and t(b+v)-t(v) at the time of death, so that the total tax payment is t(b+v), independently of the b vs v split, for given b+v. Note that although so-called "recalled gifts" ("donations rappelées") play an important role for tax computation, they are never included in the Finance Ministry estate tax statistics we used (i.e. the tax administration always compiled separate statistical tables for bequests and gifts, both before and after 1942). The bequest flow reported on col. (1) of Table B1 is the bequest flow strictly speaking, excluding recalled gifts ("hors donations rappelées"). In 1977-2006 DMTG micro-files, we do observe all variables necessary to reproduce the tax computations, and in particular we observed recalled gifts (together with the year of gifts), but we did not add them to the bequest flow. We did check though the reported recalled gifts are consistent with observed past gift flows (given mortality rates and other special rules applying to gifts, see below); they are consistent, i.e. the system seems to be applied relatively strictly.

[193] Also, note that within the realm of the 1942 law, donors can choose (but are not obliged) to pay the gift tax in place of the donee, and this tax gift t(v) is not recalled at the time of death (but is deducted from the tax t(b+v) paid by the heir). For large estates and high tax rates, this can be significant.

by donors aged less than 65-year-old; it is still in place today.[194] In effect, this special regime became a policy tool to favour early estate transmission to children, together with other temporary regimes enacted in the late 1990s and the 2000s.[195] Finally, the so-called "10 year rule" was introduced in 1992, which significantly altered the general principle of "recalled gifts" instituted in 1942. Since 1992, gifts made more than 10 years before the time of death are not recalled any more. I.e. they still pay gift tax at the time they are made, but they are not added any more to the estate when the bequest tax is computed. In 2006, the "10 year rule" became a "6 year rule".[196]

It is plausible that the increased tax advantages given to gifts in the 1990s-2000s did contribute to the recent rise of the gift-bequest ratio v_t. Because we do not have annual data, it is difficult however to isolate the impact of tax incentives per se, as opposed to the many non-tax-related reasons that could explain the rise in v_t. In particular, it is equally plausible that rising age expectancy alone can explain why parents start giving away larger fractions of their wealth in inter vivos gifts (e.g. so as to help their children to buy a home at a reasonably early age), quite independently from tax incentives. Given that the rise of the gift-bequest ratio v_t appears to start in the 1980s and early 1990s, i.e. before the

[194] The exact parameters have changed a lot. In 1987-1996, "sharing gifts" made by donors aged less than 65 benefited from a 25% tax rebate, and those made under age 75 had a 15% tax rebate. These same rules were extended to all inter vivos gifts in 1996; the 25%/15% rates became 50%/30% in 1998; they still apply today, except that the age limits are now 70/80 rather than 65/75.

[195] In 1998-2001 a general tax rebate of 30% was applied to all gifts (with no age condition). This was reiterated in 2003-2005 (with a 50% tax rebate, again with no age condition). In addition, a new special regime for cash gifts below 30,000€ given to children and grand-children was applied in 2004-2006 (full tax exemption); this special regime was made permanent in 2007 (but with an age condition: the donor needs to be less than 65-year-old). These frequent changes in tax incentives have generated significant short-run variations in the volume of gifts, as one might expect, and as one can see from annual gift tax receipts series (see Rapport CPO 2008, pp.240-242). Similar phenomena already occurred in the past (e.g. gift tax receipts rise in 1981, prior to the repeal of the sharing gifts regime and the creation of the wealth tax). Generally speaking, gift flows are structurally more volatile than bequest flows, and one must be careful when using non-annual gift series (which can easily be contaminated by purely temporary, tax-induced variations). Our 1964-1984-1987-1994-2000-2006 aggregate gift flow estimates appear however to be representative of the long run tendency (i.e. by using gift tax receipts annual series we did our best to ensure that these are not particularly high v_t or low v_t years; e.g. we smoothed over three years the large extra flow generated by the cash-gifts special regime in 2004-2006, see above), to the extent of course that there exists a long run tendency (see below the discussion on the long run sustainability of high v_t ratios).

[196] Together with the 2007 increase in the tax exemption threshold (from 50,000€ to 150,000€ per children, see above), this implies that one can now transmit relatively large estates to children without paying any tax, assuming one starts making gifts sufficiently early. The way the "x year rule" works is indeed that one can in effect benefits from the base exemption every x years. So, to consider an extreme case, under the "6 year rule", by starting making gifts at age 50 a parent dying at age 80 can now transmit six times 150,000€ tax free to a given children, i.e. 900,000€. Each parent can do that with each children, so in effect a sufficiently forward looking (and tax-phobic) married couple with two children can transmit 3.6 millions € tax free to its children. This is comparable to the base exemption threshold currently applied in the U.S. (2 million $ in 2008, for each parent), but requires relatively sophisticated behaviour, and high parental willingness to give away assets relatively early in life. This new legal regime has been applied only since September 2007, and it will take several decades (and much better data than that currently available) before one can estimate its full long run effects.

changes in tax incentives, one is tempted to conclude that non-tax factors played a dominant role. Also note that parents did not start making gifts earlier in life in recent decades: the average age gap between decedents and donors appears to have been relatively stable around 7-8 years since the 1960s, and in particular during the 1980s-1990s-2000s.[197] This suggests that the new tax incentives (most of which decline with age) did not play a major role, or at least did not have the impact expected by policy makers. In any case, note that whether tax factors or non-tax factors explain the observed rise of the gift-bequest ratio v_t since the 1970s, and in particular the very high levels observed in the 2000s (over 80%), is not really relevant for our purposes in this research. What is potentially more relevant is to know whether there has been some kind of "overshooting" of gifts in the recent past in France, in the sense that the relatively large bequests-plus-gifts flows observed in the 2000s might not be sustainable (i.e. because the cohorts who made unusually large gifts in the 2000s will also leave unusually small bequests in the 2010s). We address this issue when we present the results from the simulated model (see Appendix D).

B.2. Data on the age profile of wealth $w_t(a)$ and computation of the μ_t ratio

Our series on age-wealth profiles $w_t(a)$ and our resulting estimates of μ_t ratios are reported on Tables B3 to B5. Here we describe the data sources and methods used to construct these tables.

Table B3: Raw data on the age-wealth profile of decedents $w_{dt}(a)$, 1820-2006

On Table B3 we report our raw data on the age-wealth profiles of decedents. We note $w_{dt}(a)$ the average wealth at death of decedents of age a (i.e. the average estate left by decedents of age a),[198] and $w_t(a)$ the average wealth of living individuals of age a. In case decedents of each age group are a representative sample of the living, i.e. under the uniform mortality assumption, then by definition $w_{dt}(a)=w_t(a)$. However, in practice, there exists extensive empirical evidence showing that differential mortality between the rich and the poor is quantitatively important and age-varying, i.e. $w_{dt}(a)$ is smaller than $w_t(a)$ and the gap varies with age. So it is critical to correct our raw wealth-at-death age profiles $w_{dt}(a)$

[197] See Table B6 below or Appendix C…

[198] Of course with annuitized wealth the average wealth of decedents of age a (right before death) and the average estate left by decedents of age a (right after death) could differ, and so would the age profiles. However the fraction of annuitized wealth is very small in France, and this can be ignored here. On this issue, see Appendix A, section A5.

(Table B3) in order to compute corrected wealth-of-the-living age profiles $w_t(a)$ (Table B4), before we can properly compute the μ_t ratio (Table B5). For now we present the raw wealth-at-death age profiles $w_{dt}(a)$ reported on Table B3.

Our raw data on the age profile of wealth-at-death $w_{dt}(a)$ comes from published estate tax tabulations and from estate tax micro-files (see below). Given that we are solely interested in the relative age profile of wealth (and not in the absolute wealth levels per se), and in order to ensure easy comparability of the profiles over time, we choose to express our data on age-wealth profiles in terms of $w_{dt}(a)/w_{dt}^{50\text{-}59}$ ratios, i.e. we express the average wealth of all age groups as a fraction of average wealth of decedents aged 50-to-59-year-old. E.g. in 2006 the average wealth of decedents aged 80-year-old and over was equal to 134% of the average wealth of decedents aged 50-to-59-year-old, the average wealth of decedents aged 70-to-79-year-old was equal to 106% of the average wealth of decedents aged 50-to-59-year-old, etc.

Our raw data suffers from a number of limitations. First, because published estate tax tabulations used age brackets 0-9, 10-19, 20-29,…, 70-79, 80 and over, and also because of the limited sample size of DMTG micro-files, we also used these decennial age brackets to estimate μ_t ratios, i.e. we did not attempt to estimate the shape of continuous $w_{dt}(a)$ age-wealth profiles.

Next, age-wealth profiles are available only for a limited number of years. For the 1977-2006 period, we used the age-wealth profiles coming from the DMTG micro files. Unfortunately the age data from the initial DMTG 1977 file is not usable,[199] so for the post-1977 period we only report on Table B3 raw age-wealth profiles for years 1984, 1987, 1994, 2000 and 2006.[200]

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[199] The DMTG 1977 age table published by Fouquet and Meron (1982, p.88) only reports the number of estate tax returns by age bracket (not the value of these estates), which is insufficient to reliably estimate the age-wealth profile.

[200] Note that the "raw" age-wealth profiles reported on Table B3 all include a correction for non-filers (see below). For 1984-2000, we report profiles coming from our own computations using DMTG micro-files. For 2006, we use the age table (indicating numbers and values of estates by age bracket) computed from the DMTG 2006 survey and published in Rapport CPO 2008, p.251. Note that this table actually reports average individual bequest shares ("parts successorales moyennes") by age-of-decedent brackets (rather than average estate). To the extent that the average number of successors rises with decedent age (which we observe for other years), this suggests that we under-estimate somewhat the steepness of the 2006 age-wealth profile (and hence the μ_t ratio in 2006).

For the 1902-1964 period, we can use the Finance Ministry tabulations broken down by age bracket. Similarly to the tables indicating the number and value of estates broken down by estate bracket, the Finance Ministry tables indicating the number and value of estates broken down by age bracket rely on the exhaustive set of all estate tax returns during a given year, so the resulting age-wealth profiles are extremely reliable. Unfortunately, while the estate-bracket tables were compiled and published by the French tax administration on a quasi-annual basis during the entire 1902-1964 period, the age-bracket tables were established solely during the 1943-1964 period.[201] Prior to 1943, age tables were compiled and published in 1906, 1908, 1928 and 1934, but they solely report the number (and not the value) of estates broken down by age bracket.[202] So for the pre-1943 period, the age-wealth profiles reported on Table B3 rely primarily the exhaustive micro files of all individual estate tax returns filed in Paris in 1807, 1812, 1817, etc., 1937 compiled every 5 years by Piketty, Postel-Vinay and Rosenthal (2006). This is certainly a very rich data base (we know full individual-level details about assets, decedents and heirs), and Paris alone was a pretty big part of France wealth-wise during the 19th century and the first half of the 20th century.[203] However there is no reason to believe that Paris age-wealth profile are representative of the whole of France, so we used several other sources in order to carefully convert our observed Paris profiles into the national profile estimates reported on Table B1 for the pre-1943 period.

First, thanks to the Finance Ministry 1943-1964 tabulations and to the DMTG 1977-2006 micro files, we do observe separately Paris and France-minus-Paris age-wealth profiles for the whole post-1943 period.[204] We find that the Paris profile has always been more strongly upward sloping than the national profile (relatively to the 50-to-59-year-old, the 60-to-69, 70-to-79 and 80-and-over-year-old groups have always been richer in Paris than in the rest of France), but that the gap is relatively constant over time, so that one can relatively easily estimate the national profile from the Paris profile and from the Paris share

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[201] Tables broken down by age brackets compiled during the 1943-1964 period were published in the same statistical bulletins (BSLC, BSMF and S&EF) as the tables broken down by estate brackets. See Piketty (2001, Appendix J, p.749) for exact references. Just like the estate-bracket tables, these age-bracket tables were also compiled and published at the department-level (not only at the national level). Between 1943 and 1954, the tax administration also compiled and published cross tabulations indicating the number and values of estates broken down by estate and age cross brackets.

[202] The 1906-1908-1928-1934 age tables were published in the same BSLC bulletins as other tables.

[203] Around 1890-1930, the Paris share in the aggregate national bequest flow was over 25%. Earlier in the 19th century, it was about 15%-20%. See Piketty et al (2006, table 1, p.240).

[204] The 1943-1964 age-bracket tables compiled by the tax administration were also compiled and published at the department-level (about 90 departments in France, including Paris) (see above). The 1977-2006 DMTG micro samples are too small in size to compute reliable department-level age tables, but sufficiently large to compare Paris profiles with France-minus-Paris profiles, which we did with the 1984-2000 micro-files.

in the national bequest flow, which we know from our Paris 1807-1937 micro files. In order to test the accuracy of this method, we used the Finance Ministry 1906-1908-1928-1934 national age tables, as well as the detailed cross tabulations by estate and age brackets (with numbers and values of estates) compiled for Paris and for the Manche department (relatively representative of rural France, according to later years) compiled in a special survey organized by the tax administration in 1931.[205] These tests show that our Paris-France extrapolation method is consistent. So we feel that the estimated national age-wealth profiles $w_{dt}(a)$ reported on Table B1 for the period going from the 1890s to the 1930s are as reliable as the post-1943 profiles.[206] For the earlier parts of the 19th century, there exists no Finance Ministry national age table, and one must be careful about the fact the relative importance and wealth structure of Paris vis-à-vis the rest of France changed extensively between the 1820s and the 1890s. We used the same data sources as those used by Piketty et al (2006) in order to convert Paris wealth concentration estimates into national wealth concentration estimates.[207] The resulting national age-wealth profiles reported on Table B3 for the 1820s to 1880s are certainly less precise than for the 1890-2006 period, and they ought to be improved. However we tried several alternative assumptions and found that these had little consequence for the 19th century levels and patterns of the μ_t ratio (the key parameter of interest in the context of this research), which appear to be reliable.[208]

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[205] The full results of this 1931 special survey were published by Danysz (1934).

[206] The raw Paris profiles (see Piketty et al (2006, table 5, p.253)) display the same evolution as the national profiles reported on Table B3 for the 1890-1930 period, except that they are always more steeply upward sloping. E.g. around 1890-1910, we estimate that w_{dt}^{80-89} was about 200%-250% of w_{dt}^{50-59} for the whole of France, vs as much as 350%-400% in Paris; in the 1920s-1930s, we estimate that w_{dt}^{80-89} was about 150%-180% of w_{dt}^{50-59} for the whole of France, vs 200%-300% in Paris. The steeper Paris profiles reflect the extremely high level of wealth concentration prevailing in Paris at that time, and the fact that most top wealth holders were very old (see Piketty et al (2006)).

[207] Namely, housing tax tabulations (which are available for Paris and France throughout the 19th century) and the TRA survey (which includes representative samples of estate tax returns for the all of France starting in the 1820s). Note that the survey suffers from insufficient sample size to properly measure top estates, but is reliable for over 90% of the population; it offers an imperfect but useful source to evaluate how the gap between the Paris wealth structure and the France-minus-Paris wealth structure has evolved over the 19th century. See Piketty et al (2006, pp.248-249).

[208] In particular, the fact that the age-wealth profile gradually became more steeply upward sloping between the 1820s-1850s and the 1870s-1880s (with a corresponding rise in the μ_t ratio) seems to be extremely robust. Note that according to our computations the age-wealth profile was actually more steeply rising in the whole of France than in Paris in the early 19th century (while the opposite occurs in the late 19th century and in the 20th century). E.g. the raw Paris profiles even show hump shaped profiles in 1817 and 1827 (with w_{dt}^{70-79} and w_{dt}^{80-89} around 60%-90% of w_{dt}^{50-59} 1817 and 1827; the Paris profiles then take the standard upward sloping shape from the 1830s onwards; see Piketty et al (2006, table 5, p.253)), while our national estimates show upward-sloping profiles from the 1820s towards. This is consistent with the view that old, wealthy Parisians were hit by strong negative shocks during the Revolutionary years, while in the rest of France the old and wealthy were hit by less strong shocks. However, given the data limitations we face, it is certainly possible that we over-estimate somewhat the steepness of the national age-wealth profile in the 1820s-1830s (i.e. that the old of the 1820s-1830s were somewhat poorer than what it suggested on Table B3).

Finally, note that another limitation of our raw age-wealth data throughout the 1820-2006 period is that by construction we only observe the wealth of estate tax filers. Since the proportion of decedents filling a tax return (whose heirs filled a tax return) varies with age (generally it rises with age, especially among the younger age groups), it is critical to correct for this, otherwise the age-wealth profiles could be severely biased.[209] We proceed as follows. In our raw estate tax data – both in the Finance Ministry 1943-1964 tabulations and in the 1807-1937 Paris micro-files and 1977-2006 DMTG micro-files – we observe the number of estate tax returns $N_{dt}^{f}(a)$ filled for decedents of age group a, as well as the corresponding total estate value $W_{dt}^{f}(a)$ and average reported estate $w_{dt}^{f}(a) = W_{dt}^{f}(a)/N_{dt}^{f}(a)$. We also know from basic demographic data (see Appendix C) the total number of decedents of age group a $N_{dt}(a)$, from which we know the number of non-filers $N_{dt}^{nf}(a) = N_{dt}(a) – N_{dt}^{f}(a)$, and the proportion of filers $n_{dt}^{f}(a) = N_{dt}^{f}(a)/N_{dt}(a)$. What we do not directly observe is the average wealth of non-filers $w_{dt}^{f}(a)$. In the same way as for the computation of the non-filers correction to the aggregate fiscal bequest flow (see section B.1 above), we make simple assumptions about the value of the wealth ratio $z_{dt}^{nf} = w_{dt}^{nf}(a)/w_{dt}^{f}(a)$. I.e. we assume that z_{dt}^{nf} = 5% for years 1820-1964 and z_{dt}^{nf} = 15% for years 1977-2006. We then compute average wealth $w_{dt}(a)$ of all age-a decedents (filers and non-filers) by applying the following equation:

$$w_{dt}(a) = [\ n_{dt}^{f}(a) + (1-n_{dt}^{f}(a))\ z_{dt}^{nf}\ ]\ w_{dt}^{f}(a) \qquad \text{(B.2)}$$

It is apparent from equation (B.2) that the exact value of z_{dt}^{nf} has a limited impact on the overall age-wealth profile $w_{dt}(a)$, and even less on the resulting μ_t ratio.[210] The dominant effect comes from the filers fraction $n_{dt}^{f}(a)$. Typically, when the aggregate fraction n_{dt}^{f20+} of tax filers among adult decedents is about 50%, the observed age-level fraction of tax filers $n_{dt}^{f}(a)$ can be as large as 60%-70% for the older groups (60-69, 70-79 and 80-and-over), and as low as 30%-40 for the younger groups (20-29, 30-39 and 40-49). The pattern varies over time, and generally tends to reinforce the effects of the average reported

[209] In the data reported in Piketty et al (2006, table 5, p.253), we forgot to make this correction for year 1994. As a consequence, the reported national profile does not look as upward sloping as it is really (the reported profile even looked – wrongly – slightly hump shaped at high ages). Also, for year 1947, we wrongly reported the Paris profile (in spite of the fact that we refer to it as the national profile), which was at that time slightly upward sloping (while the national profile was hump shaped). For consistent national age-wealth profiles, one should use the new, revised estimates reported on Table B3 of the present paper rather than the estimates reported in Piketty et al (2006) for 1947 and 1994 (the 1807-1902 Paris profiles reported in this paper are correct, though).

[210] We performed several alternative computations with z_{dt}^{nf} varying in the 5%-15% range (and varying with age), and the resulting impact on the level and pattern of μ_t ratios was less than 1%.

wealth pattern.[211] Note that for very young decedents (0-9 and 10-19), the fraction of tax filers is very small (less than 5% in the postwar period, and generally less than 10% in the earlier periods): it is quite rare that children die, and it is even rarer that they die after having already inherited an estate at such an early age; so most of the time for children decedents there is no estate to report to the tax administration. As a consequence, the average wealth estimates for children decedents rely on a limited number of observations and should be viewed as approximate (they are very small anyway).[212]

Table B4: Corrected age-wealth profile $w_t(a)$, 1820-2006

On Table B4 we report our corrected age-wealth-of-the-living profiles $w_t(a)$. These were obtained from the raw age-wealth-at-death profiles $w_{dt}(a)$ reported on Table B3, by applying the differential mortality parameters indicated on Table B4.[213]

On the basis of available empirical evidence (see below), we model differential mortality as follows. For each age group a, we assume that the poor (defined as the bottom half of the wealth distribution for this age group) have a higher mortality rate than the rich (defined as the upper half of the wealth distribution for this age group). That is, we note $m_t^P(a)$ the mortality rate of the poor, $m_t^R(a)$ the mortality rate of the rich, and $\delta_t(a) = m_t^P(a)/m_t^R(a) > 1$ the differential mortality ratio. By construction, $(m_t^P(a)+m_t^R(a))/2 = m_t(a)$, where $m_t(a)=N_{dt}(a)/N_t(a)$ is the mortality rate of age group a during year t, $N_{dt}(a)$ is the number of decedents of age a, and $N_t(a)$ is the number of living individuals of age a. So we have:

$$m_t^P(a)/m_t(a) = 2\delta_t(a)/(1+\delta_t(a)) \quad (> 1) \qquad (B.3)$$
$$m_t^R(a)/m_t(a) = 2/(1+\delta_t(a)) \quad (< 1) \qquad (B.4)$$

[211] E.g. in the immediate postwar period, when the pattern of average wealth $w_{dt}^f(a)$ reported to the tax administration is hump shaped, the pattern of tax filers fractions $n_{dt}^f(a)$ is also hump shaped (the very old more often with wealth so small that it does not get reported), thereby making the pattern of $w_{dt}(a)$ (which we report on Table B3) even more hump-shaped.

[212] Because $N_{dt}^f(a)$ is usually extremely small for age groups 0-9 and 10-19, the corresponding average wealth $w_{dt}^f(a)$ can be very volatile across years (especially with the DMTG samples). So the $w_{dt}(a)$ estimates reported on Table B3 for age groups 0-9 and 10-19 are based upon approximate moving averages (e.g. for 1984-2006 we report the averages obtained for all years 1984-2006). We checked in the simulated model that these young-age $w_{dt}(a)$ estimates were consistent with the observed patterns of parental age at death and children age at parental death over the entire 1820-2006 period; they are consistent, in the sense that the relative wealth that we attribute to children for various time periods (e.g. during the 19th century) is approximately equal to what they should own according to the simulation model (see Appendix C and D).

[213] See formulas in the excel file.

We also note $sh_t^P(a)$ the poor's share in total wealth of age group a at time t. By construction, the average wealth of the poor $w_t^P(a)$ is equal to $2sh_t^P(a)w_t(a)$, and the average wealth of the rich $w_t^R(a)$ is equal to $2(1\text{-}sh_t^P(a))w_t(a)$. For a given age group the ratio $w_{dt}(a)/w_t(a)$ between the average wealth of decedents and average wealth of the living can then be computed as follows:

$$w_{dt}(a)/w_t(a) = [2\ sh_t^P(a)\ m_t^P(a) + 2(1\text{-}sh_t^P(a))\ m_t^R(a)]\ /\ [m_t^P(a) + m_t^R(a)]$$

I.e.:

$$w_{dt}(a)/w_t(a) = m_t^P(a)/m_t(a)\ sh_t^P(a) + m_t^R(a)/m_t(a)\ (1\text{-}sh_t^P(a)) \qquad \text{(B.5)}$$

Our preferred differential mortality parameters are reported on the upper part of Table B4. That is, we assume that throughout the period of study the differential mortality ratio $\delta_t(a)$ is equal to 200% for age groups 0-9 to 40-49 year-old, and then declines to 180% for 50-59 year-old group, 150% for 60-69 year-old group, 130% for 70-79 year-old and 110% for 80-year-old and over. I.e. the mortality rate of the poor is twice as large as that of the rich below 50-year-old, and then the gap slowly declines towards 10% for the very old. Next, for simplicity we assume that throughout the period of study the wealth share of the poor is equal to $sh_t^P(a)$=10% for all age groups.[214]

Applying these parameters and the above formulas, we obtained ratios $w_{dt}(a)/w_t(a)$ ratios equal to 73% below 50-year-old, and then rising until 96% for the 80-year-old and over (see Table B4). I.e. because the poor are over-represented among decedents (especially among young-age decedents), the average wealth of decedents at any given age is below the average wealth of the living (and especially so at young age). Alternatively, one can see that the $w_t(a)/w_{dt}(a)$ ratios are above 100% for all age groups and declining with age, from 136% below 50-year-old to 104% for the 80-year-old and over (see Table B4). I.e. if one observes the average wealth of decedents of a given age group, then one needs to upgrade this value by a factor ranging from 136% to 104% (depending on age) in order to compute the average wealth of the living for this given age group.

[214] As was already discussed (see section B1 above), the bottom 50% wealth share probably rose somewhat in the long run (say, from less than 5% in the 19th century early 20th century to about 5%-10% today), and it also rises slightly with age (within the 5%-15% range). These are relatively small variations, however, on which we do not have very good data, so we thought it was clearer to make this simplifying assumption ($s_t^P(a)$=10% for all years and age groups). We checked that the resulting μ_t ratio estimates are very robust with respect changes in the assumed patterns $s_t^P(a)$. E.g. if one assumes that $s_t^P(a)$ rises over time and/or with age (within the 5%-15% range), then the pattern of μ_t ratios hardly changes, as one can check by changing the parameters in the corresponding excel file.

Multiplying this profile of $w_t(a)/w_{dt}(a)$ ratios by the raw age-wealth-at-death profiles $w_{dt}(a)$ reported on Table B3 yields the corrected age-wealth profiles $w_t(a)$ reported on Table B4. Unsurprisingly, the corrected profiles look less strongly upward-sloping (or more hump-shaped, in the immediate postwar period) than the raw profiles: the differential mortality correction leads to increase the wealth of the 50-to-59-year-old relative to the 80-to-89-year-old (because the poor are more massively over-represented in the former group than in the latter).

The way we model differential mortality is relatively standard in the literature,[215] and is consistent with the best available empirical evidence. In particular, Attanasio and Hoynes (2000) compute mortality rates broken down by wealth quartiles and by age groups. They find that bottom quartile mortality rates are significantly larger than those of other quartiles, and that the mortality ratio is a strongly declining function of age (i.e. differential mortality is larger at low age). The differential mortality parameters used in our computations are directly taken from this paper.[216] We also tried several alternative formulations (e.g. mortality differentials defined at the wealth quartile level, rather than at the bottom half vs upper half level), but we found that this made very little difference in terms of final μ_t ratios estimates, and decided that the extra complexity associated to there more sophisticated formulations was not really justified given our purposes in this research.

There is also an issue as to whether the quantitative importance of differential mortality has changed significantly in the long run. Here we simply assumed constant differential mortality parameters over the entire 1820-2006 period. In order to test for the consistency of this assumption, we computed the average age at the death of the poor and the rich predicted by our differential mortality parameters, given observed average mortality rates by cohort since 1820. We found that the predicted age-at-death gap between rich and poor was relatively stable at about 4-5 years over the 1820-2006 period, and that the predicted

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[215] See e.g. Kopczuk and Saez (2004), who carefully review the evidence, and adopt the following age profile of differential mortality: they assume that the ratio between the mortality rate of the rich and the aggregate mortality rate is equal to about 60%-70% below age 50, up to about 80%-90% at age 70 and 100% above age 90 (see Kopczuk-Saez, 2004, working paper version, pp.37-39 and Figure A4). This is very close to the profile adopted here (see Table B4).

[216] See Attanasio and Hoynes (2000, p.9, table 4). They find that the ratio between the bottom quartile mortality rate and the other three quartiles mortality rate can be as high as 200%-300% at low age (below 50-60), and then declines towards 150% at higher ages (70-80). Within the top three quartiles, differential mortality seems to be more limited (gaps are usually not significant). If one computes the ratio between the bottom half and upper half mortality rates from these Attanasio-Hoynes results, one finds the pattern reported on Table B4 (i.e. from 200% below age 50 to 110%-130% at age 70-80).

gap between the rich and the average was relatively stable at about 2-3 years, both with a slight downward time trend.[217] Using estate tax data, we can compute the average age at death of tax filers (i.e. approximately the upper half of the wealth distribution) over the 1906-2006 period, and compare it the average age of decedents. We again find a relatively stable rich vs average gap of about 2 years over the past century.[218] We conclude from this that our simple assumption of stable differential mortality parameters is acceptable as a first approximation. If anything, we might slightly overstate differential mortality, especially in the recent period, which would imply that our μ_t ratio is slightly underestimated for the recent decades.

Table B5: Computation of μ_t and μ_t* ratios in France, 1820-2006

On Table B5 we report our estimates for the ratio μ_t. By definition, μ_t is the ratio between average wealth of decedents and average wealth of the living, so it can easily be computing by weighting by the relevant population the age-wealth-at-death profiles $w_{dt}(a)$ reported on Table B3 and the age-wealth-of-the-living profiles $w_t(a)$ reported on Table B4, and by dividing one by the other. As we explained in the working paper (section 3.1), we find it more convenient to exclude children from our basic accounting equation relating the aggregate bequest flow to aggregate wealth, which we wrote as follows:

$$B_t = \mu_t \ m_t \ W_t \qquad \text{(B.6)}$$

With: B_t = annual bequest flow

W_t = aggregate private wealth

m_t = adult mortality rate = N_{dt}^{20+}/N_t^{20+} = $[\sum_{a\geq20} N_{dt}(a)] / [\sum_{a\geq20} N_t(a)]$

We chose to do so because children usually own very little wealth (except in the few cases where they have already inherited). The advantage of this formulation is that this makes both the levels and evolutions of the coefficients μ_t and m_t easier to interpret. In particular this allows us to abstract from the large historical variations in infant mortality (which was

[217] See appendix do-file domortadiff.txt.

[218] See Appendix C, Table C7, col. (1)-(3). The age-at-death gap between the rich and the average seems to be somewhat lower in the mid 20th century (as little as 0.5-1 years) than at the beginning and at the end of the century (2-2.5 years). Note however the average ages for tax filers were computed using the Finance Ministry tables with decennial age brackets, and are therefore not very precise. Note also the abnormally high age gap of 4 years in 1943: this is clearly due to the abnormally high number of relatively young age decedents in this year (most of which did not file a tax return). All average ages reported on Table C7 of course solely refer to adult decedents (20-year-old and over).

much higher in the 19th century than it is today). However strictly speaking children wealth is not exactly equal to zero (because children sometime inherit), so in order to ensure the full consistency of the accounting equation (B.6) we need to introduce a small correction factor cf_t in the definition of the μ_t ratio so as to correct for the existence of positive children wealth. Taking children into account, the accounting equation is actually the following:

$$B_t = \mu_t^{0+} m_t^{0+} W_t \qquad (B.7)$$

Where $m_t^{0+} = N_{dt}^{0+}/N_t^{0+} = [\sum_{a\geq 0} N_{dt}(a)] / [\sum_{a\geq 0} N_t(a)]$ is the average mortality rate for the entire population (including children), and μ_t^{0+} is the ratio between average wealth of the deceased and average wealth of the living computed for the entire population (including children), i.e.:

$$\mu_t^{0+} = w_{dt}^{0+}/w_t^{0+} \qquad (B.8)$$

With: $w_{dt}^{0+} = (\sum_{a\geq 0} N_{dt}(a)w_{dt}(a))/N_{dt}^{0+}$ = average wealth of all decedents (incl. children)
$w_t^{0+} = (\sum_{a\geq 0} N_t(a)w_t(a))/N_t^{0+}$ = average wealth of all living individuals (incl. children)

This differs from the μ_t^{20+} ratio defined over adults (20-year-old and over):

$$\mu_t^{20+} = w_{dt}^{20+}/w_t^{20+} \qquad (B.9)$$

With: $w_{dt}^{20+} = (\sum_{a\geq 20} N_{dt}(a)w_{dt}(a))/N_{dt}^{20+}$ = average wealth of adult decedents
$w_t^{20+} = (\sum_{a\geq 20} N_t(a)w_t(a))/N_t^{20+}$ = average wealth of adult living individuals

By combining equations (B.6) and (B.7), one obtains a simple formula for the children-wealth correction factor cf_t:

$$\mu_t = cf_t \ \mu_t^{20+} \qquad (B.10)$$

$$\text{With:} \qquad cf_t = [W_t^{20+}/W_t] / [B_t^{20+}/B_t] \qquad (B.11)$$

With:
W_t^{20+} = total wealth of adult living individuals = $\sum_{a\geq 20} N_t(a)w_t(a)$
W_t = total wealth of all living individuals (incl. children) = $\sum_{a\geq 0} N_t(a)w_t(a)$

B_t^{20+} = total bequests left by adult decedents = $\sum_{a\geq 20} N_{dt}(a)w_{dt}(a)$

B_t = total bequests left by all decedents (incl. children) = $\sum_{a\geq 0} N_{dt}(a)w_{dt}(a)$

I.e. the correcting factor cf_t is equal to the ratio between the share of living individuals aged 20-year-old-and-over in aggregate private wealth W_t^{20+}/W_t and the share of decedents aged 20-year-old-and-over in the aggregate bequest flow B_t^{20+}/B_t. Of course if children own no wealth at all, then both shares are equal to 100%, the correcting factor cf_t is also equal to 100%, and the μ_t ratio is simply equal to the μ_t^{20+} ratio defined over the adult population: i.e. there is no need for a correction factor.

Applying the equations above to the age-wealth-at-death profiles $w_{dt}(a)$ and the age-wealth-of-the-living profiles $w_t(a)$ reported on Tables B3-B4, and to the demographic series $N_{dt}(a)$ and $N_t(a)$ provided in Appendix C, we obtain the series for the various ratios reported on Table B5 (col. (6)-(12)).[219] As one can see, adult shares are not exactly equal to 100%, but they are very close. According to our computations, the share owned by adults in the aggregate wealth of the living W_t^{20+}/W_t gradually grew from about 95% in the 19th century to about 99% in the early 21st century (see Table B5, col. (10)). The fact that the children wealth share declines over time reflects the fact that children successors have become rarer over time. According to our computations, the share left by adults in aggregate bequest flow B_t^{20+}/B_t also grew in the long run, from about 98% in the 19th century to almost 100% in the 20th century (see Table B5, col. (9)). The fact that the latter is always somewhat smaller than the former reflects the fact that children leave bequests even more rarely than they receive bequests. Consequently, the correcting factor cf_t is always slightly smaller than 100%. As expected, it is however very close to 100%: about 97% in the 19th century, and about 98%-99% during the 20th century (see Table B5, col. (8)). The children wealth correction factor is virtually irrelevant for our aggregate series.[220]

Multiplying the adult ratio μ_t^{20+} (col. (7)) by the children correction factor cf_t (col.(8)), we get our children-corrected ratio μ_t (col.(11)). We find that μ_t was about 120%-140% from 1820 to 1913, then dropped to less than 90% in the immediate postwar period, then gradually increased to over 120% in the 2000s. Multiplying μ_t by $1+v_t$, where v_t is the gift-bequest ratio (see section B.1 above), we get our gift-corrected ratio $\mu_t^*=(1+v_t)\mu_t$ (col. (12)). We find

[219] See excel file for formulas.

[220] An alternative strategy would have been to forget about children wealth altogether (and to attribute to adults the small share of national wealth owned by children in the real world). However in our simulations we do model explicitly the full age structure of decedents and heirs (see Appendices C and D), and it would have been somewhat arbitrary to truncate distributions of heirs age at 20.

that μ_t* was about 150%-160% from 1820 to 1913, then dropped to little more than 100% in the immediate postwar period, then gradually increased to over 220% in the 2000s. We use these μ_t* series to compute the economic inheritance flow in Appendix A (section A.2).

We also report on Table B5 the ratio between the average wealth of living individuals aged 50-to-59-year-old w_t^{50-59} and the average wealth of all adults w_t^{20+} (see col. (13). Note that the average wealth w_t^{20+} is slightly smaller than per adult wealth w_t, which we defined in Appendix A (Tables A1-A2, col. (8)) as aggregate private wealth W_t divided by the number of adults N_t^{20+}: $w_t^{20+} = w_t \times [W_t^{20+}/W_t]$. On Table B5 we also report the ratio between w_t^{50-59} and per adult wealth w_t (see col. (14)). We use it in the simulated model (see Appendix D).

Finally, we also report on Table B5 (col.(1)-(5)) the estimates for the μ_t ratios that one would obtain under uniform mortality assumptions, i.e. ignoring differential mortality. Col. (1)-(5) of Table B5 were obtained by applying the same formulas as above, but by assuming that the age-wealth-of-the living profile $w_t(a)$ is the same as the age-wealth-at-death profile $w_{dt}(a)$.[221] As one can see, differential mortality has a strong impact on estimated μ_t ratios. Under uniform mortality assumptions, the μ_t^{20+} ratio would be as large as 160%-180% from 1820 to 1913 (instead of 120%-130% under differential mortality assumptions), and would be over 150% in the 2000s (instead of over 120%) (see Table B5, col. (2) vs col.(7)). Throughout the period 1820-2006, the μ_t ratio would be about 25%-30% larger under uniform mortality assumptions. I.e. according to our computations, differential mortality (the fact that the rich dies less often than the poor) makes aggregate bequest flows about 25%-30% lower than they would otherwise be.

[221] See formulas in excel file.

Appendix C: Demographic Data

In addition to national accounts data and estate tax data, this research also relies intensively on demographic data. First, at various points we need a relatively complete demographic file with annual numbers of living individuals $N_t(a)$ and decedents $N_{dt}(a)$ by exact age and cohort, which we constructed using available historical population tables for the 1820-2009 period and existing population projections for the 2010-2100 period. The way we assembled this basic demographic data base is described in section C1. Next, in order to simulate the age-level dynamics of wealth accumulation and inheritance, we also need relatively complete data on the age structure of decedents, successors, donors and donees. The way we constructed this supplementary data base is described in section C2.

C.1. Basic demographic data (population tables)

We report on Tables C1-C4 a number of demographic series which we use repeatedly in Appendix A and B.[222] These series are directly extracted from our basic demographic data base, which takes the form of a Stata format data base, which we describe below.

Notations

We use the same demographic notations as in Appendices A and B:

N_t = total living population in France on 1/1 of year t (t=1820,1821,…,2100).
By convention population is always estimated on 1/1 (January 1st) of year t.

$n_t = N_{t+1}/N_t - 1$ = population growth rate during year t

N_t can be decomposed by birth cohort x:

$$N_t = \sum_{x<t} N_t^x \qquad \text{(C.1)}$$

With: N_t^x = total living population on 1/1 of year t and born during year x<t

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[222] In the same way as in Appendix A and B, the decennial averages reported on Table C2 refer to years 1820-1829 for "1820", 1830-1939 for "1830". The only exceptions are the 1910s (we took the average of years 1910-1913) and the 1940s (we took the average of years 1946-1949): excluding war years clarifies long run evolutions of demographic ratios (particularly mortality rates). Of course in all annual series and in the simulated annual models, we use all yearly data, including war years.

Since N_t is measured on 1/1 of each year t, then by convention $N_t^t = 0$, i.e. nobody is born during year t and alive on 1/1 of year t.

Alternatively, N_t can be decomposed by age group a:

$$N_t = \sum_{a\geq 0} N_t(a) \qquad \text{(C.2)}$$

With: $N_t(a)$ = total living population aged a-year-old on 1/1 of year t.

By convention, we measure age on 1/1 of each year, so that a(t,x) = t-x-1
With: a(t,x) = age on 1/1 of year t of individuals born during year x
Alternatively: x(t,a) = t-a-1

E.g. in 1900, the individuals aged 0-year-old are the individuals born during year 1899, the individuals aged 1-year-old are the individuals born during year 1898, etc., and the individuals aged 99-year-old are the individuals born during year 1800.

Due to data limitations, the age distribution is censored at a=99: the age of all individuals with age a≥99 is set to a=99; the birth cohort all individuals with birth cohort x≤t-100 is set to x=t-100 (see below).

We also note $N_t^{20+} = \sum_{a\geq 20} N_t(a)$ the total number of living individuals aged 20-year-old and over on 1/1 of year t.

We use similar notations for decedents:

N_{dt} = total number of decedents in France during year t

N_{dt} can be decomposed by birth cohort x or by age group a:

$$N_{dt} = \sum_{x\leq t} N_{dt}^x = \sum_{a\geq -1} N_{dt}(a) \qquad \text{(C.3)}$$

With: N_{dt}^x = number of individuals born during year x and deceased during year t

$N_{dt}(a)$ = number of individuals aged a-year-old on 1/1 of year t (a=t-x-1) and deceased during year t

$N_{dt}^{20+} = \sum_{a\geq 20} N_{dt}(a)$ = total number of individuals aged 20-year-old on 1/1 of year t (a=t-x-1) and deceased during year t

Note that $N_{dt}^{t} = N_{dt}(-1) > 0$: these are the individuals born during year t and deceased during year t (such individuals are therefore not counted in populations N_t or N_{t+1}).

We can then define mortality rates:

$$m_t = N_{dt}^{20+}/N_t^{20+} = \sum_{a\geq 20} N_{dt}(a) \,/\, \sum_{a\geq 20} N_t(a) \qquad \text{(C.4)}$$

(= aggregate mortality rate of individuals aged 20-year-old and above during year t)

$$m_t^{0+} = N_{dt}/N_t = \sum_{a\geq -1} N_{dt}(a) \,/\, \sum_{a\geq 0} N_t(a) \qquad \text{(C.5)}$$

(= aggregate mortality rate of the entire population (including the population aged 0 to 19-year-old) during year t)

$$m_t^{x} = N_{dt}^{x}/N_t^{x} \qquad \text{(C.6)}$$

(= mortality rate of birth cohort x during year t)

$$m_t(a) = N_{dt}(a)/N_t(a) \qquad \text{(C.7)}$$

(= mortality rate of individuals aged a-year-old during year t)

We use similar notations for birth and migrants.

N_{bt} = total number of births in France during year t

$f_t = N_{bt}/N_t$ = fertility rate in France during year t

N_{it} = net number of immigrants entering France during year t

$N_{it} = \sum_{x\leq t} N_{it}^{x} = \sum_{a\geq -1} N_{it}(a)$

With: N_{it}^{x} = net number of immigrants born during year x and entering during year t

$N_{it}(a)$ = net number of immigrants aged a-year-old on 1/1 of year t (a=t-x-1) and entering during year t

$i_t = N_{it}/N_t$ = net migration rate during year t

$i_t^x = N_{it}^x/N_t^x$ = net migration rate of birth cohort x during year t

$i_t(a) = N_{it}(a)/N_t(a)$ = net migration rate of individuals aged a-year-old during year t

By construction, our data base is dynamically consistent:

$$N_{t+1} = N_t + N_{bt} - N_{dt} + N_{it} \qquad \text{(C.8)}$$

I.e. $n_t = f_t - m_t + i_t$

(population growth rate = fertility rate – mortality rate + net migration rate)

Similarly, by birth cohort:

For $t-99<x<t$, $N_{t+1}^x = N_t^x - N_{dt}^x + N_{it}^x$

For $x=t$, $N_{t+1}^x = N_{bt} - N_{dt}^t + N_{it}^x$

For $x=t-99$, $N_{t+1}^x = N_t^x + N_t^{x-1} - N_{dt}^x - N_{dt}^{x-1} + N_{it}^x + N_{it}^{x-1}$

Or, alternatively, by age group:

For $0<a<99$, $N_{t+1}(a) = N_t(a-1) - N_{dt}(a-1) + N_{it}(a-1)$

For $a=0$, $N_{t+1}(a)= N_{bt} - N_{dt}(a-1) + N_{it}(a-1)$

For $a=99$, $N_{t+1}(a) = N_t(a-1) + N_t(a) - N_{dt}(a-1) - N_{dt}(a) + N_{it}(a-1) + N_{it}(a)$

Raw data sources for 1900-2050

The raw data for our demographic data base comes primarily from Insee official population tables for the 1900-2007 period and Insee official population projections for the 2008-2050 period. We then extended this 1900-2050 data base to the past (down to 1820) and to the future (up to 2100) (see below).

Current population tables are published every year by Insee.[223] Complete retrospective 1900-2007 population tables for living individuals $N_t(a)$,[224] for decedents $N_{dt}(a)$,[225] and for births N_{bt}, are easily available on-line.[226]

[223] See e.g. "La situation démographique en 2007" (Insee-Résultats août 2008, Société n°84, C. Beaumel and M. Vatan, www.insee.fr). The relevant table for the age structure of the living population ($N_t = \sum_{x<t} N_t^x =$

Note that all these population and demographic series refer to mainland France (i.e. excluding overseas territories), and more specifically to the historical territory of mainland France, i.e. the current territory for the 1820-1870, 1920-1938 and 1946-2007 periods, and the current territory minus Alsace-Moselle for the 1871-1919 and 1939-1945 periods. This territorial change explains the large population growth in 1920 and 1946, and the large fall in 1871 and 1939 (see Table C1). Note also that migration figures are to a large extent residual estimates, and should be used with caution, especially during the war years.[227]

Overall, the only missing data for the 1901-2007 period is the data on the age structure of the living population for the 1915-1919 period. We completed this missing data by using the data on the age structure of the living population for 1914, the age structure of decedents for the 1914-1918 period, the number of births for the 1914-1918, and by assuming zero migration during the 1914-1918 period. In effect, this is assuming that all cumulated migrations during the 1914-1919 period occurred in year 1919. This approximation has no impact on subsequent years.

$\sum_{a\geq-1} N_{dt}(a)$) is “Tableau 6 : Population totale par sexe, âge et état matrimonial au 1er janvier …”. The relevant table for the the age structure of decedents ($N_{dt} = \sum_{x\leq t} N_{dt}^{x} = \sum_{a\geq-1} N_{dt}(a)$) is “Tableau 71: Décès par sexe, année de naissance, âge et état matrimonial du décédé“.

[224] The full set of retrospective tables for the living population covering the 1901-2007 period (with the exception of years 1915-1919) is available on-line at www.insee.fr.

[225] The tables for decedents are also available on-line at www.insee.fr, but only since 2002. For previous years we used the Vallin-Mesle data base on decedents. This data base is available on-line at www.ined.fr, and is fully consistent with the more recent Insee tables. See J. Vallin and F. Mesle, “Décès par âge et par génération, de 1899 à 1997 » (www.ined.fr). One additional advantage of the Vallin-Mesle data base is that they attempt to include all decedents during war years (while Insee official estimates only refer to civilian decedents). For the 1997-2002 period we used the decedents tables published in the paper publications “La situation démographique en …”.

[226] See "Un siècle de fécondité française" (INSEE-Résultats juin 2007, Société n°66, www.insee.fr). This publication includes updated series from F. Daguet, "Un siècle de fécondité française, Caractéristiques et évolution de la fécondité de 1901 à 1999", 2002, INSEE Résultats, Société n° 8, 2002. Table 35 of this publication provides 1901-2007 series on the number of births broken down by gender. However these series refer to the current territory (as opposed to the historical territory) for the 1901-1919 and 1939-1945 periods. So in order to make these series consistent with the series on living population and decedents, we did the following. For 1901-1913 and 1939-1945, we used the historical-territory total number of births reported on table 1.1B, and we assumed that the gender decomposition of births reported on table 35 for the current territory also applied to the historical territory. For 1914-1919, the total number of births reported on table 1.1B was not usable (it refers to an even smaller territory), so we had to estimate the total number of births assuming that the 94.4% (historical territory)/(current territory) ratio observed in 1913 also applied to 1914-1919 (in practice, this ratio is pretty stable around 94%-95%); we also assumed that the gender decomposition of births reported on table 35 for the current territory applied to the historical territory.

[227] Like most statistical institutes, Insee independently computes living population estimates from censuses and household surveys, while decedents and birth estimates come from administrative, etat-civil data ; although direct sources on migrations are also used for control purposes, aggregate migration figures are basically obtained by differentiating these two sources.

Regarding the 2008-2050 period, we used the latest official population projections, which were published by INSEE in 2007, mostly for pension planning purposes.[228] These projections include a full set of annual tables on the age structure of the living population and decedents and on births. This set of projected population tables for 2008-2050 is also available on-line,[229] and is fully consistent with the pre-2008 demographic series.[230]

Finally, because the raw data uses varying top age censoring over the 1900-2050 period (from 100 to 120), we recoded all the series using a uniform maximum age a=99. I.e. the age of all individuals with age a≥99 was set to a=99, the birth cohort all individuals with birth cohort x≤t-100 is set to x=t-100, and we assumed zero migration for this age group. In effect, the age group (a=99, x=t-100) is a terminal point where individuals can spend several years. Given the relatively small numbers of individuals involved, and the very high annual mortality rate for this category (from about 50% in the early 20th century to about 30% in the early 21st century), this approximation is innocuous for our purposes.

Resulting data base 1900-2050 and extension to 1820-2100

The data base resulting from official population tables and projections takes the form of a rectangular Stata file pop19002050.dta. At various points in this research, and particularly for the simulations, we also need population projections running until year 2100, and a population data base starting in 1820. We used an extended population file pop18202100.dta, which was obtained from file pop19002050.dta by assuming that aggregate fertility and age-level mortality rates and migration rates remain the same during the 2050-2100 as those projected for 2050, and by using available mortality tables and birth data prior to 1900. All details are provided in the do-file dopopulation18202100.txt, which transforms the basic population file pop19002050.dta into the extended population file pop18202100.dta. According to these future projections, French population will be almost stationary after 2050, with total population rising from 70.2 millions in 2050 to 72.2 millions in 2100. Our 1820-1900 data replicates by construction observed total population

[228] See "Projections de population active pour la France métropolitaine 2006-2050" (Insee-Résultats avril 2007, Société n°63, www.insee.fr). Note that these projections take into account the higher-than-expected fertility figures observed since 2001, and were therefore revised upwards as compared to the previous population projections published in 2001.
[229] See www.insee.fr and www.ined.fr.
[230] More precisely : the 2008-2050 projections published in 2007 used 2005 as a base year and underestimated somewhat base year population, so that total living population on 1/1 2007 is equal to 61.365949 millions according to the projections series, vs 61.538322 millions according to the latest Insee estimates. In order to ensure full continuity, we therefore multiplied all 2008-2050 projected series by a uniform factor equal to 1.00281=61.535322/61.365949 (i.e. projections series were upgraded by 0.281%).

during the 19th century (30.3 millions in 1820, 35.4 millions in 1850, vs 38.5 millions in 1900) and observed trends in mortality rates by age group. However, because raw mortality data is not available at the age level prior to 1900 (we had to use raw mortality rates for 5-year-wide age groups), our demographic data base is less precise for the pre-1900 period, especially for the early cohorts born during the 18th century (annual fertility data starts in 1800 in France, so we do not know very precisely the size of earlier cohorts).[231]

We use the population file pop18202100.dta as background demographic data at various points in this paper. In particular, by applying the do-file dotableC1.txt to this data base, one can obtain the summary statistics on population growth and mortality rates reported on Table C1. By applying the do-files dotableC3-C4.txt, one can obtain the summary statistics on the age structure of living individuals and of decedents reported on Tables C3 and C4.

The population file pop19002050.dta contains 15,251 observations (151 years x 101 cohorts = 15,250 year x cohort pairs) and 15 variables.

The population file pop18202100.dta contains 25,351 observations (281 years x 101 cohorts = 28,381 year x cohort pairs) and 15 variables.

The list of 15 variables is the following:

year = t = 1900, 1901, …, 2050 = year of observation

cohort = x = year of birth of the cohort under consideration; for a given *year*, *cohort* takes the following values: *year-100*, *year-99*,…, *year*. I.e. in *year* = 1900, we observe cohorts born in *cohort* = 1800, 1801, …, 1900.

age = a= t-x-1 = *year - cohort - 1* = age a on 1/1 of the year. I.e. *age* = -1,0,1,..,99

[231] We started from an estimate of the age structure of the 1820 population computed from the survival tables published in AR 1966 pp.80-81. We used the 1820-1900 series on annual numbers of births published in AR 1966 pp.66-69. We used the age-level mortality rates observed in 1900-1910 and assumed that each age-level mortality rate followed the same linear evolution during the 1820-1900 period as the five-year-age-group-level mortality rates published in AR 1966 p.77. Starting with the 1851 census, we have detailed age-group data (see AR 1966 p.43), and we find that our date base replicates very well this observed data. See do-file dopopulation18202100.txt.

ntot = N_t^x = number of individuals born during year x and alive on 1/1 of year t

nmen = number of male individuals born during year x and alive on 1/1 of year t

nwomen = number of female individuals born during year x and alive on 1/1 of year t

ndec = N_{dt}^x = number of individuals born during year x and deceased during year t

ndecmen =number of male individuals born during year x and deceased during year t

ndecwomen =number of female indiv. born during year x and deceased during year t

nbirth = N_{bt} = total number of births during year t

nbirthmen = total number of male births during year t

nbirthwomen = total number of female births during year t

nmigr = N_{it}^x = net number of individuals born during year x and migrating to France during year t (i.e. living in France on 1/1 t+1 and not living in France on 1/1 t)

nmigrmen = net number of male individuals born during year x and migrating to France during year t

nmigrwomen = net number of female individuals born during year x and migrating to France during year t

C.2. Supplementary data on age of decedents, heirs, donors and donees

In order to simulate the age-level dynamics of wealth accumulation and inheritance, we need relatively complete data not only on the age structure of decedents, but also on the age structure of heirs (the successors receiving bequests from decedents), donors (the living individuals making inter vivos gifts) and donees (the living individuals receiving inter vivos gifts). This information is not available in standard demographic data, so we had to construct our own data base. Some of raw material and resulting series are reported on Tables C5 to C8. The complete data base is available in the form of Stata format data sets bequestshares.dta and giftshares.dta. Here we describe how we constructed these data sets.

Estimating the age structure of decedents and heirs

The annual inheritance flow B_t can be decomposed in two different ways, either from the decedents' or from the heirs' perspective:

$$B_t = \sum_{x\leq t} B_t^x = \sum_{y\leq t} B_{ty} \qquad \text{(C.9)}$$

With: B_t = total inheritance flow transmitted/received during year t
B_t^x = inheritance flow transmitted during year t by cohort $x\leq t$
B_{ty} = inheritance flow received during year t by cohort $y\leq t$

The decedents decomposition $B_t = \sum_{x\leq t} B_t^x$ is known from estate tax data (see Appendix B2). However the heirs decomposition $B_t = \sum_{y\leq t} B_{ty}$ is harder to estimate. Ideally, one would like to have systematic demographic data base relating directly the cohort of the decedents and the cohort of the heirs. I.e. one would like to know for each decedents' cohort x the distribution of heir's cohorts y(x). Unfortunately, it seemed overly complicated to estimate such distributions on an annual basis over two centuries. Because not all heirs are children, purely demographic data is not enough: one needs very detailed data from estate tax returns. We actually do have individual-level data relating decedents' cohorts and heirs' cohorts in a systematic way for the recent period, thanks to the DMTG micro-files 1977-1984-1987-1994-2000-2006. But no such data exists for the earlier periods. The estate tax statistics published by the tax administration during the 1902-1964 period include tabulations by decedents age (see Appendix B2), but never include tabulations by heirs age (not to mention cross-tabulations by decedents age and heirs age). Therefore we decided to adopt a more modest strategy, namely we estimated the decomposition $B_t = \sum_{y\leq t} B_{ty}$ without attempting to relate directly which decedents cohort gives to which heirs cohorts. I.e. we took as given the aggregate inheritance flow B_t, and estimated the shares $bshare_{ty} = B_{ty}/B_t$ of aggregate inheritance flow received by each cohort $y\leq t$. There are several steps in our estimation strategy.

First, available demographic data on fertility shows that the average age at which men and women have children has been relatively stable since the 19th century, around 33-year-old for men and around 29-year-old for women (see Table C5, col. (1) and (2)).[232] Available fertility data published by Insee also provides for each female cohort starting in 1870 the full distribution of fertility rates broken down by female age.[233] We used this detailed data to compute the average age of parenthood as a function of parental year of birth (see Table C5, col. (4) and (5)), and as a function of parental year of death (see Table C5, col. (6) and (7)).[234] This detailed data also shows that the standard deviation of the distribution of age at parenthood has been fairly stable over the 20th century, around 5.5-6.5 years.[235]

Next, we used this data to compute the evolution of the average age of children heirs at the time their parents die (see Table C6, col. (4)-(6)), and the average age difference between parents and children heirs at the time of inheritance (see Table C6, col. (8)).[236] Unsurprisingly, we find that the average age difference has been relatively stable around 30 years: average age of decedents has gone up from about 60-year-old in 1900 to 75-year-old in 2000 and 85-year-old by 2050, while the average age of children heirs has gone up from about 30-year-old in 1900 to 45-year-old in 2000 and 55-year-old by 2050 (see Table C6). Unsurprisingly, children heirs tend to be older when they inherit from their mothers than when they inherit from their fathers, simply because the former tend to die later (and also because they tend to have children at an earlier age). Note that although these computations are based solely upon pure demographic data, they deliver estimates of average age of children heirs which are fully consistent with the estimates one can obtain using the DMTG micro-files of estate tax returns available for the recent period.[237]

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[232] This data is taken from F. Daguet, "Un siècle de fécondité française, Caractéristiques et évolution de la fécondité de 1901 à 1999", INSEE-Résultats, 2002, Societe n° 8 (updated version available at www.insee.fr), table 1. Note that on table C5 age at birth of children is defined as the generational diffence (i.e. children birth year minus parental birth year). The average age at parenthood has actually been following a slight U-shaped curve in the long-run: both men and women had children slightly earlier in life at mid 20th century than in the early 20th century and early 21st century.

[233] See F. Daguet, op.cit., table 4.4.

[234] Strictly speaking, the Insee data (Daguet, op.cit., table 4.4) provides complete age-level fertility data only for female cohorts born between 1885 and 1955. However assuming stationary evolutions of age-level fertility rates one can use this data to compute average age at parenthood for female cohorts born between 1870 and 1980. For cohorts born before 1870 we assumed that the average age at parenthood was the same as for cohort 1870; for cohorts born after 1980 we assumed that the average age at parenthood was the same as for cohort 1980.

[235] See F. Daguet, op.cit., table 2.2.

[236] Note that on table C6 average age of decedents and heirs is defined in the usual way, i.e. a=t-x-1. The detailed computations leading to tables C5 and C6 are provided in do-files dotableC5.tx and dotableC6.txt.

[237] The average age of children heirs that we obtain by using 1984-2000 DMTG micro-files are slightly higher (about 0.5-1 year higher) than those reported on table C6 (col.(4)), which corresponds to the fact that the average age of decedents with estate tax returns is slightly higher than the average age of all decedents (see Table C7). We did not attempt to correct for this.

Finally, available estate tax data shows that the fraction of the aggregate inheritance flow B_t received by children has been relatively stable over the 20th century, around 70%. More precisely, if we divide heirs into three categories, i.e. children, surviving spouses, and other heirs, then we find that the decomposition of the aggregate inheritance flow B_t into these three categories has been relatively stable around 70% for children, 10% for spouses, and 20% for others.[238] This is true both when we compute this decomposition using the DMTG micro-files 1984-1987-1994-2000 and when we use the available tabulations by heir category published by the tax administration during the 1902-1964 period. There are slight variations in this decomposition, but there is no clear trend, and given that this data is available for a limited number of years, it seems pointless to attempt to give precise estimates of the time variations of this decomposition.[239]

We therefore proceed in the following manner. We estimate the average age of all heirs (see Table C6, col. (7)) by computing a weighted average of the average age of children heirs (with weight 70%), the average age of surviving spouses (with weight 10%), and the average age of other heirs (with weight 20%). In the absence of better data, we assumed the average age difference between decedents and surviving spouses to be equal to 7 years (this is the stable difference observed with the 1984-2000 DMTG micro-files),[240] and the average age difference between decedents and other heirs to be equal to 20 years (this is the stable difference observed with the 1984-2000 DMTG micro-files). By construction, this method delivers series on average age of heirs that are fully consistent with those observed in DMTG micro files over the 1984-2006 period.[241] For the rest of the

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[238] We define "other heirs" as all non-children, non-surviving spouse heirs. In practice, these are mostly brothers/sisters and nephews/nieces.

[239] Note that the proportion of spouses in the total number of heirs (about 15%) is typically larger than the share of spouses in the aggregate inheritance flow (about 10%); this corresponds to the fact that the average bequest received by spouses is typically lower than that received by children. Note also that the share of spouses in the aggregate inheritance flow seems to have been somewhat larger at mid-century than at both extremes of the 20th century. The "children vs spouses vs others" decomposition of the inheritance flow was 72%-10%-19% in 1902 (using data from BSLC oct.1903 tome 5 p.38), 70%-16%-14% in 1962 (using data from S&EF dec.1965 supp. n°204 pp.1696-1697), and 68%-11%-21% in 2000 (using the DMTG 2000 micro-file). Given our aggregate perspective in this paper, it did not seem worth trying to take into account such time variations, and we chose to simplify matters by assuming a constant 70%-10%-20% sharing rule (we re-did all simulations using a 70%-15%-15% constant sharing rule, and no result was significantly affected). It would be interesting however to explore this spouse issue in more details in the future.

[240] Note that 7 years is the average age gap between the average age of all decedents (including those with no surviving spouses) and the average age of surviving spouses. This 7-year gap can be decomposed between a 4.5-year gap between the average age of decedents and the average age of decedents with surviving spouses (who unsurprisingly tend to be younger than average) an a 2.5-year gap between those decedents and their surviving spouse.

[241] Except for the slight bias described above. Note that the estimates reported on table C5 are also fully consistent with the 1984-2006 estimates of average age of children heirs and all heirs recently published by

period, one would need to gather relatively sophisticated demographic data (data on the distribution of age differences at marriage, on the age patterns of remarriage, on the distribution of age difference with siblings and nephews/nieces, not to mention the heirs that are fully exterior to the extended family) in order to detect possible historical changes in the pattern of age difference between decedents and surviving spouses and other heirs. Given that we are primarily concerned with aggregate trends, and given that the remaining uncertainty can only affect a relatively small part of the aggregate estate flow (70% of the flow goes to children, on which we have very reliable information), we felt that this was not worth it. Our resulting estimates show that the average age difference between decedents and heirs has been stable around 25 years, as opposed to 30 years if one only considers children heirs (see Table C6, col. (8)-(9)).

We use the same methodology to estimate the full distribution of heirs age. That is, we estimated separately the distributions $bshare_{ty}^{c}$, $bshare_{ty}^{s}$, $bshare_{ty}^{o}$ using DMTG micro-files, and we then computed then computed the distribution $bshare_{ty}$ as a weighted average of the three distributions:

$$bshare_{ty} = 0.7\ bshare_{ty}^{c} + 0.1\ bshare_{ty}^{s} + 0.2\ bshare_{ty}^{o} \qquad \text{(C.10)}$$

With: $bshare_{ty} = B_{ty} / B_t$ = fraction of aggregate inheritance flow received by cohort y
$bshare_{ty}^{c}$ = fraction of the children inheritance flow received by cohort y
$bshare_{ty}^{s}$ = fraction of the spouse inheritance flow received by cohort y
$bshare_{ty}^{o}$ = fraction of the other inheritance flow received by cohort y

According to the DMTG 1984-2000 micro-files, the three distributions $bshare_{ty}^{c}$, $bshare_{ty}^{s}$, $bshare_{ty}^{o}$ follow relatively simple and stable functional forms approximately centered around their respective mean. Regarding children we find that the best fit is obtained with the following functional form:

$$bshare_{ty}^{c} = bshare_{t}^{c}(a) = b^{cmaxt}/[1 + ((a-a_{t}^{c}-a^{0c})/a_{sd}^{c})^{\delta c}] \qquad \text{(C.11)}$$

with: a = t-y-1 = age of cohort y at time t

the tax administration (see Rapport 2008 du Conseil des Prélèvements Obligatoires, nov.2008, p.279). The tax administration estimates for all heirs are slightly higher than our estimates (about 1 year higher), presumably because they did not weight their estimates by average bequest.

a_t^c = average age of children heirs at time t

and where b^{cmaxt}, a_{sd}^c , a^{0c}, δ_c are parameters satisfying the following condition: $\sum_{0\leq a\leq 80}$ $bshare_t^c(a)$ =1

The parameters minimizing the average age-level gap with the observed distributions turn out to be the following: a_{sd}^c =14.3, a^{0c}=2.5, δ_c=3, and b^{cmaxt} computed each year so as to meet condition $\sum_{0\leq a\leq 80}$ $bshare_t^c(a)$ = 1 (in practice b^{cmaxt} is always very close to 3.1%).[242]

Regarding spouses and other heirs, we use similar functional forms:

$$bshare_{ty}^s = bshare_t^s(a) = b^{smaxt}/[1 + ((a-a_t^s-a^{0s})/a_{sd}^s)^{\delta s}] \qquad \text{(C.12)}$$

with: a_t^s = average age of spouse heirs at time t

and where b^{smaxt}, a_{sd}^s , a^{0s}, δ_s are parameters satisfying the condition $\sum_{20\leq a\leq 99}$ $bshare_t^s(a)$ =1. The gap minimizing parameters are: a_{sd}^s =16.0, a^{0c}=-1.0, δ_s=4, and b^{smaxt} computed each year so as to meet condition $\sum_{20\leq a\leq 99}$ $bshare_t^s(a)$ =1 (in practice b^{smaxt} is always very close to 2.9%).

$$bshare_{ty}^o = bshare_t^o(a) = b^{omaxo}/[1 + ((a-a_t^o-a^{0o})/a_{sd}^o)^{\delta o}] \qquad \text{(C.13)}$$

with: a_t^o = average age of spouse heirs at time t

and where b^{omaxo}, a_{sd}^o , a^{0o}, δ_o are parameters satisfying the condition $\sum_{0\leq a\leq 99}$ $bshare_t^o(a)$ =1. The gap minimizing parameters are: a_{sd}^o =20.0, a^{0o}=5.5, δ_o=3.5, b^{omaxt} computed each so as to meet condition $\sum_{0\leq a\leq 99}$ $bshare_t^o(a)$ =1 (in practice b^{cmaxt} close to 2.3%).

The details of the computations are given in the do-file dobequestshares.txt, and the resulting series are given in the Stata file bequestshares.dta.

Estimating the age structure of donors and donees

[242] Note that it is important to include the a^{0c} term in the functional form, because in practice the distribution $b_y^c(a)$ is not exactly centered around mean age a_t^c. This is mostly due to the fact that average bequest varies with heir age. For instance, older children heirs tend to receive slightly bigger bequests, so that the weighted average age of children heirs $\sum_{0\leq a\leq 80}$ a $b_t^c(a)$ is slightly larger than a_t^c by about 2 years. The gap is about 4 years of other heirs. For surviving spouses, the gap is slightly negative: older surving spouses have slightly lower average bequests. All these effects are relatively small quantitatively, but we decide to take them into account in order to fit as closely as possible the observed distribution of heirs age.

We also use similar computations to estimate the distribution of donors and donees age. Inter vivos gifts are relatively simpler to deal with than bequests, because the recipients of gifts are almost exclusively children.[243] Moreover, available estate tax data shows the average age of donors has always been about 7 years below the average age of decedents (see Table C7), so we make this assumption for the entire 1900-2050 period (see Table C8, col. (1)).[244] Using DMTG 1984-2000 micro-files, we adopt the following functional form for the distribution of donors age:

$$donor_{ty} = donor_t(a) = donor^{maxt}/[1 + ((a\text{-}adonor_t\text{-}a^{donor})/a_{sd}^{donor})^{\delta donor}] \qquad (C.14)$$

with: $donor_{ty}$ = share of total gift flow at time t given by donors from cohort y
a = t-y-1 = age of cohort y at time t
$adonor_t$ = average age of donors at time t
and where $donor^{maxt}$, a_{sd}^{donor} , a^{0donor}, δ_{donor} are parameters satisfying the condition $\sum_{0\leq a\leq 99}$ $donor_t(a)$ =1

The parameters minimizing the average age-level gap with the observed distributions turn out to be the following: a_{sd}^{donor} =12.0, a^{0donor}=0.5, δ_{donor}=5.0, and $donor^{maxt}$ computed each year so as to meet condition $\sum_{0\leq a\leq 99}$ $donor_t(a)$ = 1 (in practice $donor^{maxt}$ is always very close to 3.9%).

Regarding donees age, available estate tax data shows the difference with average donors age is unsurprisingly very close to the average age at parenthood for the relevant donors' cohorts, so we make this assumption for the entire 1900-1950 period (see Table C8, col.(2)).[245] Using DMTG 1984-2000 micro-files, we adopt the following functional form for the distribution of donees age:

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[243] Decomposition by donee category are not available on a yearly basis, but whenever we have data, either through the DMTG micro-files for the 1977-2006 period or through published tabulations for the 1900-1964 period (see e.g. S&EF déc. 1965 pp.1698-1699), we find that the children share in the total gift flow is about 97%-98%.

[244] Available historical data on donors age is limited, so we cannot exclude the possibility of significant historical changes in the age difference between decedents and donors. For the recent period, DMTG-based evidence seems to suggest that this age difference might have been rising somewhat, from about 6 years in the 1977-1984 to about 9 years in 1994-2000; however the most recent data indicates an age difference of 7 years for 2006 (see Table C7), so it is clear whether there is a time pattern or not. In the absence of better data, and as a first approximation, we choose to assume a stable 7-year age difference between dedecents and donors.

[245] Our DMTG computations, as well as the most recent published data from the 2006 DMTG survey (see Rapport du Conseil des prélèvements obligatoires, nov.2008, pp.268 and 279), shows during the 1990s-2000s the average age of donees stabilized at about 37-38 year-old, while on the basis of the rising age of donors and of the age at parenthood of relevant donors' cohorts, it should have increased by about 2 years.

$$donee_{ty} = donee_t(a) = donee^{maxt}/[1 + ((a\text{-}adonee_t\text{-}a^{donee})/a_{sd}^{donee})^{\delta donee}] \qquad (C.15)$$

with: $donee_{ty}$ = share of total gift flow at time t received by donees from cohort y

a = t-y-1 = age of cohort y at time t

$adonee_t$ = average age of donees at time t

and where $donee^{maxt}$, a_{sd}^{donee} , a^{0donee}, δ_{donee} are parameters satisfying the condition $\sum_{0\leq a\leq 99}$ $donee_t(a)$ =1

The parameters minimizing the average age-level gap with the observed distributions turn out to be the following: a_{sd}^{donee} =12.0, a^{0donee}=0.0, δ_{donee}=3.5, and $donee^{maxt}$ computed each year so as to meet condition $\sum_{0\leq a\leq 99}$ $donee_t(a)$ = 1 (in practice $donee^{maxt}$ is always very close to 3.7%).

The details of the computations are given in the do-file dogiftshares.txt, and the resulting Stata file is giftshares.dta. Note that these estimates of the average of donors and donees also allow us to compute the average of “givers” (decedents and donors) and “receivers” (heirs and donees) for any given year, simply by weighting the relevant age averages by the (gift flow)/(bequest flow) aggregate ratio. Given the large increase in the gift/bequest aggregate ratio during the 1980s-1990s (see Appendix B, Table B1), the average age of “receivers” appears to have stabilized during this period (see Table C8, col. (5)). Post-2008 series on Table C8 (col. (5) to (7)) were computed assuming the gift/bequest ratio remains constant after 2008, which of course is uncertain (we explore this further in Appendix D).

Note that although we used this same methodology to compute age-level bequest shares and gift-shares for the entire 1820-2100 period, it is clear that the 19th century and early 20th century estimates rely on a number of approximations. In particular, the assumption of a constant age gap between decedents and donors throughout the period is probably not valid in the very long run. E.g. our series indicate that donees were very young in the early

This might be due to the fact that donors have started to give slightly earlier, or to give to slightly younger children. In order to fit the observed age distributions of donors and donees as closely as possible, we assumed a gradual 2.4-year downward adjustment on the average of donees over the 1994-2006 period; for the pre-1994 and post-2006 period, we just assumed the average age of donees followed the series implied by age of donors and average age at parenthood of the relevant donors’ cohors. Computation details are given in do-files dogiftshares.txt and dotableC6.txt.

19th century (less than 20-year-old on average, see Table C8), which is probably an exaggeration. We return to this in the simulations.

C.3. List of Stata format data files and do-files

pop19002050.dta : basic population data file containing numbers of living individuals and decedents by year and birth cohort

pop18202100.dta: population date file extended to the 1820-2100 period

dopop18202100.txt: do-file generating pop19002050.dta from pop18502100.dta

dotableC1.txt and dotableC3-C4.txt: do-files generating Tables C1 and C3-C4 from pop18202100.dta

ageatbirth.dta: data file with series on parental age at the birth of their children

dotableC5.txt and dotableC6.txt : do-file generating Table C5 from pop18202100.dta and ageatbirth.dta

bequestshares.dta and giftshares.dta: data files containing estimates of the shares of aggregate bequest and gift flows received by each cohort

dobequestshares.txt and dogiftshares.txt: do-files generating bequestshares.dta and giftshares.dta from pop18202100.dta and ageatbirth.dta

dotableC8.txt: do-file generating Table C8 from giftshares.dta

dodiffmort.txt = do-file generating poor vs rich average age at death implied by differential mortality parameters (using data file pop18502100.dta)[246]

[246] See Appendix B, section B2.

Appendix D: Simulations

In this appendix we present the results of our simulations of the age-level dynamics of wealth accumulation and inheritance. The main conceptual issues and conclusions related to these simulations are presented in the working paper (sections 6 and 7). Here we provide additional information about the methodology and we present the detailed results.

The transition equations and simulation parameters are presented in section D1. The simulation results under various variants are described separately for the 1820-1913 period (section D3) and the 1900-2100 period (section D4). We then present supplementary simulation results on the structure of lifetime resources by cohort (section D5) and on the share of capitalized and non-capitalized inheritance in aggregate wealth accumulation (section D6).

D.1. Transition equations and simulation parameters

The basic principle of our simulations is the following. We start from the observed age-wealth profile $w_t(a)$ for a given base year $t=t_0$ (in practice, either $t_0=1820$ or either $t_0=1900$). We then write down a transition equation for age-level wealth $w_t(a)$. We want to know whether we can correctly predict the future evolution of the age-wealth profile and of the aggregate inheritance flow. By construction, since we use the observed rates of aggregate savings (and capital gains), we always predict perfectly well the evolution of aggregate private wealth. The name of the game is to see whether simple assumptions on saving behaviour (such as uniform savings or class savings) can also allow us to correctly predict the age structure of wealth, and therefore the macroeconomic magnitude of inheritance flows, via the μ_t effect. More precisely, the transition equation can be written as follows:

$$W_{t+1}(a+1) = (1+q_{t+1}) [W_t(a) + s_{Lt}Y_{Lt}(a) + s_{Kt}\, r_t\, W_t(a) + d_t\, W_t(a)] \\ - B_t^T(a) + B_t^R(a) - V_t^T(a) + V_t^R(a) \qquad (D.1)$$

With:

$W_t(a)$ = aggregate wealth of individuals of age a at time t

$w_t(a) = W_t(a)/N_t(a)$ = average wealth of individuals of age at time t

$Y_{Lt}(a)$ = aggregate labor income of individuals of age a at time t

$y_{Lt}(a) = Y_{Lt}(a)/N_t(a)$ = average labor income of individuals of age a at time t

$B_t^T(a)$ = aggregate bequest flow transmitted by individuals of age a at time t
$B_t^R(a)$ = aggregate bequest flow received by individuals of age a at time t
$V_t^T(a)$ = aggregate inter vivos gift flow transmitted by individuals of age a at time t
$V_t^R(a)$ = aggregate inter vivos gift flow received by individuals of age a at time t

The simulation parameters are reported on Tables D1 and D3-D4. These parameters were converted into Stata format data files simulationparameters18201913.dta and simulationparameters19002100.dta. The do-files dosimul18201913.txt and dosimul19002100.txt use these data files, together with the demographic data files described in Appendix C, in order to generate the simulation results reported on Tables D5-D6. In principle all results can be easily reproduced by anyone using these files.

The simulation parameters include macroeconomic series (see Tables D1 and D3) and series on age-labor income profiles (see Table D4). We describe them in turn.

The macroeconomic series are directly taken from the national accounts tables reported Appendix A (see formulas in excel files). The only noticeable feature is that in order to run annual level simulations we annualize the decennial-averages macro series of the 1820-1913 period. We did so by assuming constant growth rates, saving rates and rates of return within each decade (see Table D1). Of course the decennial averages of annualized series do not perfectly coincide with the initial decennial averages. But the gaps due to non linearities are extremely small (see Table D2), and irrelevant for our purposes.

The age-labor income profiles reported on Table D4 should be viewed as approximate. We checked that simulation results are robust with respect to alternative assumptions about these profiles; they are robust. For the recent period, income tax return micro files provide us with very reliable data on age-labor income profiles. We started from the observed tax profile in 2006. In the same way as in the theoretical model (see working paper, section 5), the profiles refer to “augmented labor income”, i.e. the sum of net-of-payroll-tax labor income and replacement income (pension income and unemployment benefits). We assumed a constant profile over the 2006-2100 period. Given the observed tax profile appears to be relatively stable during the 1990s-2000s, this seems to be the most reasonable assumption as a first approximation. For the 1820-2006 period we proceeded as follows (all details are given on the excel file).[247] We assumed that the profile below age 60 was constant throughout the period. Thanks to our national accounts series, we know the annual 1896-2006 evolution of aggregate replacement income and net-of-payroll-tax labor income. We then used historical estimates on labor force participation rates of individuals aged 60-to-69-year-old in order to allocate aggregate replacement income to the 60-to-69, 70-to-79 and 80-and-ver age groups.[248]

In order to compute the bequest and gift terms entering into transition equation (E.1), we proceeded as follows. We start from the age-level mortality rates coming from our demographic data base: $m_t(a)=N_{dt}(a)/N_t(a)$. We use the same modelling of differential mortality as that introduced in Appendix B2. That is, we assume that:

$$m_t^P(a) = 2\delta_t(a)m_t(a)/(1+\delta_t(a))$$
$$m_t^R(a) = 2m_t(a)/(1+\delta_t(a))$$
$$m_t^*(a) = sh_t^P(a)\, m_t^P(a) + [1- sh_t^P(a)]\, m_t^R(a)$$

With: $m_t^P(a)$ = mortality rate of the poor (bottom 50%)
$m_t^R(a)$ = mortality rate of the rich (upper 50%)
$m_t^*(a)$ = wealth-weighted average mortality rate

We use the same differential mortality parameters $\delta_t(a)$ and $sh_t^P(a)$ as in Appendix B2.

We then compute the predicted aggregate bequest flow $B_t^T(a)$ transmitted by individuals of age a at time t by multiplying their aggregate wealth by the wealth-weighted mortality rate:

$$B_t^T(a) = m_t^*(a)W_t(a)$$

We then compute the aggregate bequest flow transmitted at time t: $B_t = \sum_{a\geq 0} B_t^T(a)$

[247] Income tax return micro files are not available prior to the 1970s-1980s, and historical tax tabulations published by the tax administration since 1915 do not break down taxable income by age bracket (only by income bracket). So unfortunately there exists no direct historical data source on age-labor income profiles.

[248] The labor participation rate among the 60-to-69 was about 60%-70% in France in the 1950s-1960s, and then declined quasi linearly to about 20% in 1995, and then stabilized (and is currently rising somewhat, which does not make a big difference for our profiles, given that replacement rates are very high). See e.g. Bozio (2006, figure 3.1, p.117). So we simply assume a linear downward trend from 100% in 1910 (when there was virtually no pension system) to 65% in 1960 and 20% in 1995. We assume that nobody works above age 70, and that nobody receives pension income prior to age 60.

We then compute the aggregate bequest flow $B_t^R(a)$ received by individuals of age a at time t by multiplying B_t by the shares $bshare_t(a)$ computed in Appendix C2:

$$B_t^R(a) = bshare_t(a)\ B_t$$

We do the same for inter vivos gifts. We take as given the aggregate ratio $v_t=V_t/B_t$ (either we take the observed v_t, or we run simulations for alternative v_t values, see below). We then use the shares $donor_t(a)$ and $donee_t(a)$ computed in Appendix C2 in order to compute the aggregate gift flows $V_t^T(a)$ and $V_t^R(a)$ transmitted and received by individuals of age a at time t:

$$V_t = v_t\ B_t$$
$$V_t^T(a) = donor_t(a)\ V_t$$
$$V_t^R(a) = donee_t(a)\ V_t$$

Finally, the gift-corrected aggregate bequest flow is given by: $B_t^* = B_t + V_t$.

We have now fully described our dynamic system. Starting from a given age-wealth profile $w_t(a)$ at time $t=t_0$, we compute the endogenous sequence of aggregate bequest flows B_t^* for all $t \geq t_0$ and age-wealth profiles $w_t(a)$ for all $t>t_0$, by applying the transition equation (D.1) and the above equations to simulation parameters. We are particularly interested in the endogenous evolution of the inheritance flow-national income ratio $b_{yt} = B_t^*/Y_t$ and of the ratio $\mu_t^* = b_{yt}/m_t\beta_t$ (as well as the pre-gift ratio $\mu_t = \mu_t^*/(1+v_t)$). The economic forces at play in this dynamic process are exactly the same as those analyzed in the theoretical model with exogenous saving model (see working paper, section 5.2), except that we are now out of steady-state, and except that we take into account all macroeconomic and demographic shocks (on the basis of observed data), as well as inter vivos gifts.[249]

[249] We also attempted to simulate endogenous saving behaviour, as predicted by the utility maximizing models analyzed in sections 5.3 and 5.4 of the working paper (dynastic model and wealth-in-the-utility-function model). However the short run and medium run predictions of utility maximizing models are very sensitive to the assumptions one makes about agents' expectations on future growth rates and rates of return, particularly during the chaotic 1914-1945 period (for which it would not make much sense to assume perfect foresight). Also these models generally tend to predict far more age variations in consumption profiles and savings rates than one typically observes (actual age-saving rates profiles are not very far from being flat, just like in the exogenous saving model). In order to obtain plausible predictions, authors using utility maximizing models often end up making simplifying ad hoc assumptions, e.g. they directly assume that the growth rate g_c of consumption profiles is the same as the income growth rate g (see for instance Gokhale and Kotlikoff (2001)). Given that there are already so many other effects going on in our two-century-long dynamic model, we find it more natural to simply assume exogenous saving rates, see how much one can explain with such assumptions, and leave the issue of endogenous saving behaviour to future research.

D.2. Simulation results for the 1820-1913 period

Simulation results for the 1820-1913 period are summarized on Table A5. The detailed simulation results, with the endogenous annual dynamics of the age-wealth profile $w_t(a)$, are reported on separate tables (one for each scenario).

The main findings from these simulations are discussed in the working paper (section 6). Here we discuss additional technical details. First, in all variants, we approximately reproduce the relative stability of b_{yt} around 20% of national income during the 1820-1913 period. This simply shows that the 19th century was close to a steady-state, and that with low growth rates and high rates of return, the steady-state inheritance flow tends to be close to 20% irrespective of the specific saving behaviour. The key assumption here is the flatness of the age-saving rates of profile: with dissaving at old age, one would never be able to reproduce such levels of inheritance flows.

Next, if one wants to obtain a better fit for the b_{yt} pattern, and most importantly if one wants to be also able to reproduce the full observed age-wealth profile $w_t(a)$, then one needs to assume class saving. The observed age-wealth profile at the end of the period is steeply rising at old age: around 1900-1910, individuals aged 70-to-79 and 80-and-over own as much as 180%-200% of the average wealth owned by the 50-to-59-year-old (see Appendix B, Table B5). By comparing with the simulated age-wealth profiles under scenario a1-a3 and b1-b3, one can see that the only way to get close to this is to assume that savings entirely come from capital income. With uniform saving, and even more so with reverse class saving, the simulated profile is far too flat, and the resulting pattern of b_{yt} and μ_t ratios is somewhat too low.

In fact, in order to fully reproduce the steepness of the age-wealth profile and the very high levels of b_{yt} around 1900-1910, one would need to assume not only that (most) savings come from capital income, but also that the average saving rate $s_K(a)$ actually rises with age. This would be consistent with a simple consumption satiation effect among elderly wealth holders. This interpretation is also consistent with the fact that the age-wealth profile in Paris (where top wealth levels were particularly high) was in 1900-1910 even more steeply rising than in the rest of France: the average wealth of the 70-to-79 and 80-

and-over age groups was as large as 300% of that of the 50-to-59 age group, which cannot be accounted for without a steeply rising $s_K(a)$ profile.[250]

Finally, we are particularly interested in the simulation results under the zero gift assumption. In scenario a1-a3, we take as given the observed gift-bequest ratio v_t. This generates bizarre predictions on the age-wealth profiles at mid-19th century. For instance, the 50-to-59 age group appears to be unplausibly poor. This seems to be due to the fact that gifts are very important at that time (v_t is about 40% in the 1840s, and then gradually falls to about 20% in the 1860s-1870s), and that we probably attribute an excessive fraction of these gifts to donors in their 50s (our estimates on the age structure of donors and donees are highly approximate for the 19th century). So in order to abstract entirely from the issue of inter vivos gifts (which raise interesting and complex issues on their own right), we assume in scenario b1-b3 that there was no gift at all throughout the 1820-1913 period (v_t=0%). Of course this implies that we significantly underestimate the aggregate bequest flow at the beginning of the period (since by assumption we miss the gift part). But this clarifies considerably the dynamics of the age-wealth profile, and confirms that one needs to assume class saving in order to reproduce observed profiles.

Most importantly, by the end of the period (around 1900-1910, and in fact as early as the 1850s-1860s), we generate as much total bequests under the zero gift assumption than with the observed gift ratio (compare scenario b1-b3 with a1-a3). In other words, if wealth holders stop making gifts and hold on to their wealth until their death, then their wealth at death will be higher, and total wealth transmission will eventually be approximately the same as what it would have been in the presence of gifts. This finding justifies the fact that as a first approximation we chose to simply add up cross sectional gifts and bequests in order to compute the total flow of wealth transmission.

D.3. Simulation results for the 1900-2100 period

Simulation results for the 1900-2100 period are summarized on Table A6. The detailed simulation results, with the endogenous annual dynamics of the age-wealth profile $w_t(a)$, are reported on separate tables (one for each scenario).

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[250] It is possible that we underestimate somewhat the importance of differential mortality at high age around 1900-1910. But differential mortality would have to be enormous in order to explain such a steeply rising age-wealth profile, which would be consistent with the fact that the fraction of zero-wealth decedents is almost flat (i.e. there seems to be almost as many poor people among the very old decedents than among younger decedents). See Piketty, Postel-Vinay and Rosenthal (2006).

The main findings from these simulations are discussed in the working paper (section 6). Here we give additional technical details. First, by comparing the results obtained under scenario a1-a3, one can see that the fact that pensions and replacement rates were relatively low at mid 20^{th} century does contribute to make inheritance flows smaller, but that this is a relatively small effect. This is consistent with the theoretical results obtained in the exogenous saving model.

Next, by comparing scenario a1 with scenario b1-b2, one can see that class saving is no longer adequate to account for 20^{th} century patterns. Uniform saving offers a better fit. As far as reproducing the 1950s nadir is concerned, reverse class saving offers an even better fit. In order to fully reproduce the extremely low inheritance flow observed in the 1950s (about 4% of national income), one would actually need to assume non-age-neutral war-induced capital shocks (i.e. the elderly might have suffered from more than proportional shocks, e.g. because they held a larger fraction of their wealth in public bonds or other nominal assets), and/or negative saving from wealth holders (e.g. because a number of rentiers did not adjust downwards their living standards sufficiently fast following the fall in asset values and returns), and/or negative saving from the elderly in general (because of particularly low pensions around that time). In order to settle the issue, one would need to explicitly introduce distributional considerations into the analysis and to use micro level data, which we plan to do in future research.

Maybe the most interesting simulations are scenario c1 (where we freeze the gift parameter v_t at its 1980 level for the 1980-2100 period) and scenario c2 (where we set v_t=0% throughout the 1900-2100 period). The key finding is we still reproduce observed patterns relatively well. This shows that the large rise in gifts which occurred since the 1980s is not driving the recent rise in measured inheritance flows. In particular, by looking at the predicted age-wealth profiles, one can see that if gifts had not risen since 1980 (or if they had been absent throughout the period) then the age-wealth profile would have been substantially more steeply rising at old age by 2000-2010, thereby generating large extra wealth transmission at death, thereby compensating the absence of a larger gift flow. Note however that the compensation is not complete (i.e. long run levels of b_y* are somewhat smaller in the small-gift or zero-gift scenarios c1-c2), which suggests that there was a little bit of overshooting in the rise of gifts since the 1980s (possibly due to tax incentives), and that a (small) fraction of the observed gift level is not sustainable.

Regarding the 2010-2100 period, we explored several scenarios corresponding to various assumptions about future growth rates g, net-of-tax rates of returns $(1-\tau)r$, and saving rates s (which might or might not react to changes in g and $(1-\tau)r$). Variants a1-c3 correspond to our baseline scenario: g=1.7% (average 1979-2009 growth rate), $(1-\tau)r$=3.0% (capital share fixed at 2008 level), and s=9.4% (average 1979-2009 saving rate). In variants d1-e4 we explore the consequences of growth slowdown (g=1.0%) and/or rise in the net-of-tax rate of return ($(1-\tau)r$=5.0%). In variants f1-g4 we explore the consequences of rise in the growth rate (g=5.0%), possibly accompanied by a rise in the net-of-tax rate of return. The main findings are discussed in the working paper. Here we mention two additional points.

First, it is equivalent in the model whether the rise in the net-of-tax rate of return comes from a rise in the capital share or from a decline in the capital tax rate. This is because we assume in all variants that the overall tax rate remains constant after 2010 (i.e. the disposable income-national income ratio is supposed to be fixed), so in effect any capital tax cut must be compensated by a corresponding rise in labor taxes.

Next, if saving rates do not adjust in our model, then changes in the growth rate will have large long run impact on the wealth-income ratio $\beta^*=s/g$. In the baseline scenario, with g=1.7% and s=9.4%, the long run β^* is about 560%, i.e. approximately the same level as in 2008-2009. In other scenarios, we consider variants where the saving rates adjust so as to keep the long-run wealth-income ratio approximately constant around 500%-600%. This allows us to disentangle the impact of g on b_y going through changes in β from the impact of g on b_y going through changes in μ.

For instance, a substantial part of the rise of b_y to 22%-23% in scenarios d1-d2 is due to the fact that the growth slowdown leads to a rise in the wealth-income ratio to about 650% by 2050 and 750% by 2100 (about two thirds of the rise comes from this channel). This is a plausible outcome. But in order to separate the various effects, we also consider in scenarios d3-d4 the possibility that the saving rate adjusts downwards to 6%, so that the wealth-income ratio remains stable at 550%-600%. The rise of b_y is then limited to 17%-18% in 2100. By comparing scenario a1 with scenarios d3-d4, one can also compute the relative impacts of g and r on steady-state μ^*. E.g. in the baseline scenario a1, b_y=16.0% in 2050. In scenario d3, b_y=16.9%; in scenario d4, b_y=17.3%. That is, the growth slowdown appears to explain about two thirds of the total rise of μ^* by 2050, while the rise in the rate of return explains about one third. This is consistent with the theoretical results obtained with the exogenous saving model: the r effect is multiplied s_k, and is therefore smaller than the g effect. Note however that other saving specifications would deliver different results. E.g. in the wealth-in-the-utility-function models, where individuals save a fixed fraction of their lifetime resources, changes in g and r have the same quantitative impact (only the difference r-g matters). This should be more closely investigated in future research.

D.4. Estimation and simulation results on lifetime resources by cohort

We use the simulated model in order to compute lifetime resources by cohort $\tilde{y}^x = \tilde{b}^x + \tilde{y}_L{}^x$, for all cohorts born between 1800 and 2030. The main findings from these computations are discussed in the working paper (sections 7.1-7.2). Here we present the full results (see Table D7-D8) and provide technical details (see the do-file dolifetimecohorts18002000.txt for the corresponding computer code).

For all cohorts $x \in [1800,2030]$, we compute the aggregate value of inherited resources and labor incomes resources received during the entire lifetime of cohort x (i.e. between age a=0 and age a=100), capitalized at age 50:

$$\tilde{B}^x = \sum_{x \leq t \leq x+100} (1+r_{ts}) \ (B_t^x + V_t^x) \qquad \text{(D.2)}$$

$$\tilde{Y}_L{}^x = \sum_{x \leq t \leq x+100} (1+r_{ts}) \ Y_t^x \qquad \text{(D.3)}$$

With: B_t^x = aggregate value of bequest flows received at time t by cohort x

V_t^x = aggregate value of inter vivos gift flows received at time t by cohort x

Y_t^x = aggregate value of labor income flows received at time t by cohort x

$1+r_{ts}$ = cumulated rate of return between year t and year s=x+50

We then compute average values $\tilde{b}^x = \tilde{B}^x/N^x$ and $\tilde{y}_L{}^x = \tilde{Y}_L{}^x/N^x$ by dividing aggregate values by cohort size N^x (we use cohort size at birth). The corresponding values, expressed in 2009 euros, are reported on Table D7 (benchmark scenario) and Tables D8 (low-growth, high-return scenario).[251] We also report the inheritance share in total lifetime resources

[251] We use the simulated inheritance flows to do these computations, not the observed flows. Since they are very close, this makes little difference. However we overpredict somewhat the levels of inheritance flows in the 1950s-1960s, this implies that our lifetime resources series tend to overestimate the share of inheritance

$\hat{\alpha}^x = \tilde{b}^x/(\tilde{b}^x + \tilde{y}_L^{\ x})$ and the inheritance-labor ratio $\psi^x = \tilde{b}^x/\tilde{y}_L^{\ x}$, as well as the capitalization factors λ_B^x and λ_L^x (i.e. the ratios between capitalized lifetime resources and the uncapitalized resources obtained by replacing $1+r_{ts}$ by 1 in equations (D.2)-(D.3)), and the ratio $\lambda^x = \lambda_L^x/\lambda_B^x$. For 19th century cohorts, age 50 happens relatively late in life, so the capitalization factors λ_B^x and λ_L^x are far above 100%. For 20th century cohorts, age 50 is closer to mid life, so the capitalization factors are closer to 100%. In both cases, the ratio $\lambda^x = \lambda_L^x/\lambda_B^x$ is always relatively close to 100%.[252] Of course the choice of age a=50 has no consequence on the ratios $\hat{\alpha}^x$, ψ^x and λ^x, since we use the same rates of return for inheritance and labor ressources.[253]

We also report on Tables D7-D8 the values for the two-dimensional inequality indicators discussed in the working paper (section 7.2). These were computed by applying directly the ratio ψ^x to the intra-cohort distributions of inherited wealth and labor income indicated in the working paper (Table 4).[254] In order to compute ε^x (i.e. the proportion of cohort x with inheritance ressources larger than bottom 50% labor resources), we assume that the fraction fraction $p^x(b)$ of cohort x with inheritance resources larger than b can be approximated by a simple type-1 Pareto distribution, and we borrow Pareto coefficients from our previous work on wealth concentration.[255]

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in the lifetime resources of the cohorts who inherited in the 1950s-1960s. I.e. the true U-shaped pattern is somewhat more marked than what our series indicate.

[252] The fact that we assume the same age-labor income profile throughout the 1820-2008 period (below age 60) probably leads us to overestimate the value of λ^x for the recent cohorts (as compared to 19th century and early 20th century cohorts), and therefore to overestimate the labor share of the lifetime resources of recent cohorts (and underestimate the inheritance share), again relatively to earlier cohorts. It is indeed very likely that the age-labor income profile has become more and more upward sloping over time (i.e. in the 19th century workers in their 20s and 30s were probably not earning much less than workers in their 40s and 50s, and in a large number of cases they were actually earning more). I.e. all resources (not only inheritance, but also labor resources) now tend to accrue later in life. Unfortunately we have little systematic information on age-labor income profiles in the 19th century, so we did not try to correct for this.

[253] For the benchmark estimates reported on Tables D7-D8, we did not include real rates of capital gains $1+q_{ts}$ into the capitalization factors (i.e. the $1+r_{ts}$ factors only include the normal after-tax rates of return, see do-file). We also re-did all computations with real rates of capital gains: the λ^x ratio remains very close to 100%, and the shares $\hat{\alpha}^x$ and ψ^x are virtually unaffected (unless of course one applies capital gains and losses only to inheritance resources; in which case inheritance shares would be substantially larger to the recent period, and substantially lower for the mid-20th century; but we do not want in this paper to deviate from the assumption of a common rate or return for all individuals and types of ressources). The monetary values $\tilde{b}^x$ and $\tilde{y}_L^{\ x}$ however would be affected and would become more volatile and difficult to interpret, for purely artificial reasons (i.e. cohorts who happen to turn 50 in a year with high asset prices would appear as having higher lifetime resources – both inheritance and labor income resources – in euros 2009 than cohorts who happen to turn 50 in a year with low asset prices), so we prefer to present the results this way. The same remarks apply to the rates of capital destructions $1+d_{ts}$ (which were not included in the benchmark estimates reported on Tables D7-D8. One can easily redo the computations by adding the q and d factors in the corresponding line of the do-file.

[254] See formulas in excel file.

[255] Namely, we assume that the inverted Pareto coefficient is equal to 5 for cohorts 1820-1870 and equal to 3 for cohorts 1930-2030 (and declined linearly in between), which approximately corresponds to the observed

D.5. Estimation and simulation results on inheritance shares in wealth accumulation

We also use the simulated model in order to compute the non-capitalized and capitalized inheritance shares in aggregate wealth φ_t^M and φ_t^{KS} for all years between 1850 and 2100. The main findings from these computations are discussed in the working paper (section 7.3). Here we present the full results (see Table D9-D10) and provide technical details (see the do-file doinheritanceshare18502100.txt for the corresponding computer code).

For all years t$\in$[1850,2100], we compute φ_t^M and φ_t^{KS} by dividing the cumulated value of past bequests and gifts $\hat{B}_t$ and $\tilde{B}_t$ by aggregate wealth W_t :

$$\varphi_t^M = \hat{B}_t/W_t \text{ , with: } \hat{B}_t = \textstyle\sum_{t-100\leq s\leq t} (B_{st}+V_{st}) \qquad \text{(D.4)}$$

$$\varphi_t^{KS} = \tilde{B}_t/W_t \text{ , with: } \tilde{B}_t = \textstyle\sum_{t-100\leq s\leq t} (1+r_{st}) (B_{st}+V_{st}) \qquad \text{(D.5)}$$

With: B_{st} = aggregate bequests received at time s by individuals who are still alive at time t
V_{st} = aggregate gifts received at time s by individuals who are still alive at time t
$1+r_{st}$ = cumulated rate return between year s and year t

In order to compute B_{st} and V_{st}, we need to know which fraction of the various cohorts is still alive at in year t, so we computed survival rates using the same differential mortality parameters as in the general simulations (see do-file). The corresponding wealth aggregates, expressed in 2009 billions euros, are reported on Table D7 (benchmark scenario) and Tables D8 (low-growth, high-return scenario). We actually report three series $\hat{B}_{t0}$, $\hat{B}_t$ and $\tilde{B}_t$ for cumulated inherited wealth, as well as the three corresponding series φ_{t0}^M, φ_t^M and φ_t^{KS} for shares in aggregate wealth. The raw series $\hat{B}_{t0}$ correspond to nominal inherited wealth and are reported only for illustrative purposes: we did not even adjust past bequest and gifts flows B_{st} and V_{st} for price inflation, so of course the corresponding wealth shares φ_{t0}^M are artificially low, especially following periods of rapid inflation (e.g. φ_{t0}^M is less than 10% around 1950). The uncapitalized series $\hat{B}_t$ were

values at the 90th percentile level (see Piketty et al (2006, data appendix, Tables A3-A6)). See formulas in excel file. In order to obtain a better fit, one should use type-2 Pareto distributions for wealth distributions rather than type-1 Pareto distributions. But here this would have little effect on the pattern of ε^x.

computed by adjusting past bequest and gifts flows B_{st} and V_{st} by cumulated consumer and asset price inflation $1+p_{st}$ and $1+q_{st}$ between year s and year t. This seems to be the most reasonable way to define uncapitalized inherited wealth $\hat{B}_t$, and the corresponding uncapitalized inheritance wealth share in aggregate wealth φ_t^M.[256] In the capitalized series $\tilde{B}_t$ and φ_t^{KS}, we also apply to past bequest and gift flows the cumulated rate of return $1+r_{st}$, as indicated by equation (D.5).

We also report on Tables D7-D8 the average capitalization factor, i.e. the ratio $\tilde{B}_t/\hat{B}_t$. Note that with the deterministic demographic structure used in the stylized model, everybody inherits at age a=I, so the capitalization factor only depends on r, g and generation length H, and is given by simple steady-state formulas.[257] However here we use the observed demographic structure of bequests and gifts, with full demographic shocks, i.e. individuals receive bequests and gifts at all ages. So for instance in each cohort there is a fraction of individuals who inherited very early in life (much before age I, e.g. because their parents died early), and a fraction of individuals who inherited very late (or even died before their parents). Because capitalized returns are a convex function of time, this tends to push upwards the average capitalization factor $\tilde{B}_t/\hat{B}_t$, i.e. the few individuals in each cohort who inherited very early in life have an enormous capitalized inherited wealth and can have a substantial impact on aggregate capitalized inherited wealth. In effect, non-deterministic demography makes the capitalized definition φ_t^{KS} even more sensitive to r-g than it naturally is. We see no obvious reason why we should exclude or truncate early successors, however, so the series reported on Table D7-D8 do not make any such truncature. For illustrative purposes, we indicate the shares of non-capitalized and capitalized inherited wealth which were received more than 30 years or 50 years before the current year. So for instance, as of 2010, bequests and gifts received before 1980 represent 13% of non-capitalized inherited wealth $\hat{B}_t$ and 39% of capitalized inherited wealth $\tilde{B}_t$; those received before 1960 represent 2% of non-capitalized inherited

[256] If we were only adjusting for consumer price inflation $1+p_{st}$, then the corresponding wealth shares φ_t^M would be artificially high following periods of relative asset price decline (such as the 1950s-1960s). For the same reason, we also adjust past bequest and gift flows by cumulated capital destruction rates $1+d_{st}$ (otherwise the corresponding wealth shares φ_t^M would be artificially high following war periods: in effect we would be including in inherited wealth assets that were destroyed during wars). One can easily redo the computations by adding or substracting the p, q and d factors in the corresponding line of the do-file.

[257] See formulas (7.6)-(7-7) (working paper, section 7.3) and Table E12 for illustrative computations.

wealth $\hat{B}_t$ and 15% of capitalized inherited wealth $\tilde{B}_t$ (see Table D9). As compared to the steady-state formulas, there are other effects going in the opposite direction: we take into account all observed bequests and gifts, so for instance this includes bequests and gifts to surviving spouses and/or siblings, which often occur not very long before the receiver's death (and typically less than H=30 years before the receiver's death, so that average capitalization length is effectively less than 30). Overall, the average capitalization factor $\tilde{B}_t / \hat{B}_t$ is relatively close to the theoretical steady-state level in the benchmark scenario (it is actually a bit lower), and it is significantly higher in the low-growth, high-return scenario (reflecting the strength of the convexity effect).[258]

D.6. List of Stata format data files and do-files

simulationparameters18201913.dta: data file containing the parameters used for the 1820-1913 simulations (see Tables D1 and D4)

simulationparameters19002100.dta: data file containing the parameters used for the 1900-2100 simulations (see Tables D3 and D4)

dosimul18201913.txt: do-file generating 1820-1913 simulation results

dosimul19002100.txt: do-file generating 1900-2100 simulation results

simulresults18201913.dta, simulresults19002100.dta, simulwealth18201913.dta, simulwealth19002100.dta, simulwealth19002100(scenariod2), simulwealth18202100.dta: data files containing 1820-1913 and 1900-2100 simulation results

dolifetimecohorts18002000.txt: do-file generating lifetime resources by cohort

lifetime18002000.dta: data file containing the results on lifetime resources by cohort

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[258] With H=30, g=1.7%, and (1-τ)r=3.0%, the theoretical steady-state capitalization factor should be 207% (see Table E12), and we find that the average capitalization factor converges towards about 195% during the 21st century (see Table D9). With H=30, g=1.0% and (1-τ)r=5.0%, the theoretical level is 269% (see Table E12), and we find convergence towards 350% during the 21st century (see Table D10).

doinheritanceshare18502100.txt: do-file generating capitalized and non-capitalized inheritance shares in aggregate wealth accumulation

inheritanceshares18502100.dta: data file containing the results on inheritance shares

Appendix E: Steady-state inheritance formulas

In the working paper we develop a stylized model of wealth accumulation, inheritance and growth, and present a number of theoretical results and steady-state formulas on inheritance flows (see section 5). Omitted proofs for these results and formulas are provided here, together with a number of tables and figures illustrating how the various steady-state formulas can be used with real numbers (sections E1-E5). We then show how the main theoretical results and formulas can be extended to more general demographic structures, and in particular to the case with population growth (section E6).

E.1. Proof of Proposition 3 (section 5.2)
(exogenous savings model, closed economy)

With exogenous saving rates $s_L \geq 0$ & $s_K \geq 0$, the steady-state wealth-income ratio $\beta_t = w_t/y_t$ is equal to $\beta^* = s/g$ and the steady-state rate of return r_t is equal to $r^* = \alpha/\beta$ (see Proposition 1). We are looking for the steady-state ratio $\mu_t = b_t/w_t = w_t(D)/w_t$.

(i) Case ρ=1. First consider the case ρ=1 (i.e. 100% replacement rate). Because savings are assumed to be linear, the average wealth $w_t(a)$ of a-year-olds at time t can be broken down into two components, i.e. an inherited wealth component $w_{Bt}(a)$, and a labor wealth component $w_{Lt}(a)$:

If $a \in [A,I[\quad w_t(a) = w_{Lt}(a) = \int_{t+A-a \leq s \leq t} s_L \, y_{Ls} \, e^{s_K r^*(t-s)} \, ds$

If $a \in [I,D] \quad w_t(a) = w_{Bt}(a) + w_{Lt}(a) = b_{t+I-a} \, e^{s_K r^*(a-I)} + \int_{t+A-a \leq s \leq t} s_L \, y_{Ls} \, e^{s_K r^*(t-s)} \, ds$

Since y_{Lt} and b_t grow at rate g in steady-state, we have: $y_{Ls} = y_{Lt} \, e^{-g(t-s)}$ and $b_{t+I-a} = b_t \, e^{-g(a-I)}$.

Therefore we have:

If $a \in [A,I[\quad w_t(a) = w_{Lt}(a) = s_L \, y_{Lt} \, [1-e^{-(g-s_K r^*)(a-A)}]/(g-s_K r^*)$

If $a \in [I,D] \quad w_t(a) = w_{Bt}(a) + w_{Lt}(a) = b_t \, e^{-(g-s_K r^*)(a-I)} + s_L \, y_{Lt} \, [1-e^{-(g-s_K r^*)(a-A)}]/(g-s_K r^*)$

Since $g - s_K r^* = (s - \alpha s_K)/\beta = (1-\alpha)s_L/\beta$, one can replace $s_L y_{Lt}/(g-s_K r^*)$ by w_t and obtain:

If $a \in [A,I[\quad w_t(a) = w_{Lt}(a) = [1-e^{-(g-s_K r^*)(a-A)}] \, w_t$

If $a \in [I,D] \quad w_t(a) = w_{Bt}(a) + w_{Lt}(a) = b_t \, e^{-(g-s_K r^*)(a-I)} + [1-e^{-(g-s_K r^*)(a-A)}] \, w_t$

It follows that the steady-state ratio $\mu_t = b_t/w_t = w_t(D)/w_t$ is given by:

$$\mu^* = \mu(g) = \frac{1-e^{-(g-s_K r^*)(D-A)}}{1-e^{-(g-s_K r^*)H}} \qquad \textbf{(E.1)}$$

Alternatively, instead of assuming that $\mu_t=b_t/w_t$ is in steady-state, we can write the transition equation for μ_t as a function of the μ_{t-H} of the previous generation:[259]

$$\mu_t = \mu_{t-H}\, e^{-(g-s_K r^*)H} + [1-e^{-(g-s_K r^*)(D-A)}]$$

As long as $s_L>0$, $g-s_K r^*>0$, so this dynamic process converges: $\mu_t \rightarrow \mu^* = \mu(g)$ as $t\rightarrow+\infty$.
Since $g-s_K r^*=g(1-\alpha)s_L/s$, formula (E.1) can also be rewritten as follows:

$$\mu^* = \mu(g) = \frac{1-e^{-\frac{(1-\alpha)s_L}{s}g(D-A)}}{1-e^{-\frac{(1-\alpha)s_L}{s}gH}} \qquad \textbf{(E.2)}$$

Note that as $s_L\rightarrow 0$, $\mu(g)\rightarrow \overline{\mu}=(D-A)/H$: we are back to the class savings case.

$\neg$ $s_L>0$, $\mu'(g)<0$, with $\mu(g)\rightarrow \overline{\mu}$ as $g\rightarrow 0$ and $\mu(g)\rightarrow 1$ as $g\rightarrow+\infty$.

Note also that for given g, a rise in $(1-\alpha)s_L/s$ (the share of total savings coming from labor income) has the same impact on steady-state μ^* as a rise in g (and conversely for a rise in the share of total savings coming from capital income $\alpha s_K/s$).
In the uniform savings case ($s_L=s_K=s$), $g-s_K r^*=(1-\alpha)g$, so we have:

$$\mu^* = \mu(g) = \frac{1-e^{-(1-\alpha)g(D-A)}}{1-e^{-(1-\alpha)gH}} \qquad \textbf{(E.3)}$$

For illustrative purposes, numerical examples of age-wealth profiles for bequest wealth $w_{Bt}(a)/w_t$ and labor wealth $w_{Lt}(a)/w_t$ are represented on Figures E1-E2.[260] Because $g-s_K r^*>0$, i.e. the growth effect dominates the savings effect, bequest wealth $w_{Bt}(a)/w_t(a)$ declines with age a (above inheritance age). Labor wealth $w_{Lt}(a)/w_t(a)$ naturally rises with age. By definition, the total wealth profile $w_t(a)/w_t$ is the sum of the two profiles: it is rising until age a=I and then declining (see Figure E3). I.e. the cross-sectional wealth profile is hump-shaped (this is because of the growth effect), in spite of the fact that longitudinal profiles are always upward sloping in the exogenous saving model (there is no old-age dissaving).[261] As $g\rightarrow 0$, both profiles become almost flat and the relative importance of

[259] Here we implicitly assume that the aggregate wealth accumulation process has already converged, i.e. $\beta_t=w_t/y_t$ is permanently equal to $\beta^*=s/g$, so that w_t and y_t grow exactly at rate g. Otherwise one could not replace $s_L y_{Lt}/(g-s_K r^*)$ by w_t. and the transition equation for μ_t would look more complicated.
[260] We used the following parameters for Figures E1-E2 (see formulas in excel file): A=20, H=30, D=70, 1-α=70%. We assumed uniform savings (in which case s has no impact on age-wealth profiles). With g=2%, we get μ*=147% (see Figure E1); with g=5% we get μ*=127% (see Figure E2).
[261] The fact that growth effects can mechanically deliver hump-shaped cross-sectional profiles, even in the absence of any lifecycle saving behaviour, was first pointed out by Shorrocks (1975). Here however the growth effect comes from inheritance (older individuals are poorer because they received lower bequest)

labor wealth declines; so that in the limit the total age-wealth profile looks like the profile prevailing in the class saving case (see working paper, Figure 12), and $\mu^* \to \bar{\mu}$. Conversely, as $g \to +\infty$, the bequest wealth profile becomes more and more downward sloping, and the labor wealth profile more and more upward sloping; so that in the limit the total wealth profiles displays two peaks at ages I and D, and $\mu^* \to 1$.

<u>(ii) Case ρ<1.</u> Now consider the case $\rho \leq 1$ (less than 100% replacement rates). Labor wealth $w_{Lt}(a)$ of retired individuals ($a \in [R,D]$) is now given by:

$$w_{Lt}(a) = \frac{D-A}{(R-A)+\rho(D-R)} \left[\int_{t+A-a\leq s\leq t+R-a} s_L y_{Ls} e^{s_K r^*(t-s)} ds + \int_{t+R-a\leq s\leq t} s_L \rho\, y_{Ls} e^{s_K r^*(t-s)} ds \right]$$

That is:

$$w_{Lt}(a) = \frac{D-A}{(R-A)+\rho(D-R)}\, w_t \left[[e^{-(g-s_K r^*)(a-R)} - e^{-(g-s_K r^*)(a-A)}] + \rho\,[1-e^{-(g-s_K r^*)(a-R)}] \right]$$

And the steady-state ratio $\mu_t = b_t/w_t = w_t(D)/w_t$ is given by:

$$\mu^* = \mu(g,\rho) = \frac{D-A}{(R-A)+\rho(D-R)} \; \frac{e^{-(g-s_K r^*)(D-R)} - e^{-(g-s_K r^*)(D-A)} + \rho(1-e^{-(g-s_K r^*)(D-R)})}{1-e^{-(g-s_K r^*)H}} \qquad \textbf{(E.4)}$$

With $\rho=1$, we are back to the above formula. $\neg$ $\rho<1$, $\mu(g,\rho) \to \hat{\mu}$ as $g \to 0$.
Note also that $\mu(g,\rho) \to \mu_0(\rho) < 1$ as $g \to +\infty$, with $\mu_0(\rho)$ given by:

$$\mu_0(\rho) = \frac{\rho(D-A)}{(R-A)+\rho(D-R)}$$

$\mu_0'(\rho)>0$. $\mu_0(\rho) \to 1$ as $\rho \to 1$. $\mu_0(\rho) \to 0$ as $\rho \to 0$.

The key difference with the case $\rho=1$ is that retired individuals now do not fully benefit from labor income growth, so for ρ small and g high the labor wealth profile $w_{Lt}(a)/w_t$ becomes downward sloping above retirement age. In the extreme case $\rho=0$ and $g \to +\infty$, the relative

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rather than from labor income: with $\rho=1$ older individuals have the same labor income as younger ones. When we introduce $\rho<1$, then the Shorrocks labor income effect shows up (older indidivuals are poorer because they do not fully benefit from labor income growth), which reinforces the total growth effect and makes the cross-sectional profile even more hump-shaped (see below).

wealth of the elderly μ^* tends toward 0% (they become infinitely poor as compared to workers), and inheritance vanishes. But for this effect to be quantitatively significant, growth rates need to be enormous. E.g. for ρ=0% and g=10%, then μ^*=40% and b_y^*=4%. For more reasonable values of g, μ^* and b_y^* are much closer to class saving levels. On Table E1 we provide numerical illustrations of the $\mu(g)$ and $\mu(g,\rho)$ formulas for various parameter values.[262]

E.2. Proof of Proposition 4 (section 5.2)

(exogenous savings model, open economy)

(i). Wealth-income ratio. We first solve for the long-run $\beta_t=W_t/Y_t$. In the closed economy case, steady-state $\beta^*=s/g$ and $r^*=\alpha g/s$ (with $s=(1-\alpha)s_L+\alpha s_K$) follow directly from the wealth accumulation equation $dW_t/dt=sY_t$, i.e. $d\beta_t/dt=s-g\beta_t$. The open economy case introduces two complications into this equation: the capital share (incl. net foreign asset income) generally differs from α, so the aggregate saving rate is now endogenous (except in the uniform saving case $s=s_L=s_K$); the long run growth rate of national income can differ from g, in case the world rate of return r is larger than $\grave{r}=g/s_K$ (in which case it will be equal to $s_K\grave{r}>g$). To solve the model, we use the following notations. Private wealth W_t is now equal to the sum of the domestic capital stock K_t and net foreign assets W_{Ft} (≥ 0 or ≤ 0): $W_t=K_t+W_{Ft}$. National income Y_t is equal to the sum of domestic income (domestic output, i.e. net domestic product) $Y_{pt}=F(K_t,H_t)$ and net foreign asset income rW_{Ft}: $Y_t=Y_{pt}+rW_{Ft}=(1+r\beta_{Ft})Y_{pt}$ (with $\beta_{Ft}=W_{Ft}/Y_{pt}$ = foreign wealth-domestic income ratio). We maintain the Cobb-Douglas assumption for domestic production ($Y_{pt}=F(K_t,H_t)=K_t^{\alpha}\,H_t^{1-\alpha}$), so the domestic capital/output ratio $\beta_{Kt}=K_t/Y_{pt}$ is permanently equal to $\beta_K^*=\alpha/r$, and the growth rate of domestic output is permanently equal to g. The wealth-national income ratio $\beta_t=W_t/Y_t$ is equal to $\beta_{pt}/(1+r\beta_{Ft})$, where $\beta_{pt}=W_t/Y_{pt}=\beta_{Kt}+\beta_{Ft}$ is the wealth-domestic income ratio. The wealth accumulation equation can be written as follows:

$$dW_t/dt = dW_{Ft}/dt + dK_t/dt = s_L Y_{Lt} + s_K Y_{Kt}$$

$$\text{With: } Y_{Lt} = (1-\alpha)Y_{pt} = \text{labor income}$$

$$Y_{Kt} = rW_t = \alpha Y_{pt} + rW_{Ft} = \text{corrected capital income (incl. net foreign asset income)}$$

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[262] The values for $\mu^*=\mu(g,\rho)$ reported in Table E1 apply for all savings rates s_K and s_L (all what matters for $\mu^*=\mu(g,\rho)$ is the relative share of labor income savings in total savings $(1-\alpha)s_L/s$). In order to illustrate the impact of μ^* on b_y^*, we also report the corresponding $b_y^*=\mu^*m^*\beta^*$ assuming a fixed β^*=600% (this is implicitly assuming that levels of s_K and s_L adapt to changes in g so as to keep β^* constant; this allows us to focus upon the pure age profile effect on b_y^*, shutting down the aggregate wealth accumulation β^* effect).

Note that $dK_t/dt = gK_t = g\alpha Y_{pt}/r$.

By differentiating $\beta_{Ft} = W_{Ft}/Y_{pt}$ one then obtains the following dynamic equation:

$$d\beta_{Ft}/dt = (1-\alpha)s_L + (\alpha + r\beta_{Ft})s_K - g\alpha/r - g\beta_{Ft} = (s - g\alpha/r) - (g - s_K r)\beta_{Ft}$$

Where $s=(1-\alpha)s_L + \alpha s_K$ is again the aggregate closed economy saving rate.

If $r>\grave{r}=g/s_K$ t, i.e. if the rate of wealth reproduction $s_K r$ is larger than g, then this dynamic equation admits no (stable) steady-state: $\beta_{Ft}\to+\infty$ as $t\to+\infty$. I.e. in the long run domestic output Y_{pt} becomes negligible as compared to foreign asset income rW_{Ft}, and national income $Y_t \approx rW_t$ grows at rate $s_K r>g$. It follows that $\beta_t=W_t/Y_t \to 1/r$ as $t\to+\infty$.

If $r<\grave{r}$, i.e. if $g-s_K r>0$, then this dynamic equation admits a unique steady-state β_F^*, with:

$$\beta_F^* = (s-g\alpha/r)/(g-s_K r).= s(1-r^*/r)/(g-s_K r)$$

Note that $\beta_F^*>0$ if $r>r^*$ and $\beta_F^*<0$ if $r<r^*$ (where $r^*=\alpha g/s \leq \grave{r}$ is the closed-economy steady-state). $\beta_F^*\to+\infty$ as $r\to \grave{r}$.

Finally, we have: $\beta^{**} = (\beta_K^*+\beta_F^*)/(1+r\beta_F^*) = s_L/[g-r(s_K-s_L)]$.

With uniform savings ($s=s_L=s_K$), then $\beta^{**} = s/g = \beta^*$. I.e. the open economy steady-state wealth-income ratio is the same as the closed economy ratio, and does not depend on the world rate of return r.

If $s_K>s_L$ (resp. $s_K<s_L$), then β^{**} is an increasing (resp. decreasing) function of r, and is larger than the closed economy ratio iff $r>r^*$ (resp. $r<r^*$).

Note that $\beta^{**}\to 1/r$ as $r\to \grave{r}$. Note also that the class saving case ($s_L=0$ & $s_K>0$) is degenerate: if $r>\grave{r}=r^*$ then $\beta_{Ft}\to+\infty$ as $t\to+\infty$; if $r<\grave{r}$ then $\beta_{Ft}\to 0$ as $t\to+\infty$.

<u>Example</u>: g=2%, $s=s_K=s_L$=12%, α=30%. In the closed-economy, $\beta^*=s/g$=600% and $r^*=\alpha/\beta^*$=5%. Now assume the economy opens up and faces a world rate of return r=6%. Then the domestic capital-output ratio β_K^* declines to α/r=500%, and wealth-holders accumulate foreign assets equivalent to β_F^*=156% of domestic output, bringing $r\beta_F^*$=9% additional income. The wealth-income ratio $\beta^{**}=(\beta_K^*+\beta_F^*)/(1+r\beta_F^*)$ is unchanged at 600%.

Now assume g=2%, s_K=24% and $s_L$such that s=12% ($s_L\approx$9%). Then as the economy opens up, β_K^* still declines from 600% to 500%, but β_F^* now rises to 250% (bringing that $r\beta_F^*$=15% additional income), so that the wealth-income ratio β^{**} rises to 652%.

<u>(ii). Inheritance ratios</u> The formulas for the age-wealth profile $w_t(a) = w_{Bt}(a) + w_{Lt}(a)$ are the same as in the closed economy case, except that r* needs to be replaced by r. In the case $r>\bar{r}$, then labor income y_{Lt} vanishes in the long run, i.e. $y_{Lt}/w_t \to 0$ as $t\to+\infty$, and so does labor wealth $w_{Lt}(a)$ relatively to bequest wealth $w_{Bt}(a)$. So we are back to the class saving case, and $\mu_t \to \bar{\mu}$ as $t\to\infty$. In the case $r<\bar{r}$, $\mu_t \to \mu^*=\mu(g,r,\rho)$ as $t\to\infty$, with $\mu(g,r,\rho)$ given by the same formulas as in the closed economy case (except that r* is replaced by r):

$$\mu(g,r) = \frac{1-e^{-(g-s_K r)(D-A)}}{1-e^{-(g-s_K r)H}} \qquad \textbf{(E.5)}$$

The limit results as $g\to 0$ apply for the same reasons. The limit results as $r\to\bar{r}$ follow from the fact that $\beta_F^*\to+\infty$ as $r\to\bar{r}$, so labor wealth $w_{Lt}(a)$ again vanishes relatively to bequest wealth $w_{Bt}(a)$. The results for b_{wt} and b_{yt} follow directly from the μ_t results.[263] On Table E2 we provide numerical illustrations of the $\mu(g,r)$ for various parameter values.[264]

<u>E.3. Proof of Proposition 6 (section 5.3)</u>
(dynastic model, ρ=1, with borrowing)

We assume ρ=1. In case young agents (A≤a<I) cannot borrow against future inheritance receipts, then the cross-sectional age-wealth profile $w_t(a)$ is the same as with class saving:

If If $a\in[A,I[$ $w_t(a) = 0$

If If $a\in[I,D]$ $w_t(a) = \overline{w}_t = \bar{\mu}\, w_t$

But in case young agents are allowed to borrow against future inheritance, then they will choose to do so as soon as they become adult. We assume perfect foresight and solve for the corresponding steady-state. Consider agents belonging to a given cohort x. They know from the beginning of their adult life that they will receive (average) inheritance b^x at age a=I. They choose a consumption path $c^x(a)$ (A≤a≤D) in order to maximize dynastic utility, anticipating that their offspring will do the same.[265] Utility maximization implies that they will choose a consumption path growing at rate $g_c=(r_t-\theta)/\sigma$, i.e. at rate $g_c=g$ since we look

[263] Here we refer to $b_{yt}=\mu_t m_t \beta_t$, i.e. to the inheritance flow-national income ratio. If we were instead using domestic income as the denominator, then the inheritance-income ratio would naturally $\to+\infty$ in the case $r>\bar{r}$ (in the same way as in the wealth-in-utility-function model, see below).

[264] The values for $\mu^*=\mu(g,r)$ reported in Table E2 were computed for a fixed s_K and for ρ=1. In order to illustrate the impact of μ^* on b_y^*, we again report the corresponding $b_y^*=\mu^* m^* \beta^*$ assuming a fixed β*=600% (this is implicitly assuming that s_L adapt to changes in g so as to keep β* constant).

[265] Here we write wealth and consumption equations at the aggregate cohort level, but because of linearity they are exactly the same at the same at the individual level (i.e. everything applies proportionally to each dynasty i with inheritance b_i^x within cohort x). Also note that we do not need to worry about labor income since with ρ=1 it is entirely consumed (irrespective of the bequest level, again because of linearity).

at steady-state paths with $r_t=r^*=\theta+\sigma g$. That is, they choose a consumption path of the form: $c^x(a) = c^x(A)\ e^{g(a-A)}$. One possibility would be to set their initial consumption level $c^x(A)$ equal to a fraction r^*-g of the present value of their bequest ($c^x(A)=(r^*-g)\ b^x\ e^{-r^*(I-A)}$), which by definition corresponds to a consumption path that can be sustained for ever. But they need to anticipate that their children will do the same, i.e. they will also start consuming their bequest before they receive it.[266] So the time-consistent utility maximizing consumption path $c^x(a) = c^x(A)\ e^{g(a-A)}$ must be such that cohort x leaves bequest b^{x+H} allowing the next generation to pursue the same consumption path, i.e. such that $b^{x+H} = b^x\ e^{gH}$. Now, for given $c^x(A)$, the wealth path $w^x(a)$ followed by cohort x will look as follows:

If $a \in [A,I[\quad w^x(a) = - \int_{A \le a' \le a} c^x(A)\ e^{g(a'-A)}\ e^{r^*(a-a')}\ da'$

If $a \in [I,D]\quad w^x(a) = b^x\ e^{r^*(a-I)} - \int_{A \le a' \le a} c^x(A)\ e^{g(a'-A)}\ e^{r^*(a-a')}\ da'$

That is:

If $a \in [A,I[\quad w^x(a) = - c^x(A)\ \dfrac{e^{r^*(a-A)} - e^{g(a-A)}}{r^* - g}$

If $a \in [I,D]\quad w^x(a) = b^x\ e^{r^*(a-I)} - c^x(A)\ \dfrac{e^{r^*(a-A)} - e^{g(a-A)}}{r^* - g}$

Note that wealth $w^x(a)$ is negative until inheritance age (as cohort x borrows against future inheritance) and positive afterwards. Wealth at death $w^x(D)$ left by cohort x is by definition equal to the bequest b^{x+H} received by the next generation:

$$b^{x+H} = w^x(D) = b^x\ e^{r^*H} - c^x(A)\ \frac{e^{r^*(D-A)} - e^{g(D-A)}}{r^* - g}$$

The consumption path is time consistent iff $b^{x+H} = b^x\ e^{gH}$, i.e. iff:

$$c^x(A) = (r^*-g)\ \frac{e^{r^*H} - e^{gH}}{e^{r^*(D-A)} - e^{g(D-A)}}\ b^x$$

Note that this is lower than $(r^*-g)\ b^x\ e^{-r^*(I-A)}$), i.e. time consistency forces to choose lower consumption path.

We can now compute the resulting cross-sectional wealth profile $w_t(a)$. Individuals who are a-year-old at time t belong to cohort x=t-a, and they will receive (or have received) inheritance $b^x = b_t\ e^{g(I-a)}$ at time t+I-a (at age I). So we have:

If $a \in [A,I[\quad w_t(a) = - b_t\ e^{g(I-A)}\ \dfrac{e^{r^*H} - e^{gH}}{e^{r^*(D-A)} - e^{g(D-A)}}\ [e^{(r^*-g)(a-A)}-1]$

[266] Parents derive utility only from what their children consume after age I; but children also care about what they consume between age A and age I; in perfect foresight steady-states, parents end up internalizing this borrowing behavior (this is of course assuming that parents cannot disallow their children to borrow against inheritance, which does not seem very realistic; see discussion in working paper, section 7.3).

If $a \in [I,D]$ $w_t(a) = = b_t \left[e^{(r^*-g)(a-I)} - e^{g(I-A)} \dfrac{e^{r^*H} - e^{gH}}{e^{r^*(D-A)} - e^{g(D-A)}} [e^{(r^*-g)(a-A)}-1] \right]$

If we now compute average wealth $w_t = \int_{A \le a \le D} w_t(a)\, da$ and the ratio $\mu_t = \dfrac{w_t(D)}{w_t}$, we obtain the following formula for steady-state μ^* :

$$\mu^* = \frac{e^{(r^*-g)(D-A)} - 1}{e^{(r^*-g)H} - 1} \qquad \textbf{(E.6)}$$

Note that $\forall\, r^*-g>0$, $\mu^* > \overline{\mu} = (D-A)/H$.

Steady-state μ^* is an increasing function of r^*-g.

$\mu^* \rightarrow \overline{\mu}$ as $r^*-g \rightarrow 0$ (which can occur only if $\theta \rightarrow 0$ and $g \rightarrow 0$).

The reason why μ^* is always larger than $\overline{\mu}$ when we allow for borrowing is fairly intuitive. Because young agents borrow against future inheritance, the cross-sectional age-wealth profile $w_t(a)$ is negative and downward sloping from age A to I, then jumps to positive levels at age I, and then is upward sloping from age I to age D (see Figure E4).[267] This pushes upwards the relative wealth of decedents μ_t and the steady-state inheritance flows. A larger r^*-g differential will make the downward-sloping part more steeply declining, and the upward-sloping part more steeply rising, thereby pushing μ_t further up. The formula shows that the effect can become very large and push steady-state μ^* towards very high levels, especially if life expectancy is large. E.g. with A=20, D=70, H=30, g=2% and r*=5%, one gets μ^*=239% (instead of $\overline{\mu}$=167% in the case with no borrowing). But with A=20, D=80, H=30, g=0% and r*=5%, one gets μ^*=548% (instead of $\overline{\mu}$=200% in the case with no borrowing). With β^*=600%, this would imply an aggregate inheritance flow equal to b_y^*=55% of national income (instead of 20% with no borrowing). We view this extreme case merely as an intellectual curiosity.

E.4. Proof of Proposition 7 (section 5.3)

(dynastic model, ρ≤1, no borrowing)

(i) Computation of lifecycle saving rates. We now consider the case $\rho<1$. That is, instead of getting (average) labor income y_{Lt} during their entire adult lifetime (R≤a≤D), agents now receive $y_{Lt}(a)=(1-\tau_\rho)\,\hat{y}_{Lt}$ when they are working (A≤a<R) and $y_{Lt}(a)=\rho(1-\tau_\rho)\,\hat{y}_{Lt}$ when they are retired (R≤a≤D), with:

[267] We used the following parameters for Figure E4 (see formulas in excel file): A=20, H=30, D=70, g=2%, r*=5%. Applying the formulas we get $\overline{\mu}$=167% (no borrowing) and μ^*=239%.

$\hat{y}_{Lt} = \frac{D-A}{R-A}\, y_{Lt}$ = average pre-tax labor income of adult workers at time t

τ_ρ = budget-balanced pension tax rate = $\frac{\rho(D-R)}{R-A+\rho(D-R)}$

With ρ=0 they receive no pension at all. With ρ=1 we are back to the previous case.

Consider agents belonging to a given cohort x. We assume that young agents cannot borrow against future inheritance. More precisely, we assume that they behave until age I as if they were not going to receive any inheritance; so in effect they maximize twice their dynastic utility function: once at age A (under the anticipation that they will receive no inheritance), and once at age I (in case they receive inheritance they revise their plan).[268] Utility maximization at age A implies that they will choose a consumption path $c^x(a)=c^x(A)e^{g(a-A)}$ growing at rate g during their entire lifetime.[269] To achieve this goal, they save a fraction $s_L y_L{}^x(a)$ of their labor income when they are working ($A\leq a<R$), in order to finance an extra consumption flow $c_L{}^x(a)$ when they are retired ($R\leq a\leq D$). Since $y_L{}^x(a)=y_L{}^x(A)e^{g(a-A)}$ for $a\in[A,R[$ and $y_L{}^x(a)=\rho y_L{}^x(A)e^{g(a-A)}$ for $a\in[R,D]$, the extra consumption flow picked by utility maximizing agents will simply be equal to $c_L{}^x(a)=c_L y_L{}^x(A)e^{g(a-A)}$, with $c_L+\rho=1-s_L$, i.e. agents will save whatever it takes in order to complement the pension replacement rate and ensure an effective replacement rate of 100% at retirement age.[270] In order to equilibrate the saving and dissaving flows, the saving rate s_L must be such that:

$$s_L \int_{A\leq a\leq R} e^{(r^*-g)(R-a)}\, da \;=\; c_L \int_{R\leq a\leq D} e^{(r^*-g)(R-a)}\, da \qquad \text{(E.7)}$$

Replacing c_L by $1-s_L-\rho$, we get the following formula for the lifecycle saving rate s_L:

$$s_L = (1-\rho)\,\overline{s}_L$$

$$\text{With: } \overline{s}_L = \frac{1-e^{-(r^*-g)(D-R)}}{e^{(r^*-g)(R-A)} - e^{-(r^*-g)(D-R)}} \qquad \textbf{(E.8)}$$

In case the pension system offers 100% replacement rate (ρ=1), then there is no need for lifecycle saving (s_L=0). Conversely if there is no pension system (ρ=0), then lifecycle

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[268] If we instead assume perfect foresight on inheritance receipts and full maximization at age A, then individuals with high expected inheritance will save less between age A and I (and possibly not at all) than the formulas below (which would only apply to individuals with zero inheritance). This might be relevant empirically (borrowing against future inheritance is difficult in practice, but lowering saving is easy). So the formulas below should be viewed as an upper bound for lifecycle wealth.

[269] We look at steady-state paths ($r^*=r+\theta\sigma$), so the desired consumption growth rate $g_c=(r-\theta)/\sigma$ is equal to g.

[270] Here we again write wealth and consumption equations at the aggregate cohort level, but because of linearity they are exactly the same at the same at the individual level (i.e. everything applies proportionally to each dynasty i with labor income y_{Li} within cohort x). Again because of linearity we can look separately at lifecycle wealth and forget about bequest wealth.

savings takes its maximal value $\overline{s}_L$. Note that as long as r*-g>0, $\overline{s}_L < \overline{\tau}_p = \frac{D-R}{D-A}$, i.e. the private savings rate delivering 100% replacement rate at retirement is less than the pension tax rate delivering the same outcome. This is a direct consequence of the fact that the internal rate of return of unfunded pay-as-you-go pension system is equal to g, while the rate of return on private savings is equal to r*. In case $r^*-g \to 0$ (which in the steady-case of the dynastic model requires r*=g=θ=0), then $\overline{s}_L \to \overline{\tau}_p = \frac{D-R}{D-A}$. Conversely, other things equal, the higher r*-g, the lower $\overline{s}_L$.

Example: Take A=20, R=60, D=80. Then if r*-g=0, $\overline{s}_L = \overline{\tau}_p$=33%. If r*-g=1%, then $\overline{\tau}_p$ is still equal to 33%, but $\overline{s}_L$=27%. If r*-g=3% (say, g=2, r*=5%), then $\overline{s}_L$=16% (see Table E3).

(ii) Computation of lifecycle wealth. Once we know s_L, the longitudinal age profile of lifecycle wealth $w_L^x(a)$ follows:

If $a \in [A,R]$ $\quad w_L^x(a) = s_L\, y_L^x(A) \int_{A\leq a'\leq a} e^{g(a'-A)}\, e^{r^*(a-a')}\, da'$

If $a \in [R,D]$ $\quad w_L^x(a) = s_L\, y_L^x(A) \int_{A\leq a'\leq R} e^{g(a'-A)}\, e^{r^*(a-a')}\, da' \;-\; c_L\, y_L^x(A) \int_{R\leq a'\leq a} e^{g(a'-A)}\, e^{r^*(a-a')}\, da'$

Replacing c_L by $1\text{-}s_L\text{-}\rho$ and s_L by $(1\text{-}\rho)\overline{s}_L$, we get:

If $a \in [A,R]$ $\quad w_L^x(a) = (1-\rho)\overline{s}_L\; y_L^x(A)\, e^{g(a-A)}\; \frac{e^{(r^*-g)(a-A)}-1}{r^*-g}$

If $a \in [R,D]$ $\quad w_L^x(a) = (1-\rho)\, y_L^x(A)\, e^{g(a-A)}\; \left[\overline{s}_L\; \frac{e^{(r^*-g)(a-A)}-1}{r^*-g} - \frac{e^{(r^*-g)(a-R)}-1}{r^*-g}\right]$

With ρ=1 there is no lifecycle wealth. With ρ<1, lifecycle wealth $w_L^x(a)$ has the usual hump-shaped profile: it rises from zero at age a=A to a maximum $w_L^x(R)$ at retirement age a=R, and then declines towards zero at death age a=D. In case $r^*-g \to 0$, then we get the standard Modigliani triangle:

If $a \in [A,R]$ $\quad w_L^x(a) = (1-\rho)\overline{s}_L\; y_L^x(A)\,(a-A) = (1-\rho)\, \frac{D-R}{D-A}\, y_L^x(A)\,(a-A)$

If $a \in [R,D]$ $\quad w_L^x(a) = (1-\rho)\, y_L^x(A)\, [\overline{s}_L\; (a-A) - (a-R)\,] = (1-\rho)\, \frac{R-A}{D-A}\, y_L^x(A)\,(D-a)$

We can now compute the resulting cross-sectional age profile of lifecycle wealth $w_{Lt}(a)$. Individuals who are a-year-old at time t belong to cohort x=t-a, and they received labor income $y_L^x(A) = y_{Lt}\, e^{-g(a-A)}$ at time t+A-a (at age A). So we have:

If $a \in [A,R]$ $\quad w_{Lt}(a) = (1\text{-}\rho)\,\overline{s}_L\ y_{Lt}\ \dfrac{e^{(r^*-g)(a-A)}-1}{r^*-g}$

If $a \in [R,D]$ $\quad w_{Lt}(a) = (1\text{-}\rho)\ y_{Lt}\ \left[\overline{s}_L\ \dfrac{e^{(r^*-g)(a-A)}-1}{r^*-g} - \dfrac{e^{(r^*-g)(a-R)}-1}{r^*-g}\right]$

From these equations we can compute average lifecycle wealth $w_{Lt} = \int_{A\leq a\leq D} w_{Lt}(a)da$, and define $\beta_L=\dfrac{w_{Lt}}{y_{Lt}}$ the ratio between average lifecycle wealth and average labor income. We obtain the following formula:

$$\beta_L=\frac{w_{Lt}}{y_{Lt}} = (1\text{-}\rho)\,\overline{\beta}_L$$

With:

$$\overline{\beta}_L = \frac{1}{D-A}\left[\overline{s}_L\ \frac{e^{(r^*-g)(D-A)}-1-(r^*-g)(D-A)}{(r^*-g)^2} - \frac{e^{(r^*-g)(D-R)}-1-(r^*-g)(D-R)}{(r^*-g)^2}\right] \qquad \textbf{(E.9)}$$

β_L measures the number of years of labor income which is being accumulated in lifecycle wealth in this economy. $\overline{\beta}_L$ is the maximum value of β_L, i.e. the value prevailing in the absence of a pay-as-you-go pension system (ρ=0). As the pension system becomes more and more generous ($\rho\to1$), $\beta_L=(1\text{-}\rho)\,\overline{\beta}_L$ declines linearly towards zero.

In case $r^*\text{-}g\to0$, then $\overline{\beta}_L \to = \dfrac{1}{D-A}\left[\overline{s}_L\ \dfrac{(D-A)^2}{2} - \dfrac{(D-R)^2}{2}\right] = \dfrac{(D-R)(R-A)}{2(D-A)}$

This is the standard Modigliani triangle formula: in an economy with zero growth and zero rate of return, then in order to consume as much during retirement as during their working life, then individuals need to accumulate lifecycle wealth equivalent to (D-R)/2 years of labor income, where D-R is retirement length.[271]

<u>Example</u>: Assume r*-g=0%, A=20, R=60. With D=70, then $\overline{\beta}_L$ = 400%. That is, lifecycle wealth equals 400% of aggregate labor income. With D=80, then $\overline{\beta}_L$ = 667%. If r*-g>0, then the capitalization effect allows to lifecycle savers to save less, and the economy as a

[271] See e.g. Modigliani (1986). The intuition for this well known formula is very simple: at the top of their wealth accumulation trajectory (at age a=R), lifecycle savers need to own the equivalent of D-R years of labor income in order to finance retirement consumption; by linearity, individuals on the rising and declining segments of the triangle (below and above age a=R) own on average (D-R)/2 years. The reason why (D-R)/2 needs to be multiplied by (R-A)/(D-A) is because we divide lifecycle wealth w_{Lt} by per adult labor income y_{Lt} (rather than by per worker labor income=$(D\text{-}A)y_{Lt}/(R\text{-}A)$). This makes more sense from an aggregate wealth accumulation viewpoint. So for instance if R is fixed and D goes to infinity, then $\overline{\beta}_L \to$ (R-A)/2 (not to infinity).

whole to accumulate lower lifecycle wealth. So for instance if r*-g=3% (say, because g=2% and r*=5%), then $\overline{\beta}_L$ = 332% with D=70 and $\overline{\beta}_L$ = 568% with D=80 (see Table E3).

Note that $\overline{s}_L$ and $\overline{\beta}_L$ depends solely on the differential r*-g, and not on the absolute level of either r* or g. Also note that a rise in r*-g does not have a huge impact on the quantitative magnitudes: $\overline{\beta}_L$ declines, but not that much.[272]

Note also that if we want to compute maximal lifecycle wealth w_{Lt} as a fraction of national income y_t rather than labor income y_{Lt}, then we need to multiply $\overline{\beta}_L$ by the labor share 1-α. So for instance with α=30% and r*-g=3%, we have $(1-\alpha)\overline{\beta}_L$ = 232% with D=70 and $(1-\alpha)\overline{\beta}_L$ = 398% with D=80. If we now want to compute actual lifecycle wealth (given the existence of a pay-as-you-go pension system in the model), then we need to multiply $(1-\alpha)\overline{\beta}_L$ by 1-ρ. So for instance with ρ=80%, α=30% and r*-g=3%, we have $(1-\alpha)\overline{\beta}_L$ = 46% with D=70 and $(1-\alpha)\overline{\beta}_L$ = 80% with D=80. With ρ=50%, we get 116% and 199%.

(iii) Computation of inheritance ratios. Finally, we can compute the total age-wealth profile $w_t(a)$ and the μ_t ratio. The utility-maximizing profile of consumption must grow at rate g, so after they receive their inheritance, agents save a fraction s_K=g/r* of the corresponding flow return (and consume the rest). So the growth and saving effect again compensate each other, and the age profile of bequest wealth $w_{Bt}(a)$ is flat above inheritance age, in the same way as in the class saving model:

If $a\in[A,I[$ $\quad w_{Bt}(a) = 0$

If $a\in[I,D]$ $\quad w_{Bt}(a) = b_t$

The total age-wealth profile $w_t(a)$ is given by summing up the two profiles:

$\neg\, a\in[A,D]$ $\quad w_t(a) = w_{Bt}(a) + w_{Lt}(a)$

Computing the averages over all ages we have:

$$w_t = \frac{H}{D-A}\, b_t + w_{Lt}$$

Replacing w_{Lt} by $(1-\alpha)\beta_L y_t$ and w_t/y_t by its steady-state value β*, we obtain that $\mu_t=b_t/w_t$ has a unique steady-state level μ* given by:

[272] Intuitively, this is because a rise in r*-g makes everybody richer (workers now receive labor income and capital income, while with r*=g=0 there was no capital income at all, i.e. saving was a pure storage technology); to obtain given absolute living standards during retirement, workers could afford accumulating a lot less lifecycle wealth; but because they seek the have the same relative consumption during work and retirement years, lifecycle wealth does not decline all that much as r*-g rises.

$$\mu^* = \bar{\mu}\ [1-\frac{(1-\alpha)\beta_L}{\beta^*}] = \bar{\mu}\ [1-\frac{(1-\rho)(1-\alpha)\bar{\beta}_L}{\beta^*}] \qquad \textbf{(E.10)}$$

On Table E4 we provide numerical illustrations for this formula, using various values of the replacement rate ρ.[273] On Figures E5-E8 we show the steady-state age-wealth profiles: with ρ close to 100%, then hump-shaped lifecycle wealth is not very large and has little impact on the overall age-wealth profile; but as ρ→0% lifecycle wealth plays a larger role and the overall profile becomes more and more hump-shaped.[274] Note that μ* is always higher for lower growth rates g. This is because the ratio $\frac{(1-\alpha)\bar{\beta}_L}{\beta^*}$ (i.e. the share of maximal lifecycle wealth in aggregate wealth) is an increasing function of the growth rate.[275] The intuition is again that higher growth favours new savings relatively to inheritance. However the impact of g on μ* is smaller than in the exogenous saving model. In particular, as g→0, μ* does not converge toward $\bar{\mu}$: lifecycle wealth remains strictly positive, so μ* remains strictly below $\bar{\mu}$.

E.5. Proof of Propositions 8-9 (section 5.4)
(wealth-in-the-utility model)

(i) open economy, ρ=1, with borrowing. We start with the open economy case, which is easier to solve in the wealth-in-the-utility model, and we assume ρ=1 (i.e. we shut down the lifecycle saving motive). We also start by assuming that young agents can borrow against future inheritance, and we assume perfect foresight about future inheritance receipts. Consider agents belonging to a given cohort x. During their lifetime ($a \in [A,D]$) they receive (average) labor income flows $y_L^x(a)=y_L^x(A)e^{g(a-A)}$. At age a=I they receive

[273] We again report the corresponding $b_y^*=\mu^* m^* \beta^*$ assuming a fixed β*=600% (this is implicitly assuming that θ adapts to changes in g so as to keep β* constant).
[274] We use the following parameters for Figures E5-E8 (see formulas in excel file): A=20, H=30, R=60, D=80, g=2%, r*=5%. Applying the formulas we get μ*=173% (ρ=80%), μ*=134% (ρ=50%) and μ*=67% (ρ=0%) (see Table E4).
[275] See Table A3 for detailed computations on the lifecycle wealth share. Note that this is true both a given r* (a rise in g then implies a fall in r*-g, and therefore a rise in $\bar{\beta}_L$), and for an endogenous r*=θ+σg: a rise in g then leads to a rise in r*-g (assuming σ>1) and a decline in $\bar{\beta}_L$, but the rise in r* means an even bigger decline in β*, so that the lifecycle wealth share rises. Note also that for extreme parameter values (g above 4%-5%, endogenous r* above 10%), then lifecycle wealth may exceed aggregate wealth: strictly speaking this would imply negative bequests, i.e. borrowing from future generations, but this is impossible.

(average) bequest b^x, with capitalized end-of-life value $b^x e^{rH}$. We note $\tilde{y}_L{}^x$ the end-of-life capitalized value of their labor income flows $y_L{}^x(a)$:

$$\tilde{y}_L{}^x = \int_{A\leq a\leq D} e^{r(D-a)}\, y_L{}^x(a)\, da \;=\; y_L{}^x(A)\, e^{r(D-A)}\, \frac{1-e^{-(r-g)(D-A)}}{r-g}$$

We note $\tilde{y}^x = \tilde{y}_L{}^x + b^x e^{rH}$ their total lifetime resources (capitalized at the end of life). At age a=A they maximize $V(U_c,w^x(D))$ in order to allocate $\tilde{y}$ between their lifetime consumption flows $c^x(a)$ ($a \in [A,D]$) and their end-of-life wealth $w^x(D)$. With $U_C=[\int_{A\leq a\leq D} e^{-\theta(a-A)} c^x(a)^{1-\sigma} da\,]^{\frac{1}{1-\sigma}}$, standard first-order conditions imply that they will that they will choose a consumption path $c^x(a)=c^x(A)e^{g_c(a-A)}$ growing at rate $g_c=(r-\theta)/\sigma$ during their lifetime. Note that $(1-\sigma)g_c-\theta=g_c-r$. The utility value U_c of this consumption flow is given by:

$$U_c = [\int_{A\leq a\leq D} e^{-\theta(a-A)}\, c^x(a)^{1-\sigma}\, da\,]^{\frac{1}{1-\sigma}} = c^x(A)\,\left(\frac{1-e^{-(r-g_c)(D-A)}}{r-g_c}\right)^{\frac{1}{1-\sigma}}$$

We note $\tilde{c}^x$ the end-of-life capitalized value of consumption flows $c^x(a)=c^x(A)e^{g_c(a-A)}$:

$$\tilde{c}^x = \int_{A\leq a\leq D} e^{r(D-a)}\, c^x(a)\, da \;=\; c^x(A)\, e^{r(D-A)}\, \frac{1-e^{-(r-g_c)(D-A)}}{r-g_c}$$

The lifetime budget constraint is: $\tilde{c}^x + w^x(D) \leq \tilde{y}^x$

Maximization of $V[U_c,w^x(D)]=(1-s_B)\log(U_C)+s_B\log[w^x(D)]$ implies:

$w^x(D)=s_B\,\tilde{y}^x$ and $\tilde{c}^x=(1-s_B)\,\tilde{y}^x$.

Thanks to linearity, the consumption profile can be broken down into bequest-financed and labor-financed consumption (each flow growing at rate g_c): $c^x(a) = c_B{}^x(a) + c_L{}^x(a)$. We have:

$c^x(A)= c_B{}^x(A) + c_L{}^x(A)$

$c_B{}^x(A) = (1-s_B)\dfrac{r-g_c}{1-e^{-(r-g_c)(D-A)}}\; b^x e^{-r(I-A)}$

$c_L{}^x(A) = (1-s_L)y_L{}^x(A)$

with:[276] $$1\text{-}s_L=(1\text{-}s_B)\ \frac{(r-g_c)(1-e^{-(r-g)(D-A)})}{(r-g)(1-e^{-(r-g_c)(D-A)})} \qquad \text{(E.11)}$$

The wealth profile $w^x(a)$ can be be written: $w^x(a) = w_B{}^x(a)+w_L{}^x(a)$, where $w_B{}^x(a)$ is bequest wealth (i.e. the non-consumed part of capitalized bequest ressources at age a) and $w_L{}^x(a)$ is labor wealth (i.e. the non-consumed part of capitalized labor ressources at age a):

$$\neg\ a\in[A,D],\ w_L{}^x(a) = \int_{A\le a'\le a} e^{r(a-a')}y_L{}^x(a')da - \int_{A\le a'\le a} e^{r(a-a')}c_L{}^x(a')da$$

$$\text{I.e. } \neg\ a\in[A,D],\ w_L{}^x(a) = y_L{}^x(A)\ e^{r(a-A)}\left[\frac{1-e^{-(r-g)(a-A)}}{r-g} - (1\text{-}s_L)\ \frac{1-e^{-(r-g_c)(a-A)}}{r-g_c}\right]$$

$$\neg\ a\in[A,I[,\ w_B{}^x(a) = -\int_{A\le a'\le a} e^{r(a-a')}c_B{}^x(a')da = -\ b^x e^{r(a-I)}\ (1\text{-}s_B)\ \frac{1-e^{-(r-g_c)(a-A)}}{1-e^{-(r-g_c)(D-A)}}$$

$$\neg\ a\in[I,D],\ w_B{}^x(a) = b^x e^{r(a-I)} - \int_{A\le a'\le a} e^{r(a-a')}c_B{}^x(a')da = b^x e^{r(a-I)}\left[1 - (1\text{-}s_B)\ \frac{1-e^{-(r-g_c)(a-A)}}{1-e^{-(r-g_c)(D-A)}}\right]$$

Wealth-at-death $w^x(D)$ left by cohort x to cohort x+H follows a simple dynamic equation:

$$w^x(D) = w_B{}^x(D)+w_L{}^x(D) = b^{x+H} = s_B\,\tilde{y}^x = s_B(\tilde{y}_L{}^x + b^x e^{rH})$$

I.e. $b^{x+H} = s_B\,\tilde{y}_L{}^x + s_B e^{rH}\ b^x$

At time t, the cohort receiving bequest b_t is the cohort born at time x=t-I. This cohort started working with labor income $y_L{}^x(A)=y_{Lt}e^{-g(I-A)}$ and their lifetime labor resources can be rewritten as follows:

$$\tilde{y}_L{}^x = y_L{}^x(A)\ e^{r(D-A)}\ \frac{1-e^{-(r-g)(D-A)}}{r-g} = \lambda\ (D\text{-}A)\ e^{rH}\ y_{Lt}$$

With: $$\lambda\ =\ \frac{e^{(r-g)(I-A)}-e^{-(r-g)(D-I)}}{(r-g)(D-A)} \qquad \text{(E.12)}$$

The dynamic equation can be rewritten: $b_{t+H} = s_B\ \lambda\ (D\text{-}A)\ e^{rH}\ y_{Lt} + s_B e^{rH}\ b_t$

Noting that $y_{Lt}=(1-\alpha)y_{pt}$ (where y_{pt} is per adult domestic income, which in the open economy case differs from per adult national income $y_t=y_{pt}+r(w_t\text{-}k_t)$),[277] we find the following dynamic equation for the inheritance flow-domestic income ratio $b_{yt}=m_t b_t/y_{pt}$:

$$\mathbf{b_{yt+H} = s_B\lambda(1\text{-}\alpha)e^{(r-g)H} + s_B e^{(r-g)H}\ b_{yt}} \qquad \textbf{(E.13)}$$

This process converges iff $s_B e^{(r-g)H} <1$, i.e. iff $r<\bar{r}(g)=\bar{r}+g$, with $\bar{r} = -\log(s_B)/H$.

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[276] In case g_c=g (i.e. in case $r=\hat{r}$, where $\hat{r}=\theta+\sigma g$ is the dynastic model steady-state rate of return), then s_L=s_B. However in the wealth-in-the-utility-function model there is no reason why r should be equal to $\hat{r}$: the closed-economy steady-state r can be larger or smaller than $\hat{r}$ depending on the value of s_B (see below).

[277] We use the same open economy notations as those introduced in the proof of proposition 4 (see above).

Example: With s_B=10% and H=30, then $\bar{r}$=7.7%. With g=0% the process converges iff the world rate of return r is less than 7.7%. With g=1%, it needs to be less than 8.7%.

If $r>\bar{r}(g)$, then as $t\rightarrow+\infty$, $b_{yt}\rightarrow+\infty$.

If $r<\bar{r}(g)$, then as $t\rightarrow+\infty$, $b_{yt}\rightarrow b_y^*$, with: $b_y^* = b_y(g,r) = \dfrac{s_B\lambda(1-\alpha)e^{(r-g)H}}{1-s_B e^{(r-g)H}}$ (E.13)

One can see that $b_y'(g)<0$ and $b_y'(r)<0$. Note that the steady-state inheritance-income ratio b_y^* depends only on the gap r-g, not on the absolute levels of r and g. Numerical computations show that b_y^* is a steeply rising function of r-g. E.g. with s_B=10% and H=30, then b_y^*=8% if r-g=0%, b_y^*=26% if r-g=3% and b_y^*=81% if r-g=5% (see Table E5).
Note however that the very high values of b_y^* obtained for r-g=5% are partly due to the fact that the b_y^* ratio given by equation (E.13) uses the domestic income denominator (rather than national income, which for high r-g is much larger than domestic income). When we look at the $\widehat{b}_y^*$ ratio (which we define using the national income denominator), then we find less extreme quantitative impact of r-g.[278]

One can then use the longitudinal profile $w^x(a)$ equations in order to compute cross-sectional age-wealth profiles $w_t(a)$, and from there obtain closed form analytical formulas for the steady-state ratios $\mu_t=b_t/w_t$ and $\beta_t=w_t/y_t$. This is what we do below for the case without borrowing, which we view as more realistic. The b_y^* formula turns out to be the same without or with borrowing. However the formulas for μ^* and β^* are different. One can easily adapt the no-borrowing μ^* and β^* formulas given below to the borrowing case. In particular, in the same way as in the dynastic model, μ^* will be larger in the borrowing case. When borrowing from future inheritance is allowed, then $w_B^x(a)<0$ for a<I (i.e. young agents borrow in order to raise their consumption, see formulas above), which pushes upwards the steady-state relative wealth of decedents. The difference with the dynastic model is that there will also be less aggregate wealth accumulation (β^* will be lower in the borrowing case), so that b_y^* is unaffected.

<u>(ii) open economy, ρ=1, no borrowing</u>. We now assume that young agents cannot borrow against future inheritance. More precisely, in the same way as in the dynastic model, we assume that they behave until age I as if they were not going to receive any inheritance;

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[278] See Tables E6 and E7 below.

so in effect they maximize twice their utility function $V(U_c,w(D))$: once at age A (under the anticipation that they will receive no inheritance), and once at age I (in case they receive inheritance, they revise their consumption plans accordingly).

Consider agents belonging to a given cohort x. Utility maximization at age A implies that they again choose a consumption path $c^x(a)=c^x(A)e^{g_c(a-A)}$ growing at rate $g_c=(r-\theta)/\sigma$. The utility value U_c of this consumption flow is the same as before. The formulas for end-of-life capitalized values $\tilde{c}^x$ and $\tilde{y}_L{}^x$ are the same as before. The only difference is that $\tilde{y}^x=\tilde{y}_L{}^x$, i.e. there is no anticipated bequest, so total expected resources are equal to labor income resources. So we have $\tilde{c}^x=(1-s_B)\tilde{y}_L{}^x$, i.e. $c_L{}^x(A)=(1-s_L)y_L{}^x(A)$ (with $1-s_L$ is given by the same formula as in the borrowing case) and $c_B{}^x(A)=0$. The longitudinal age-wealth profile $w^x(a)$ can again be written $w^x(a) = w_B{}^x(a)+w_L{}^x(a)$, but $w_B{}^x(a)=0$ ($\neg\, a \in [A,I]$), so that:

$$\neg\, a\in[A,I],\ w^x(a) = w_L{}^x(A) = y_L{}^x(A)\, e^{r(a-A)} \left[\frac{1-e^{-(r-g)(a-A)}}{r-g} - (1-s_L)\,\frac{1-e^{-(r-g_c)(a-A)}}{r-g_c}\right]$$

At age a=I, cohort x receives average bequest b^x, with capitalized end-of-life value $b^x e^{rH}$. They revise their plans and choose a new consumption path $c^x(a)=c^x(I)e^{g_c(a-I)}$ growing at rate g_c. The rest-of-life budget constraint is: $\tilde{c}' + w^x(D) \leq \tilde{y}' = \tilde{y}_L' + w^x(I)e^{rH} + b^x e^{rH}$

With: $\tilde{y}_L' = \int_{I\leq a\leq D} e^{r(D-a)}\, y_L{}^x(a)da = y_L{}^x(I)\, e^{rH}\, \dfrac{1-e^{-(r-g)H}}{r-g}$,

$$\tilde{c}' = \int_{I\leq a\leq D} e^{r(D-a)}\, c^x(a)\, da = c^x(I)\, e^{rH}\, \frac{1-e^{-(r-g_c)H}}{r-g_c}$$

Utility maximization leads to: $w^x(D)=s_B\,\tilde{y}'$ and $\tilde{c}=(1-s_B)\,\tilde{y}'$, i.e.:

$c^x(I)=c_B{}^x(I)+c_L{}^x(I)$

with: $c_B{}^x(I) = (1-s_B)\dfrac{r-g_c}{1-e^{-(r-g_c)H}}\ b^x e^{rH}$

$c_L{}^x(I) = c_L{}^x(A)e^{g_c(I-A)} = (1-s_L)\ y_L(A)\ e^{g_c(I-A)}$

Note that this latter equation simply follows from time consistency: individuals who do not receive any bequest at age a=I have no reason to change their initial consumption plan. However individuals with positive bequests do adjust upward their initial consumption plan. Again thanks to linearity we can concentrate on cohort-level aggregates.

The longitudinal age-wealth profile $w^x(a)$ after age a=I is given by:

$\neg\, a\in[I,D]$, $w^x(a)=w_B{}^x(a)+w_L{}^x(a)$, With: $w_B^x(a) = b^x e^{r(a-I)} [1-(1-s_B) \frac{1-e^{-(r-g_c)(a-I)}}{1-e^{-(r-g_c)H}}]$

$w_L^x(a) = y_L^x(A) e^{r(a-A)} [\frac{1-e^{-(r-g)(a-A)}}{r-g} -(1-s_L) \frac{1-e^{-(r-g_c)(a-A)}}{r-g_c}]$

Note that for a=D, we again have: $w^x(D) = w_B^x(D)+w_L^x(D) = b^{x+H}=s_B(\tilde{y}_L^{\,x} + b^x e^{rH})$

I.e. wealth at death $w^x(D)$ is the same as in the case with borrowing: allowing young agents to borrow against future inheritance alters the time pattern of consumption and wealth accumulation, but does not affect end of life wealth, which is always equal to a fraction s_B of total lifetime resources.[279] So the dynamic equation for the inheritance-income ratio is the same as before ($b_{yt+H}=s_B\lambda(1-\alpha)e^{(r-g)H} + s_B e^{(r-g)H} b_{yt}$), and we obtain the same convergence results: if $r>\bar{r}(g)$, then as $t\rightarrow+\infty$, $b_{yt}\rightarrow+\infty$.

If $r<\bar{r}(g)$, then as $t\rightarrow+\infty$, $b_{yt}\rightarrow b_y^*$, with: $b_y^* = b_y(g,r) = \frac{s_B\lambda(1-\alpha)e^{(r-g)H}}{1-s_B e^{(r-g)H}}$

In order to compute $\mu_t=b_t/w_t$ and $\beta_t=w_t/y_t$, we now need to compute the cross-sectional age-wealth profile $w_t(a)$. At time t, a-year-old individuals belong to cohort x-a, and their beginning of life labor income was $y_L^x(A) = y_{Lt}\, e^{-g(a-A)}$. Assuming we are in (non-explosive) steady-state ($b_{yt}=b_y^*$), at age I they receive bequest $b^x = b_t e^{-g(a-I)}$. So we have:

$\forall\, a\in[A,I]$, $w_t(a) = w_{Lt}(a) = y_{Lt}\ [\frac{e^{(r-g)(a-A)}-1}{r-g} - (1-s_L)\ \frac{e^{(r-g)(a-A)}-e^{-(g-g_c)(a-A)}}{r-g_c}]$

$\forall\, a\in[I,D]$, $w_t(a) = w_{Bt}(a) + w_{Lt}(a)$

With: $w_{Bt}(a) = b_t\, e^{(r-g)(a-I)}[1-(1-s_B)\frac{1-e^{-(r-g_c)(a-I)}}{1-e^{-(r-g_c)H}}]$

$w_{Lt}(a) = y_{Lt}\ [\frac{e^{(r-g)(a-A)}-1}{r-g} - (1-s_L)\ \frac{e^{(r-g)(a-A)}-e^{-(g-g_c)(a-A)}}{r-g_c}]$

Average wealth w_t is given by: $w_t = w_{Bt} + w_{Lt} = \frac{1}{D-A}\ [\int_{I\leq a\leq D} w_{Bt}(a)da + \int_{A\leq a\leq D} w_{Lt}(a)da\ ]$

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[279] This is partly due to the specific functional form we use for utility functions: the utility value of lifetime consumption flows U_C is proportional to capitalized end-of-life consumption $\tilde{c}$, so the log form for $V(U_C,w(D))$ implies that the multiplicative term does not matter. I.e. the marginal utility derived from extra $\tilde{c}$ does not depend on the length of time available to consume $\tilde{c}$. So for instance even if one cannot consume inheritance before age I (i.e. no borrowing), agents will keep consuming the same fraction of their bequest $(1-s_B)b^x e^{rH}$, no matter how short the time span D-I left for consumption. With other functional forms (e.g. CES), one would get consumption time effects, and typically agents would end up consuming a lower fraction of their bequest in the no-borrowing case. In effect, the s_B factor will be higher for inheritance resources than for labor resources, which might be more realistic (see e.g. Masson (1988)). A higher s_B for inheritance resources could also be due to the fact that individuals might feel less comfortable eating up a large fraction of their inherited resources rather than eating up a large fraction of the product of their own labor. Here we adopt standard preferences with a single budget constraint (no separate mental account), so agents treat both types of resources identically.

We use the same open economy notations as those introduced in proposition 4. We note $\beta_t=w_t/y_t$ the wealth-national income ratio, $\beta_{pt}=w_t/y_{pt}$ the wealth-domestic income ratio, $\beta_{Ft}=w_{Ft}/y_{pt}$ the foreign wealth-domestic income ratio, and $\beta_{Kt}=k_t/y_{pt}$ the domestic capital-output ratio. By definition $w_t=k_t+w_{Ft}$ and $y_t=y_{pt}+rw_{ft}$, so $\beta_{pt}=\beta_{Ft}+\beta_{Kt}$ and $\beta_t=\beta_{pt}/(1+r\beta_{Ft})$.
We also define $\beta_{Bt} = w_{Bt}/y_{pt}$ and $\beta_{Lt} = w_{Lt}/y_{Lt}$.
By integrating the age-wealth profiles $w_{Bt}(a)$ and $w_{Lt}(a)$ and by dividing by y_{pt}, we obtain the following formulas for steady-state β_B* and β_L*:

$$\beta_B^* = b_y^* \left[\frac{e^{(r-g)H}-1}{r-g} - \frac{1-s_B}{1-e^{-(r-g_c)H}} \left(\frac{e^{(r-g)H}-1}{(r-g)} - \frac{1-e^{-(g-g_c)H}}{(g-g_c)} \right) \right] \qquad \textbf{(E.14)}$$

$$\beta_L^* = \frac{1}{D-A} \left[\frac{e^{(r-g)(D-A)}-1-(r-g)(D-A)}{(r-g)^2} - \frac{1-s_L}{r-g_c} \left(\frac{e^{(r-g)(D-A)}-1}{r-g} - \frac{1-e^{-(g-g_c)(D-A)}}{g-g_c} \right) \right] \qquad \textbf{(E.15)}$$

We can then compute $\beta_p^* = \beta_B^*+\beta_B^*$ and $\mu^*=b_y^*/m^*\beta_p^*=(D-A)b_y^*/\beta_p^*$.
Finally, the assumption of a Cobb-Douglas production function implies $\beta_K^*=\alpha/r$, from which one can compute the foreign wealth ratio $\beta_F^*=\beta_p^*-\beta_K^*$, the wealth-national income ratio $\beta^*=\beta_p^*/(1+r\beta_F^*)$, and the inheritance flow-national income ratio $\widehat{b}_y^* = b_y^*/(1+r\beta_F^*)$.

Equations (E11) to (E15) solve the open-economy model for the non-explosive case $r<\bar{r}(g)$. These are closed form solutions, but there are many effects going on. In particular the full formula for μ* is relatively complicated, and in general there is no reason that μ* is equal to the class saving level $\bar{\mu} = \frac{D-A}{H}$, or even that $\mu^*=\mu(g)\to\bar{\mu}$ as $g\to 0$. In the case g=0%, the formula for μ* still involves all other parameters, and not only D-A and H.

However by doing numerical calibrations using equations (E11)-(E15), one can see that higher growth and/or lower rates of return tend to reduce inheritance ($\mu'(g)<0$, $\mu'(r)>0$), and that for realistic parameter values, and for low growth and/or high rates of return, then μ* and b_y* are relatively close to class saving levels $\hat{\mu}$ and β*/H.

We report on Tables E6 and E7 two series of calibrations (parameters can be changed in the excel file). On Table E6 we assume r=5%, θ=2%, σ=5, s_B=10%, and we make the growth rate vary from g=0% to g=5%, and the demographic parameters vary from 19th century values (D=60, I=30) to 21st century values (D=80, I=50). As g rises, r-g declines, so the inheritance-income ratio b_y* declines: as was already noted above, b_y* is a steeply rising function of r-g. Note that the economy accumulates a lot of foreign assets when r-g is large (i.e. g small), and conversely is almost entirely owned by the rest of the world when r-g is small (i.e. g high). Consequently, as g rises, the inheritance-income ratio $\widehat{b}_y$ * declines in a less extreme and more realistic way than the ratio b_y*. E.g. $\widehat{b}_y$ * goes from 31% for g=1% to 25% for g=2% (rather than from 44% to 26%). We also find that the relative wealth of decedents μ* rises sharply as life expectancy increases (almost as sharply as in the class saving case; see Table E1). μ* is also an increasing function of r. Somewhat counter-intuitively, μ* also appears to be on Table E6 an increasing function of g. However this is entirely due to the g_c effect: i.e. as g rises from 0% to 5%, the desired consumption growth rate g_c=(r-θ)/σ remains constant at 1%. So with high growth young age agents borrow enormously against future growth, thereby raising the relative wealth of the old. We are not sure that such massive borrowing patterns are realistic.

In order to shut down this effect, on Table E7 we assume that the values of θ and σ adjust to changes in g so that g_c remains permanently equal to g as g rises from 0% to 5%.[280] We then find that μ* declines as g rises. I.e. high labor income growth raises the relative wealth of the young, in the same way as in the exogenous saving and dynastic models.

In the case $r>\bar{r}(g)=\bar{r}+g$ (with $\bar{r}= -\log(s_B)/H$), then we have an explosive path: as t→+∞, b_{yt}→+∞ and β_{Ft}→+∞. I.e. in the same way as in the explosive case of the exogenous saving model, domestic output y_{pt} becomes negligible as compared to foreign asset income rw_{Ft}, and national income $y_t \approx rw_t$ grows at rate $g_r=r-\bar{r}$ (>g). The wealth-income ratio $\beta_t \to \beta^*=1/r$ as t→+∞. One can also show that $\widehat{b}_{yt}$ and μ_t converge towards some finite values $\widehat{b}_y$ * and μ*. All income derives from wealth, so nobody has wealth before age I. However there is no reason in general that the age-wealth profile $w_t(a)$ is flat above age I, so there is no reason that $\mu^*=\bar{\mu}$. This will occur iff g_c=(r-θ)/σ=g_r. So for instance if g=0%,

[280] There are several reasons why g_c is usually relatively close to g in the real world, i.e. why consumption tends to track income much more closely that what optimising models tend to predict. First, agents might not know in advance that g is going to be equal to 5% in the next 30 years (e.g. it was pretty hard to predict in the 1930s-1940s that g was going to be 5%-6% in the 1950s-1960s), so they might adjust consumption growth to current growth. Next, even if they know in advance that g=5% and their preference parameters are such that they want a lot of consumption smoothing (say g_c=1%) they might face borrowing constraints.

θ=0% σ=+∞, then $\mu^* \to \bar{\mu}$ as $r \to \bar{r}$ (either from above or from below). But in general μ^* will be different from $\bar{\mu}$: $\mu^* > \bar{\mu}$ if $g_c < g_r$ and $\mu^* < \bar{\mu}$ if $g_c > g_r$.

<u>(iii) closed economy, ρ=1, no borrowing</u>. We now consider the closed economy case. All equations are exactly the same as in the open economy case, except that now the rate of return r is no longer a free parameter. The long run steady-state r* is determined by the equality between the supply and the demand of capital:

$$\beta_B^*(r) + \beta_B^*(r) = \beta_K^* = \alpha/r \quad \textbf{(E.16)}$$

Where $\beta_B^*(r)$ and $\beta_B^*(r)$ are given by equation (E.14) and (E.15) above. These wealth accumulation ratios are increasing functions of r, so the supply equals demand equation has a unique solution. Unfortunately there exists no closed form solution for r*. The steady-state r* and $\beta^* = \alpha/r^*$ of the wealth-in-the-utility-function model can be larger or smaller than the dynastic model steady-state values $\hat{r} = \theta + \sigma g$ and $\hat{\beta} = \alpha/\hat{r}$, depending on the various parameters. The aggregate wealth-income ratio β* is naturally an increasing function of s_B and a decreasing function of g (and conversely for r*). In case s_B=0 and ρ=1, then $r^* > \hat{r}$ (with $r = \hat{r}$ there would be no saving at all), with r* declining and $\to \hat{r}$ as life expectancy D→+∞ (in effect the model converges toward the dynastic model). In case s_B is sufficiently large, then $r^* < \hat{r}$ (as $s_B \to 1$, then $\beta^* \to +\infty$ and $r^* \to 0$). But if one wants to go beyond these qualitative statements, one needs to use numerical solutions in order to study closed economy steady-states.

We report on Tables E8 to E11 four series of calibrations (parameters can be changed in the excel file). On Tables E8 and E9, we assume that s_B adjusts so that when g rises from 0% to 5% the steady-state r* and β* remain fixed at 5% and 600%. On Tables E10 and E11, we assume that s_B is fixed at 10%, and we compute the equilibrium values of r* and β*, either with fixed θ and σ (Table E10), or by assuming that θ and σ adjust so as to keep g_c=g (Table E11), in the same way as in the open economy calibrations. The most striking finding is that in all variants the steady-state b_y^* almost does not depend on life expectancy D: i.e. the decline in m* is almost entirely compensated by the rise in μ*. For realistic low-growth parameter values (g=1%-2%, r*=4%-5%, s_B around 10%), we find that μ* and b_y^* are extremely close to the class saving levels $\bar{\mu}$ and β*/H (or if anything slightly above class saving levels).

(iv) $\rho \leq 1$. All equations above can be extended to the case with less than 100% replacement rates, in the same way as in the dynastic model (see above). There are major differences with the dynastic model, however.

First, one can easily show that the same formula for steady-state b_y* (equation (E13)) applies for any $\rho \leq 1$. This is because $\rho < 1$ adds an extra lifecycle wealth term $w_L^x(a)$ in the wealth equations (with $w_L^x(a)$ hump shaped, i.e. maximal at age a=R and going to zero for a=D), but without affecting the fact that wealth at death $w^x(D)$ is equal to a fixed fraction s_B of lifetime resources. So the dynamic equations for b^{x+H} as a function of b^x and b_{yt+H} as a function of b_{yt} are wholly unaffected, and so is the steady-state formula for b_y*. The only change in the formula is the value of λ. With $\rho \leq 1$, we have $y_L^x(a)=y_L^x(A)e^{g(a-A)}$ for $a \in [A,R[$ and $y_L^x(a)=\rho y_L^x(A)e^{g(a-A)}$ for $a \in [R,D]$, so $\tilde{y}_L$ is now given by:

$$\tilde{y}_L = \int_{A \leq a \leq D} e^{r(D-a)}\, y_L^x(a)\, da \;=\; y_L^x(A)\, e^{r(D-A)} \left[\frac{1-e^{-(r-g)(R-A)}}{r-g} + \rho\frac{e^{-(r-g)(R-A)} - e^{-(r-g)(D-A)}}{r-g}\right]$$

At time t, the cohort receiving bequest b_t is the cohort born at x=t-I. This cohort started working with labor income $y_L^x(A)=\dfrac{D-A}{R-A+\rho(D-R)}y_{Lt}e^{-g(I-A)}$. So $\tilde{y}_L=\lambda(D-A)e^{rH}y_{Lt}$, with:

$$\lambda = \frac{D-A}{R-A+\rho(D-R)}\left[\frac{e^{(r-g)(I-A)} - e^{-(r-g)(R-I)}}{(r-g)(D-A)} + \rho\frac{e^{-(r-g)(R-I)} - e^{-(r-g)H}}{(r-g)(D-A)}\right] \qquad \textbf{(E.17)}$$

If $\rho=1$, then we are back to the simpler λ formula given by equation (E12). If r-g=0, then by construction λ=1 ($\neg\, \rho \leq 1$). With r-g>0, note that $\lambda'(\rho)<0$, i.e. more generous pay-as-you-pension systems lead to lower λ factors. This simply reflects the fact that with r-g>0 pay-as-you-go pension systems have a lower rate of return than private wealth. This also implies that for given s_B and r-g, less generous pensions (lower ρ) will actually lead to higher steady-state inheritance ratios b_y* (see Table E5).

In the open economy case, both g and r are given, so ρ has no further impact on b_y*. Of course lower ρ leads to higher β* and lower μ* (because of additional hump-shaped wealth accumulation), but in the open economy this has no impact on r. In effect the additional pension wealth is entirely invested in foreign assets, so there is no crowding out at all with other forms of wealth. Note however that the rise in β* also implies a rise in national income, so the rise in $\widehat{b}_y *$ will be less strong than the rise in b_y*. Calibration results (not reported here) show that the two effects almost exactly cancel out, so that a lower ρ has virtually no impact on $\widehat{b}_y *$.

In the closed economy case, the rise in β* due to lower ρ and the rise of pension wealth will lead to lower r*, which in turn leads to lower b_y*. I.e. there will be partial crowding out between pension wealth and other forms of wealth. Calibration results (not reported here) show that this r* effect is somewhat larger than the λ effect, so the overall effect of lower ρ on steady-state b_y* is slightly negative (but much smaller than in the dynastic model, where there was full crowding out).[281]

E.6. Extension of the formulas to the case with population growth

So far, all theoretical results and formulas on inheritance flows were derived within the context of the simple stationary demographic structure introduced in section 5: everybody becomes adult at age A, has exactly one kid at age H, and dies at age D, so each cohort size is fixed (and normalized to 1), and that total (adult) population N_t is also fixed (and is equal to adult life length: N_t=D-A). Note that all propositions also make the assumption that inheritance age I=D-H was higher that adulthood age: I=D-H>A. This assumption is satisfied in modern societies (with A=20 and H=30, then I=D-H>A as long as D>50), and allowed us to ignore children altogether (they never own any wealth, nor do they receive any income) and concentrate on the analysis of the age-wealth profile $w_t(a)$ within the adult population ($a \in [A;D]$). This assumption might however not hold in some ancient societies (e.g. if A=20, H=30 and D=40, then I=D-H=10). It can easily be relaxed, and all results and formulas can be extended to the case with children inheritors, with minor changes.[282]

Next, and most importantly, all propositions can also be generalized to a model with self-sustained (positive or negative) population growth. Generally speaking, the impact of population growth on steady-state inheritance flows is similar to the impact of productivity growth, and for the most part one simply needs to replace g by g+n (where g is

[281] It is not really meaningful to push further the pension analysis without modelling explicitly the reason why pay-as-you-go systems were introduced in the first place (i.e. uninsurable uncertainty on r).

[282] E.g. in the class saving case (Proposition 2), then if I=D-H<A the steady-state age-wealth profile (incl. children) would now be : $w_t(a)=0$ if $a \in [0;I]$ and $w_t(a)=b_t$ if $a \in [I;D]$, so that $\mu^*=b_t/w_t=D/(D-I)=D/H$. Since the mortality rate (incl. children) is now given by $m^*=1/D$, we again obtain $b_w^*=\mu^* m^*=1/H$ and $b_y^*=\beta^*/H$. I.e. the basic result is unchanged. In order to fully solve the models with endogenous saving, one would need however to make assumptions as to whether children inheritors are allowed to borrow against future labor resources or are under the control of other family members until adulthood.

productivity growth and n is population growth) in the steady-state formulas. This is illustrated by Propositions 12-13 below.

Consider the following demographic structure. Everybody becomes adult at age a=A, has 1+η children at age a=H>A, and dies at age D>H. Everybody again inherits at age I=D-H>A. The only difference with the demographic structure used so far is that we allow the (average) number of children to differ from 1, i.e. η can now be positive or negative. Noting f the fertily rate (average number of children per woman), we simply have: 1+η = f/2. Each cohort size N^x now grows at rate n, and so does total adult population N_t :[283]

$$N^x = e^{nx}$$

$$N_t(a) = e^{n(t-a)}$$

$$N_t = \int_{A\leq a\leq D} N_t(a)\,da \;=e^{nt}\,\frac{e^{-nA}-e^{-nD}}{n}$$

$$\text{With: } e^{nH} = 1+\eta = f/2\text{, i.e. } \mathbf{n = \frac{\log(f/2)}{H}} \qquad \textbf{(E.18)}$$

Example: With a fertily rate f=2.0 children per woman, then n=0: we are back to the case with zero population growth. A fertily rate f=2.2 means that everybody has 1.1 children, i.e. the population rises by 10% every generation, so for generation length H=30 this corresponds a population growth rate n=0.3% per year. A fertily rate f=3.0 corresponds to n=1.4%. Conversely, a fertility rate f=1.5 corresponds to n=-1.0%.

Because we assume steady-state population growth, the mortality rate m_t is stationary:

$$m_t = \frac{N_t(D)}{N_t} = m^* = m(n) = \frac{n}{e^{n(D-A)}-1} \qquad \textbf{(E.19)}$$

Note that m'(n)<0: in growing populations, dying cohorts are smaller in size than living cohorts, so the mortality rate is lower. If n=0, we are back to the case with zero population growth: m*=1/(D-A). If n>0, m(n)<1/(D-A). If n<0, m(n)>1/(D-A).

The rest of the model is unchanged. We still assume a Cobb-Douglas production function $Y_t = F(K_t,H_t) = F(K_t,e^{gt}L_t)$ with exogenous productivity growth g≥0. With an exogenous

[283] We set initial cohort size N^0 equal to 1.

saving rate $s=\alpha s_K+(1-\alpha)s_L$, one simply needs to replace g by g+n in the Harrod-Domar-Solow closed-economy formula for steady-state β* and r*:

$$\beta^* = \frac{s}{g+n}$$

$$r^* = \frac{\alpha}{\beta^*} = \frac{\alpha(g+n)}{s}$$

In steady-state national income Y_t and aggregate wealth W_t grow at rate g+n. Per adult income $y_t=Y_t/N_t$ and per adult wealth $w_t=W_t/N_t$ grow at rate g. We are looking for a steady-state where the aggregate inheritance flow B_t also grows at rate g+n, while per decedent inheritance $b_t=w_t(D)$ grows at rate g. We again need to solve for the steady-state age-wealth profile $w_t(a)$. Note that because of population growth, average bequest left and average bequest received do not longer coincinde: at time t, decedents leave average bequest $b_t=w_t(D)=B_t/N_t(D)$, while successors receive average bequest $b_t/e^{nH}=B_t/N_t(I)$.

One can easily show that in the class saving case, the results obtained for b_w* and b_y* are wholly unaffected by the introduction of population growth:

<u>Proposition 12</u> (class saving model with population growth)
Assume pure class savings (s_L=0 & s_K>0) and population growth (n>0 or n<0). As t→+∞, μ_t→μ*, b_{wt}→b_w* and b_{yt}→b_y*. Steady-state ratios μ*, b_w* and b_y* are given by:
(1) The ratio μ* between average wealth of decedents and average adult wealth depends solely on demographic parameters: $\mu^* = \mu(n) = \frac{e^{n(D-A)}-1}{nH} = \frac{1}{Hm^*}$
(2) The inheritance flow-private wealth ratio $b_w^*=\mu^*m^*$ and the estate multiplier $e^*=1/b_w^*$ depend solely on generation length H: $\mathbf{b_w^* = \overline{b}_w = 1/H}$ and $\mathbf{e^* = \overline{e} = H}$
(3) The inheritance flow-national income ratio $b_y^*=\mu^*m^*\beta^*$ depends solely on the aggregate wealth-income ratio β* and on generation length H: $\mathbf{b_y^* = \overline{b}_y = \beta^*/H}$

<u>Proof of Proposition 12.</u> The steady-state cross-sectional age-profile $w_t(a)$ looks as follows. Since s_L=0, young individuals have zero wealth until the time they inherit. Now take the group of individuals with age a>I at time t. They inherited a-I years ago, at time s=t-a+I. They received average bequests $b_s/e^{nH}=e^{-g(a-I)}b_t/e^{nH}$. But although they received smaller bequests, they saved a fraction $s_K=(g+n)/r^*$ of the corresponding return, so at time t their inherited wealth is now equal to: $w_t(a) = e^{(g+n)(a-I)}e^{-g(a-I)}b_t/e^{nH} = e^{n(a-D)}\ b_t$. So we have the following steady-state profile:

If $a \in [A,I[$, then $w_t(a) = 0$
If $a \in [I,D]$, then $w_t(a) = e^{n(a-D)}\ b_t$

So with positive population growth n>0, the cross-sectional age-average wealth profile is now upward sloping after inheritance age: on average younger successors are poorer than older successors. However the cross-sectional age-aggregate wealth profile is still flat: younger successors are poorer, but they are more numerous, and both effects exactly compensate each other. That is, if we define $W_t(a)=N_t(a)w_t(a)$, we have:

If $a \in [A,I[$, then $W_t(a) = 0$
If $a \in [I,D]$, then $W_t(a) = e^{n(t-D)}\ b_t = B_t$

It follows that aggregate wealth $W_t=(D-I)B_t=H\ B_t$, i.e. $b_w^*=1/H$ and $b_y^*= \beta^*/H$.
Alternatively, we get the following formula for average wealth:

$$w_t = [\ \int_{A\leq a\leq D} N_t(a)\ w_t(a)\ da\ ]/N_t = \frac{nHe^{-nD}}{e^{-nA}-e^{-nD}}\ b_t$$

So we have:

$$\mu^* = \mu(n) = \frac{e^{n(D-A)}-1}{nH} = \frac{1}{Hm^*} \qquad \text{(E.20)}$$

Note that $\mu^*>\overline{\mu} = (D-A)/H$ if $n>0$, and $\mu^*<\overline{\mu}$ if $n<0$. When population grows faster, the mortality rate m* is lower, but the relative wealth of decedents μ* is higher, so that the product $b_w^*=\mu^*m^*=1/H$ is unchanged.
End of proof of Proposition 12.

Now consider the wealth-in-utility model. One can show that the formula for steady-state inheritance flows obtained under population stationarity is almost the same in the model with zero population growth (see Propositions 8-9). That is, one simply needs to replace g by g+n in the steady-state formula for b_y^*, and to add an additional term to the λ factor:

<u>Proposition 13</u> (wealth-in-the-utility model with population growth).

As $t\to+\infty$, $\mu_t\to\mu^*=\mu(g,r)$, $b_{wt}\to b_w^*=\mu^*m^*$, and $b_{yt}\to b_y^*=\mu^*m^*\beta^*=\dfrac{s_B\lambda'(1-\alpha)e^{(r-g-n)H}}{1-s_B e^{(r-g-n)H}}$

With: $\lambda' = \lambda\dfrac{(D-A)N_t(I)}{N_t} = \lambda\ \dfrac{n(D-A)e^{-nI}}{e^{-nA}-e^{-nD}}$

And: $\lambda = \dfrac{e^{(r-g)(I-A)}-e^{-(r-g)(D-I)}}{(r-g)(D-A)}$

<u>Proof of Proposition 13</u>. Consider the inheriting cohort at time t, i.e. the cohort born at time x=t-I. We again note $\tilde{y}_t=\tilde{b}_t+\tilde{y}_{Lt}$ the average lifetime resources received by this cohort, where $\tilde{b}_t$ is the average end-of-life capitalized value of their inheritance resources, and $\tilde{y}_{Lt}$ is the average end-of-life capitalized value of their labor income resources.

We have:

$\tilde{b}_t = e^{rH} B_t/N_t(I)$.

$\tilde{y}_{Lt} = e^{rH}\lambda(D-A)(1-\alpha)Y_t/N_t$

In the same way as in the zero population growth case, utility maximization implies that the average bequest b_{t+H} left by cohort x is equal to a a fraction s_B of their end-of-life capitalized lifetime resources $\tilde{y}_t$. So in aggregate terms we have:

$$B_{t+H} = s_B N_t(I)\ [\ e^{rH} B_t/N_t(I) + e^{rH}\lambda(D-A)(1-\alpha)Y_t/N_t\ ]$$

$$\text{I.e.}\qquad \mathbf{B_{t+H} = s_B e^{rH}\ [\ B_t + \lambda'(1-\alpha)Y_t\ ]}\qquad \textbf{(E.21)}$$

$$\text{With: } \lambda' = \lambda\frac{(D-A)N_t(I)}{N_t}$$

Note that the additional term $\dfrac{(D-A)N_t(I)}{N_t}$ is simply the ratio between the size of the currently inheriting cohort $N_t(I)$ and average cohort size $N_t/(D-A)$. With zero population growth this ratio is equal to 100% and this additional term disappears. More generally, if n is small, and if inheritance happens around mid-life, one can see that it will be close to 100% (the first-order term disappears, in the same way as in the λ formula).

Dividing both sides of equation (E21) by $Y_{t+H} = Y_t\ e^{(g+n)H}$, we get the following transition equation for the inheritance flow-national income ratio $b_{yt}=B_t/Y_t$:

$$\mathbf{b_{yt+H} = s_B\, e^{(r-g-n)H}\ [\ b_{yt} + \lambda'(1-\alpha)\ ]} \qquad \textbf{(E.22)}$$

Assuming $s_B\, e^{(r-g-n)H} < 1$, we have a unique steady-state $b_y^* = \frac{s_B \lambda'(1-\alpha) e^{(r-g-n)H}}{1 - s_B e^{(r-g-n)H}}$.

Higher population growth reduces the relative importance of inheritance, in the same way as higher productivity growth. Conversely, negative population growth raises the relative importance of inheritance. If n is sufficientely negative, then $s_B\, e^{(r-g-n)H} > 1$, i.e. we have an explositive path. Intuitively, in a society where individuals almost stop having children, the size of dying cohort becomes very large as compared to the size of the inheriting cohorts, and so does the inheritance flow as compared to national income.

End of proof of Proposition 13.

List of files

The folder www.jourdan.ens.fr/piketty/inheritance contains the following files:

1. Piketty2010WP.pdf = pdf file for the working paper “On the Long Run Evolution of Inheritance – France 1820-2050”, PSE, 2010

2. Piketty2010DataAppendixPart1.pdf & Piketty2010DataAppendixPart2.pdf = pdf files for the present data appendix

3. Piketty2010DataAppendix.zip = zip file containing detailed tables, figures, data files and computer codes in excel and stata formats:

- MainTablesFigues.xls = excel file containing all tables and figures included in the working paper, with linked formulas to other excel files & sheets

- AppendixTables(NationalAccountsData).xls = excel file containing all tables from appendix A, with linked formulas to other excel files & sheets

- AppendixTables(EstateTaxData).xls = excel file containing all tables from appendix B, with linked formulas to other excel files & sheets

- AppendixTables(DemoData).xls = excel file containing all tables from appendix C, with linked formulas to other excel files & sheets

- AppendixTables(Simulations).xls = excel file containing all tables from appendix D, with linked formulas to other excel files & sheets

- AppendixTables(SSFormulas).xls = excel file containing all tables from appendix E, with linked formulas to other excel files & sheets

- AppendixFigures.xls = excel file containing supplementary figures drawn from appendix tables, with linked formulas to other excel files & sheets

- AppendixDataFiles.zip: zip file containing a number of stata format data sets and do files used in the simulations (the exact list and description of these files is given in Appendix C3 and Appendix E6).